CARDIAC ARRHYTHMIA CLASSIFICATION USING WAVELET ANALYSIS OF ELECTOCARDIOGRAM

by
Nripendra Malhotra
Bachelor of Engineering, Ryerson University, 2000

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Abstract

Cardiac Arrhythmia Classification Using Wavelet Analysis of Electrocardiograms

Master of Applied Science, 2021

Nripendra Malhotra,

Computer and Electrical Engineering.

Cardiac Arrhythmias are heart rhythm abnormalities that seriously impact the quality of life and can be lethal. Four major types of cardiac arrhythmia that originate from atria and ventricles are atrial flutter (AFL), atrial fibrillation (AF), ventricular tachycardia (VT), and ventricular fibrillation (VF). Annually about 50,000 embolic strokes mostly due to AF and 35,000 sudden cardiac deaths mostly due to VF are reported in Canada. Accurate detection and segregation of these arrhythmia swiftly is an essential requirement for appropriate treatment. Automating this process is especially critical and valuable for implantable devices and long-term monitoring scenarios.

In this thesis, with the above motivation, we analyzed the electrocardiograms (ECGs) recorded during these 4 types of arrhythmia using wavelet transform. A total of 100 ECG segments containing 25 ECG segments each for AF, AFL, VT and VF, obtained from well-known open source databases were used in this study. Discriminative features were extracted from the wavelet coefficients and fed to a linear discriminant analysis based classifier. Based on the proposed scheme, best classification accuracies using library wavelets and adaptive continuous wavelets (ACW) are as follows: (i) for four group classification, Paul CW and A-pattern ACW attained **77%** and **81%** respectively, (ii) for the two group classification of AA, Paul CW and M-pattern ACW attained **76%** and **86%** respectively, (iii) for the two group classification of VA, Bump CW and M-pattern ACW attained **92%** and **94%** respectively.

Acknowledgment

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

ESSENTIAL for life, heart is one of the most vital organs of the human body. The heart is part of the cardiovascular system, which also consists of closed system of vessels called veins, arteries and capillaries. The heart receives deoxygenated blood from the body, while simultaneously supplying oxygenated blood throughout the body. Moreover, it facilitates the transport of blood and nutrients to the major and minor parts of the body. Heart pumps continuously throughout our life span. The pumping is made possible by electric impulses that cause the cardiac muscles to contract and expand, also known as depolarization and re-polarization. Electric impulse is like a wave that propagates from cell to cell, through the entire heart, causing depolarization. After depolarization is complete, heart restores to its original state which is known as re-polarization [1].

The activity of the electrical conduction system of the heart responsible for cardiac muscle contraction may be measured. An electrocardiogram (ECG) records the electrical signals of the heart. Measured along the surface of the chest, ECG signals offer precise information regarding the functioning the heart.

When the heart is pumping normally and supply of blood is normal, it is known as normal sinus rhythm (NSR). When the heart behaves erratically, i.e arrhythimically, due to pathophysiological conditions, the supply of blood is compromised. Arrhythmias occur when the normal electrical conduction system is disturbed due to an illness or damage, causing the heart to pump irregularly. Research into the functioning of the heart and its makeup can help to fully understand the underlying processes at work and their activity, as well as other factors that cause heart diseases [3].

Arrhythmias can be divided into two main groups depending on the origin, Atrial Arrhythmia (AA) and Ventricular Arrhythmia (VA). AA can be subdivided into Atrial Fibrillation (AF) and

Atrial Flutter (AFL) and VA can be subdivided into Ventricular Tachycardia (VT) and Ventricular Fibrillation (VF). These are the four well known types of arrhythmia. AA often may evolve into VA over time, if left untreated. VF is the most lethal of the cardiac arrhythmia and if not treated within minutes of its onset, can lead to sudden cardiac death (SCD).

One of the objectives of arrhythmia signal analysis is to characterize arrhythmia using ECG features in order to provide an automated detection of a particular case of arrhythmia, which can lead to proper therapy or medication. Following section will briefly discuss cardiac system in context of arrhythmia, which is the focus of this thesis.

1.1 Cardiac System

Heart is a very complex organ, almost the size of human fist. Its main function is to pump blood throughout our body. This continuous rhythmic function of the pumping of the blood causes normal blood flow. To understand any divergence from its normal function, it is important to understand the cardiac system, mechanism and its function. The information provided in this section is a brief summary of Chapter 9 of 'Textbook of Medical Physiology', 12th edition, Guyton and Hall, Structure of the heart, and course of blood flow through the heart chambers and heart valves, Page 101 - 113, © Elsevier, 2011 [4]. To get more information, please refer to book.

1.1.1 Physiology

Heart can be divided, mainly into two atria and two ventricles. Two atria can be defined as right atria and left atria which are the upper chambers, and the two ventricles can be defined as right ventricle and left ventricle which are the lower chambers of the heart. The right atrium gets the oxygen-poor blood from the peripherals, pushes it to right ventricle, and right ventricle pumps the oxygen-poor blood to the lungs. Left atrium receives the oxygenated blood from the lungs, pushes it to left ventricle and then left ventricle pumps it to the whole body.

The make up of the heart can be seen in Figure 1.1. The contractions of the atrium or ventricles are due to electric impulses. Sinoatrial Node (SA) and Atrioventricular Node (AV) are the two nodes within heart which provide electric stimuli in a rhythmic fashion to facilitate blood circulation. Cardiovascular system and its vessels are a closed circulation system, which ensures that all the blood is circulated back to heart.

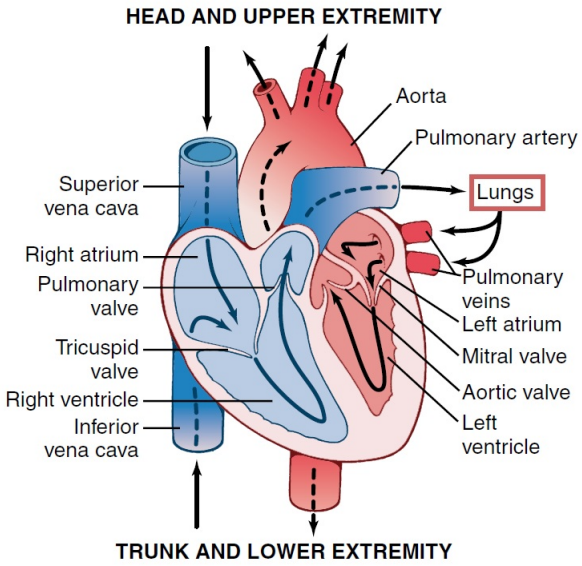

Figure 1.1: Makeup Of Heart. Source: Textbook of Medical Physiology, 12th edition, Guyton and Hall, Structure of the heart, and course of blood flow through the heart chambers and heart valves, Page 101, © Elsevier 2011.

SA node is located between superior vena cava and right atrium. AV node is located within the triangle of Koch, a region located at the base of the right atrium defined by coronary sinus ostium, tendon of todaro and the septal leaflet of the tricuspid valve. Location of SA node and AV node can be seen in Figure 1.2. SA node is responsible for initiating the electric impulses that depolarize the myocardium.

SA node is also known as a natural pacemaker and generates the rhythmic impulses that cause depolarizations. Rhythmic impulses are conducted through the atria to the less rapidly firing AV node. After a short delay in AV node, the cardiac impulses are conducted through the main conducting system of the heart, for rapid conduction of impulses. From the beginning of one rhythmic impulse of SA node, to the beginning of next rhythmic impulse, is known as a single heartbeat.

ECG records the electrical activity generated by the heart from the surface of the body. ECG of a normal sinus rhythm is a combination of P-wave followed QRS complex and finally T-wave. The P-wave is depolarization of atria, and just after P-wave, the atria restore to normal, known as atrial re-polarization. The QRS complex is the most dominating section of the cardiac cycle in ECG, which is a result of ventricular depolarization and it occurs about 0.16 seconds after the P-wave.

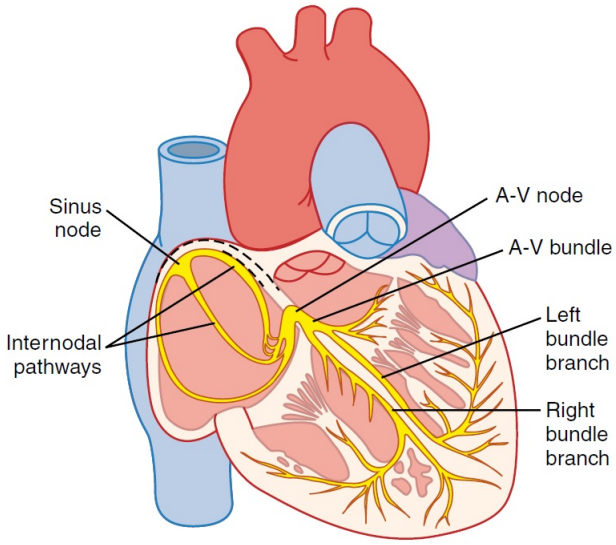

Figure 1.2: SA Node and AV Node. Source: Textbook of Medical Physiology, 12th edition, Guyton and Hall, Sinus node and Purkinje system of the heart, showing also A-V node, atrial internodal pathways, and ventricular bundle branches, Page 116 © Elsevier 2011.

Right after QRS complex, the ventricles re-polarize.

ECG of a single heartbeat is shown in Figure 1.3. Actual ECG recording is shown in Figure 1.4. There is a relationship between Atrial and Ventricular contractions in the ECG signal recording. It is important to understand the way action potential travels across the heart and to better understand its relationship with the AA and VA. It is crucial to note, that atria re-polarizes about 0.15 to 0.20 seconds after termination of the P-wave, which is also when QRS complex is being recorded. Since QRS complex wave is a dominant wave structure, it conceals the atrial re-polarization. When heart pumps in a rhythm and follows the above pattern then it is considered normal heart rate.

Normal heart rate can easily be determined from ECG recording because it is a direct representation of interval between two consecutive heartbeats. If the time between two successive QRST is 1 second (on the graph paper), the heart rate is 60 beats per minute. Usually, the normal interval between two successive beats for an adult person is 0.83 seconds, therefore total number of heartbeats can be be calculated, by calculating the difference between two successive QRS complexes. Total number of heartbeats in a minute is 60/0.83 times per minute which equals 72 beats per minute.

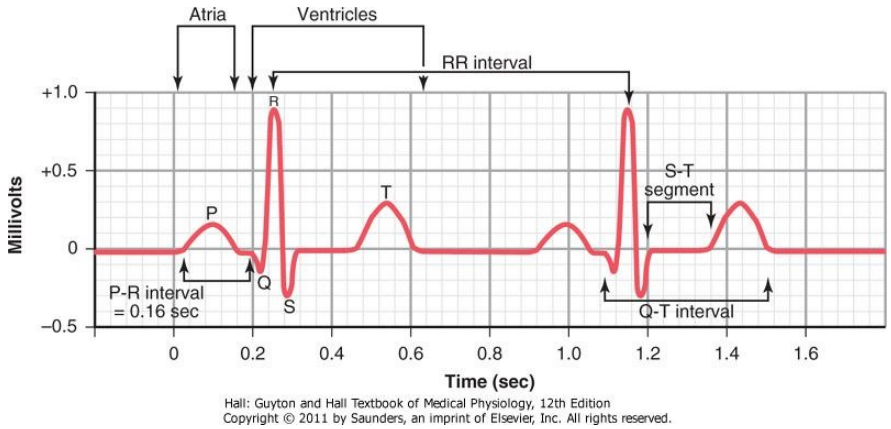

Figure 1.3: ECG: Ideal Normal Sinus Rhythm. Source: Textbook of Medical Physiology, 12th edition, Guyton and Hall, Normal electrocardiogram, Page 121, © Elsevier 2011.

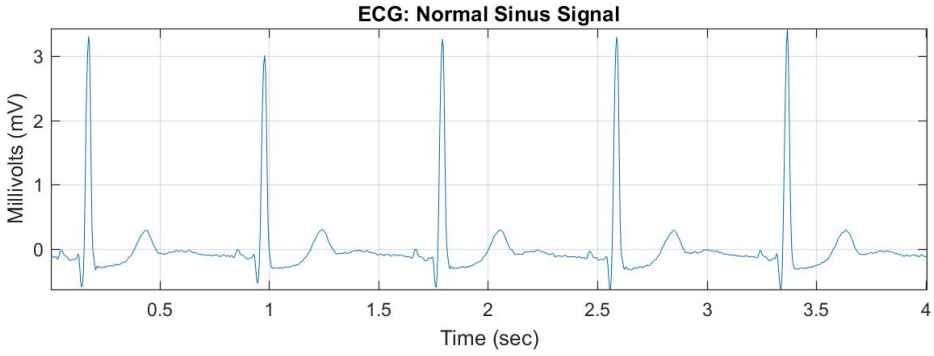

Figure 1.4: ECG: Actual Normal Sinus Rhythm ECG.

1.2 Arrhythmias

An arrhythmia occurs when the heart does not perform proper rhythmic pumping and in many cases, is unable to circulate blood efficiently. The condition could be due to injury, disease or simply genetics. There are different types of arrhythmias, and they are characterized by the location of origin within the heart, and the speed of heartbeat that it causes. When the heart beats rapidly, faster than normal, the condition is called tachycardia. When its slower than normal, the condition is known as bradycardia. Also, when the heartbeat is erratic, it is known as fibrillation. The four major kinds of arrhythmia will be discussed in the following section.

1.2.1 Atrial Arrhythmia

Two most common types of AA are AF and AFL and both can co-exist or overlap each other, even though they might generate from a different mechanism [5]. AF and AFL differ in morphology over time. Figures 1.5 and 1.6 show AF and AFL respectively. In the recent years AA has become the cause of considerable mortality. Each year more than 50,000 canadians suffer a stroke due to AF [7]. Minor effect of AF is that it can cause dizziness and dyspnea. A major effect of AF is it can cause heart failure, angina and myocardial infarction. AF has consistently been associated with acute myocardial infarction [8].

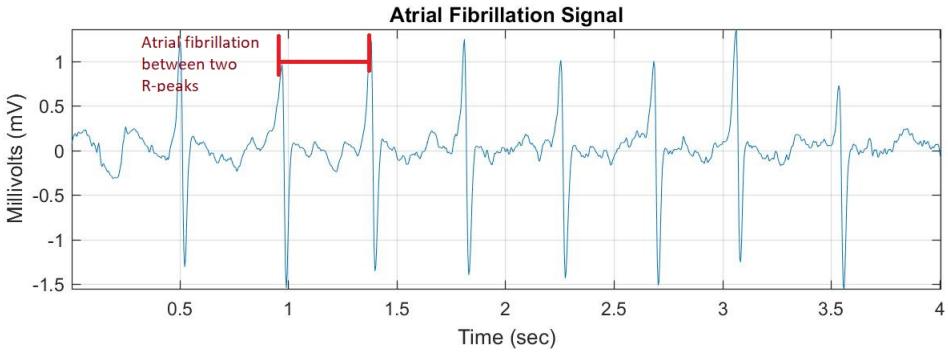

Figure 1.5: Atrial Fibrillation between two R-peaks.

Physical conditions that are associated with AF and AFL are theyrotoxicosis and pheochromocytoma. Other physical conditions associated with AA are, obesity, alcohol intoxication, hypoxia, stress and surgery. Diabetes is also associated with AF but may not be a direct contributing factor. However, it may be related to heart disease, hypertension or sudden cardiac arrest (SCA) [9].

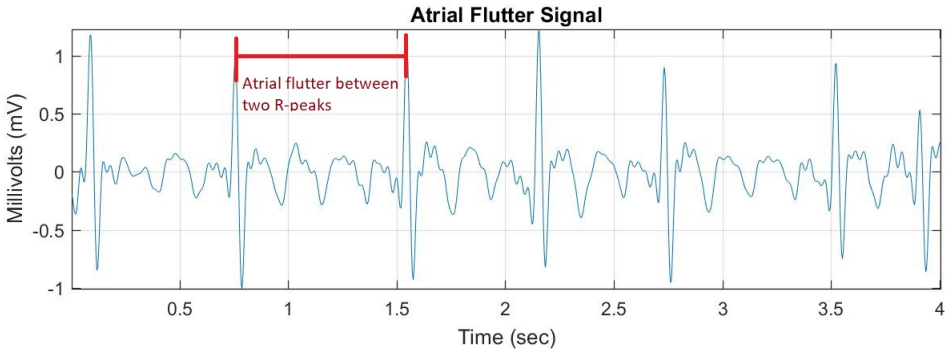

Figure 1.6: Atrial Flutter between two R-peaks.

AF is associated not only with cardiovascular mortality, but also associated with stroke. The risk of stroke in AF increases as an individual's age increases. Risk of stroke is higher for someone with chronic AF. On the contrary, risk is lower for someone with paroxysmal AF. With AF there is also a risk of thromboembolism [18].

AFL is due to the circular activation of impulse in the atrium with counter-clockwise activation of the atrial septum [10]. During AFL the heart's upper chamber beats faster than the lower chambers, causing rhythm to be out of sync. AFL can be subdivided into Type I which is in the range of 240-340 beats/min and Type II in the range of 340-430 beats/min [11].

1.2.2 Ventricular Arrhythmia

Two common types VA are VT and VF, both originating from the ventricular myocardium or His-Purkinje system [12]. The difference between these two types of arrhythmia is the duration, morphology and haemodynamics, which means functioning of heart and the way blood flows through the system and its pressure [14], [15]. VT and VF are shown in Figure 1.7 and Figure 1.8 respectively.

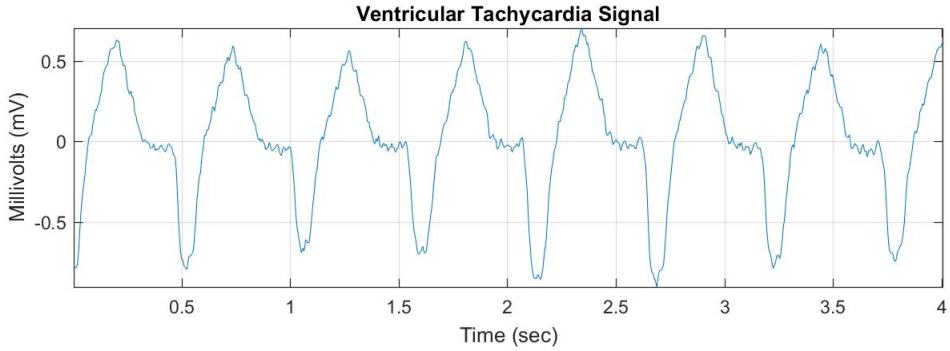

Figure 1.7: Ventricular Tachycardia.

VT can be divided into two sub-groups, non-sustained and sustained. Non-sustained VT (NSVT) is an incidence defined as 3 (sometimes 5) or more consecutive beats arising below the AV node with RR interval < 600ms (>100 beats/min) and lasting <30 sec. It has also been defined as beat rate ≥ 125 beats/min [13]. However, the sustained VT lasts longer than 30 seconds and/or requires an intervention for its termination [14]. VT refers to waveform which resembles only QRS complex, without P-wave and T-wave, as shown in Figure 1.7. VT can be monomorphic or poly-

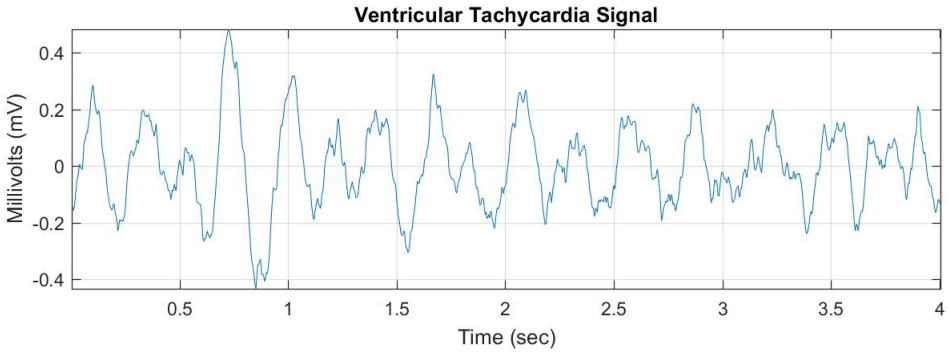

Figure 1.8: Ventricular Fibrillation.

morphic depending upon the structure. When VT signal does not vary; which means that every impulse resembles each other, it's called monomorphic. When VT signal varies in its shape; where impulses do not resemble each other, it is known as polymorphic. During VT, heart rate is higher than normal, unlike physical activity where the heart rate is high but maintains the QRST normal sinus rhythm.

VF is more dangerous than VT. This is due to the fact that during VT, heart maintains a rhythm, even if heart rate is very high. However during VF, the heart starts to act erratic. Figure 1.7 shows that even though the heart is beating rapidly, there are impulses that resemble QRS complex and it is still rhythmic. However during VF, there is lack of rhythm and the heart acts erratic and because of this erratic behavior, circulation of blood to vital organs and the heart itself is severely compromised. When blood flow is hindered, it can cause damage to the heart and can even lead to SCD.

Some of the prevalent causes of ventricular arrhythmia are angina, higher cholesterol levels, diabetes, improper and unbalanced diet, heart surgery, hypertension, smoking and genetics. If a person had a previous history of arrhythmia, that person has a high risk of having another heart attack [14]. The causes can be divided into two subcategories, lifestyle and genetics. Lifestyle causes are inclusive but not limited to lack of exercise or physical activity and diet. It can lead to obesity; another contributing factor to heart diseases. Genetics can contribute to heart disease. This is because a genetic variation can change the way a particular protein works. For example an individual processes cholesterol differently, increasing the likelihood of blocked arteries.

During VT or VF, a person can feel shortness of breath, discomfort, feel chest pressure, fullness

or pain and pain in arms, back, neck stomach or jaw and can also lead to SCD. SCD accounts for about 15% of total mortality in the US [16]. VT can lead to VF which is the most common cause of out-of-hospital SCD [14]. During VF, the heart rate is even faster than VT.

Fundamentally, the danger behind arrhythmias, especially VF is the loss of circulation to the heart and other vital organs, leading to major damage and potential death in minutes. For several decades researchers have been working together to find better treatment options for AA and VA, but currently the health outcomes for patients remain poor and mortality rate remains high. In the instance of VF, the most severe arrhythmia, currently the only available treatment is defibrillation shock. The defibrillation shock itself can cause chronic damage to patients, and finding better ways of administering it would be helpful in saving patient lives.

1.2.3 Use of Defibrillators

During VF, the heart has stopped functioning almost entirely. This requires some corrective measures. These corrective measures include cardiopulmonary resuscitation, defibrillation and cardioversion. These are the most common treatment options available [22]. Implantable cardioverter defibrillator (ICD) are useful in preventing SCD in patients with known VT and VF.

AA are common in patients who are recipients of ICDs [23], [24], [25]. AA is the most common source of embolic stroke. The two most common therapies to control AA are rate control therapy and rhythm control therapy. The aim of these two therapies is to control the rapidly beating heart.

Accurate detection of cardiac arrhythmia and providing shock therapy is the basic function of the ICD. Providing proper shock therapy in a timely manner is very critical for patients experiencing an instance of VT or VF [29], [32]. In one study, patients who were recipients of ICD, up to 13% of these patients received inappropriate shocks. This suggests that ICD provided a shock therapy to patients when it was not required, making it an inappropriate shock therapy. These inappropriate shocks can worsen the severity of VA and increase the mortality rate [46]. It was also reported that almost 80% of those shocks resulted in supraventricular tachycardia which includes AF and AFL. If not treated, usually the VA - VT or VF is preceded by AA - paroxysmal AF or AFL also known as dual tachycardia [23].

ICD is used as a protective measure for patients who have already experienced VF or patients which are at high risk of VT or VF. But patients experiencing AA require medication or medical

device to control higher than normal heartbeats. The control of AA is through rate control or rhythm control treatment.

1.2.4 Rate Control and Rhythm Control

Rate Control and *Rhythm Control* are two strategies that are being used to control the AF. Aim of both strategies is to control heart rate if there is an incidence of AF [18]. The criteria for each therapy is different, such that rhythm control is for the patients younger than 65 years of age with newly detected AF and no previous anti-arrhythmic drug failure. On the other hand, Rate control is for patients over 65 years of age with a persistent AF, hypertension, less symptomatic and no history of cardiac failure [19]. Internal electric cardioversion (ICV) and external electric cardioversion (ECV) are medical devices that provide rate control for some patients who experience AA [25].

Fundamental idea behind rate control or rhythm control is to control AA. Any medical device that is being used by the patients, requires a method of capturing signals and using signal processing to analyze that signal, then providing treatment. An improved system of differentiating arrhythmia can provide a custom solution for each patient and avoiding inappropriate shock treatment.

Patients, who experience AA are also monitored very closely in order for a medical professional to recommend medication or medical device for treatment. This monitoring is a long process and requires long ECG recordings of the patients. This recording device is known as Holter, which is used by the medical practitioner to monitor patient's heart activity.

1.2.5 Holter Recordings

Holter is a small, wearable device that keeps a track of heart rhythm. On the recommendation of a medical professional, Holter is worn by the patient for one or two days. During one or two day period this device records all of the heartbeats. Holter could also be worn by the patient, if the traditional test and the ECG recording provided inconclusive results about the functioning of the heart. Holter could also be worn by the patients who had myocardial infarction [26], [27] or to monitor the effects of antiarrhythmic drugs [28].

ECG recordings captured by the holter are long because these recording are for one or two days, maybe longer. These recordings are carefully examined by the medical professionals to monitor cardiac health and treatment options, which can be a very strenuous and tiresome task. Auto-

mated system of classifying arrhythmia can simplify ECG analysis for medical practitioners. ECG recordings can be analyzed using mathematical techniques to discriminate and classify different arrhythmias.

1.3 Existing Works on ECG Analysis and Signal Processing

Signal processing algorithms have helped analyze biomedical signals. Since most biomedical signals are time-varying signals, they require special attention to analyze. For signal analysis, time domain, frequency domain and time-frequency/time-scale tools have been used by researchers in the past.

Time domain analysis have been used by the researchers as a signal processing tool for years. There are many such time domain methods which have been used for the analysis of the arrhythmia. Most widely used feature in time domain analysis was ECG amplitude and inter-beat R-R interval and it was used to calculate heart rate variability [47]. Time domain method such as Sequential hypothesis testing was used where the signal was compared to a threshold of generating binary sequence, which was used for detecting arrhythmia using Walds Sequential hypothesis testing procedure [48], [49]. Some other techniques which were used as time domain characteristics of the ECG such as Pan-Tompkins, auto-correlation, modified exponential, signal comparison, and standard exponential algorithm [50], [49], [51].

Frequency domain analysis of a signal provides very important spectral information of the signal. Widely used frequency domain technique is Fourier transform [59]. Parameters like dominant frequency, mean frequency, edge frequency and magnitude of the frequency have been used in the past to predict cardiac arrest and its success of resuscitation of a person [30], [60].

In order, to analyze the time-varying spectral content present in biomedical signals like AA/VA, Short Time Fourier Transform (STFT), a time-frequency signal processing tool, has been used previously. The concept behind STFT is to break down the signal into smaller segments (intervals) and analyze the signal using Fourier transform [3], [30], [65]. This will provide the localized frequency of the signal. For each interval that calculates STFT, a specific spectrum value is obtained, and the combination of these spectra provides us with a sequence of time-frequency distribution.

Wavelet transform is another time-scale tool used for the analysis of signals. Different continuous wavelet bases have been used for the decomposition and classification of biomedical signals,

specially ECG signals [58]. Researchers have used different wavelet for the classification of NSR, VA and AA or arrhythmia in general.

Our group in the past has worked on the feature extraction and classification of VA using different techniques [3], [30], [31]. They have used signal processing for feature extraction and classification of VA. This thesis will focus on developing methodologies that will perform automated detection and characterization of the VA and AA. The focus is on adaptive morphological techniques to understand the four major types of arrhythmia.

1.4 Motivation

Although there have been decades of research to understand the four major types of arrhythmia and many different treatment options, discriminating these arrhythmias is still a challenge. Even with the better understanding of VA and AA and its effects on patient's survival and lifestyle, it is evident that further research is needed to improve the current treatment options.

Surviving patients of myocardial infraction (MI) can face lifestyle changes and challenges. Patients who recover from a VF or VT, require long term treatment. Current treatment options for the patients who survive VF include anti-arrhythmic drugs, controlled pacing of the heart, defibrillators and ablation of the underlying tissue substrate. Patients who receive ICD have deteriorating heart condition due to use of ICDs and its inappropriate shock therapy.

AA is common in patients who receive ICD and require further treatment to control heart rate. In a recent study, 41 patients out of 167, with ICD developed AA, 16 developed AF and 25 developed AFL. AF developed at an average of 4.1 $\pm$3 years after transplant. In these patients a total of 122 incidences of AA occurred which accounted for 50 (43.3%) incidences of AF and 70 (57.3%) incidences of AFL. The treatment of AA included medication, pacemaker implantation with rate control therapy, DC cardioversion, and some patients had combination of pacemaker and medication [23].

Shock therapy algorithms have been designed to help patients who have already suffered MI and have existing heart condition. Yet, shock therapy increased the complexity of recovery and treatment of the patients. The challenge is discriminating the four arrhythmias accurately. Better discrimination and classification of the arrhythmias can improve the shock therapy, which can be customized as compared to generic application. To address the challenge, this thesis attempts to

develop different techniques to improve the classification of arrhythmia, which can also further improve the devices which provide shock treatment. Improvement in such devices can avoid inappropriate shock and can also improve predetermined treatment depending on the type of arrhythmia. Improved treatment will provide patients better quality of life, fewer trips to the hospital, and lower strain on the healthcare system. It may also reduce the chances of SCD, cutting mortality rates for patients. Automation and improvement in classifying arrhythmia will also greatly benefit analysis and detecting arrhythmia in challenging and time consuming long ECG recording acquired from devices like Holter.

1.5 Objective

Primary objective of this thesis was to improve the discrimination and classification of all four major arrhythmia discussed. The goal is to arrive at improved methodologies that provide optimal treatment options for patients who have ICD, ICV or ECV and reduce or eliminate inappropriate shocks. Secondly, it was aimed to provide methodology that automatically and quickly discriminate and classify four arrhythmias from long ECG recordings and simplify task for medical professionals.

- Using time-scale signal analysis, an attempt will be made to develop a framework to discriminate all four major kinds of arrhythmia from surface ECG. Different wavelets will be used to decompose four major arrhythmias and to extract features for classification of AF, AFL, VT and VF.

- Develop adaptive continuous wavelet for the signal analysis to develop a framework to improve the discrimination of all four arrhythmias.

Primary goal of these methods is to design algorithm that can automatically discriminate and classify four major arrhythmia. To achieve the objectives, detailed block diagram of the proposed approach is shown in Figure 1.9.

Input signal of the groups were first divided using morphology between the VA, AA and NSR. Using wavelets transform, selected feature were extracted and using multilevel binary classifier, these signals were then classified between VT, VF, AF or AFL. Block diagram will be referred to in this thesis contribution in Chapter 3 and Chapter 4.

Figure 1.9: Block Diagram of the Work Proposed in This Thesis.

1.6 Thesis Outline

This thesis is outlined as follows:

- **Chapter 2:** This chapter will provide information on the background of signal processing including time domain, frequency domain and time-frequency domain signal processing techniques. It will also provide information about pattern based wavelet design. Moreover, it will provide background of different methods of QRST cancellation and information about feature extraction and theory behind the classification scheme that will be used in this thesis.

- **Chapter 3:** This chapter will provide analysis using different wavelets and extraction of select features and classification of the four groups of arrhythmia, using linear discriminant analysis. This chapter contains the results, review and comparison of the results between different wavelets.

- **Chapter 4:** This chapter will focus on the purpose and design of pattern based adaptive

wavelets. This chapter will provide analysis of the different adaptive wavelet and extraction of select features and classification of the four groups, using linear discriminant analysis. This chapter contains the results, and comparison of the results between different library wavelets and adaptive wavelets.

- **Chapter 5:** This chapter will summarize the thesis with conclusions and direction for future work. This chapter will also identify the potential application of the proposed work.

Chapter 2

Background

THIS chapter introduces the necessary signal analysis techniques and classification tools, that will be used in this thesis to classify the four major types of arrhythmia. This chapter will also explore the various techniques which past researchers have used to analyze and classify AA and VA. These techniques exist in various signal processing domains namely time domain, frequency domain and time-frequency domain. Different wavelets and theory behind wavelets will also be discussed. This chapter discusses signal processing techniques of these domains, and significance for using time-scale signal processing techniques. The techniques and extraction methods for different feature categories and features are also discussed. Lastly, the classification techniques and methodologies used to separate the four major types of arrhythmia are also discussed in this chapter.

2.1 Time Domain Analysis

There are many time domain signal processing methods which have been used for the analysis of the arrhythmia. Time domain analysis was only discussed as an overview, due to their prevalence in previous literature.

The most widely used signal feature in time domain analysis is ECG amplitude and inter-beat R-R interval [47]. QRS complex or R-peak is the most prominent feature of the normal ECG signal. The R-R interval is measured as the difference between one R-peak and the next R-peak, and was used to calculate another important feature, heart rate variability. The R-R interval on the ECG recording was a widely used feature to characterize the difference between normal and abnormal heart rate.

Another time domain method was sequential hypothesis testing [48], [49]. According to this method, the signal was compared to a threshold of generating binary sequence. A probability distribution of this binary sequence is used to detect Arrhythmia using Walds sequential hypothesis testing procedure [48], [49].

Some other time domain techniques which used characteristics of the ECG are Pan-Tompkins, auto-correlation, modified exponential, signal comparison, and standard exponential algorithm [50], [49], [51]. Other time domain techniques used for the purpose of classification of arrhythmia are maximal rate of sinus tachycardia compared to the onset of VT [52], changes in the cycle length at the onset of VT [53], and rate stability during VT [54]. The most widely used method for detection of VT in single chamber antitachycardia devices are rate, rate stability and sudden onset [55], [56].

The use of late potential in the time domain analysis was used for AF detection [57]. The terminals collecting QRS complex and ST segment of ECG were analyzed for the presence of late potential. The signal was considered as a ST segment, when it showed a deflection after QRS complex. The signal was gathered from three different leads and then averaged. Root-mean-square and the duration of QRS complex was used for classification.

The techniques mentioned above are some of many time domain techniques used for analyzing AA and VA in the past. Time domain signal analysis techniques run into difficulty when trying to capture morphological changes of the four major arrhythmia over time.

2.2 Frequency Domain Analysis

Frequency domain analysis of a signal provides spectral information of the signal. The Fourier transform reveals the frequency components of the time series by transforming the time domain information into frequency domain [60]. Fourier transform of a time domain signal decomposes the signal into a combination of sine and cosine functions of different frequencies [30]. Fourier transform $F(k)$ of a discrete signal $f(n)$ is expressed in (2.1):

$$F(k) = \sum_{n=0}^{N-1} f(n) e^{\frac{-i2\pi kn}{N}} \tag{2.1}$$

Where k represents the discrete frequency, n is the index of time samples and N is sample

length.

Inverse Fourier transform of $F(k)$ can be written as follows:

$$f(n) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{\frac{i2\pi kn}{N}} \tag{2.2}$$

Fourier transform has been used to study VA for the purpose of classification and prediction by many researchers. Parameters like dominant frequency, mean frequency, edge frequency and amplitude of the frequency have been used to predict cardiac arrest and its success of resuscitation of a person [61]. VF-filter leakage is another prominent feature popularly used for VF detection [49]. According to the algorithm, narrow band-stop filter was applied with a central frequency equivalent to the mean frequency of the analyzed VF signal.

Frequency domain features have also been used for the analysis of AA. Dominant Frequency of AA corresponds to the maximum amplitude in its frequency spectrum [62]. Some other frequency domain features are regularity index and spectral width of the signal [49]. Dominant frequency analysis corresponds well with NSR [63]. For the signal analysis, discrimination and classification of AA, R-peak is considered an artifact and may mask many non-dominant frequency components. For the analysis of non-dominating frequencies in an AA signal, the R-peak or QRST complex was removed.

Fourier transform has its own limitations, even though it converts the stationary time series signal into frequency domain. There is limited information about time localization of specific frequencies within a signal. Fourier transform does not provide information about occurrence of particular frequency component at a specific time. Frequency domain analysis is good for stationary signals however, non-stationary signals require both frequency component and time localization. To overcome the time localization limitation of Fourier transform, time-frequency technique is required for analysis of biomedical signals.

2.3 Time-Frequency Domain Analysis

Time-Frequency domain analysis is best for transient or non-stationary signals. This approach provides time localization for the frequency components present in the signal. There are two approaches to time-frequency domain analysis, an adaptive time-frequency decomposition based ap-

proach [3], [30], [64] and time-frequency energy distribution based approach [3], [30], [65].

First approach, adaptive time-frequency decomposition approach is ideal for classification applications because the purpose is to capture signal characteristics by representing it with a given basis function. With this approach, a signal is approximated using a variety of time-frequency basis functions that are translated, modulated and scaled. Each basis function has a definite time and frequency localization. Second approach is time-frequency energy distribution based approach, which is used for visualization with high time-frequency resolution.

There are different signal processing techniques that are used for complex feature extraction and pattern recognition of non-stationary signal. There are many different time-frequency methods, but only STFT and wavelets are discussed in this thesis.

2.3.1 Short Time Fourier Transform STFT

STFT has been used in the past for the analysis of non-stationary signals [59], [66], [67]. The concept behind STFT is to break down the signal into smaller segments and analyze each segment using Fourier transform. This method provides the time localized frequency content of the signal [58]. It provides information about frequencies and how they vary over time. To understand the basic concept of STFT, lets look at windowing function representation of a signal.

$$y_n = \sum_{n=-\infty}^{\infty} f(n)g(n-m) \tag{2.3}$$

Where $f(n)$ is the signal and $g(n-m)$ is the window function. Using the same windowing function to Fourier transform gives us STFT.

$$S\,f(l,m) = \sum_{n=0}^{N-1} f(n)g(n-m)e^{\frac{-i2\pi ln}{N}} \tag{2.4}$$

Where the windowing function is *g(n-m)* applied to the discrete signal *f(n)*. Signal energy S_E of the windowed function at frequency l and time instance m is given as,

$$S_E = \mid S\,f(l,m) \mid^2 \tag{2.5}$$

STFT is a widely used method for the analysis of non-stationary signals, but its weakest point is its resolution, which is based on the size of its window. The window size introduces a compromise between frequency and time resolution because time resolution and frequency resolution

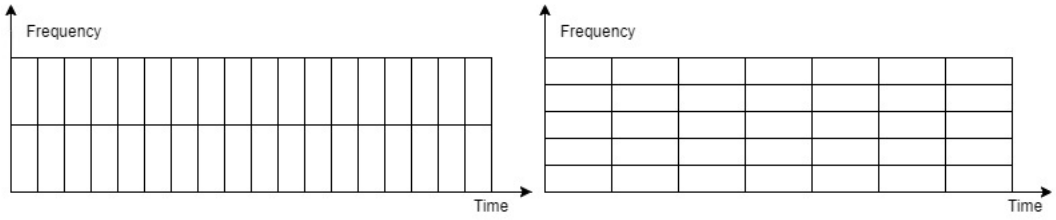

Figure 2.1: Time Vs Frequency Tradeoff.

are inversely proportional. A wide window or a large window reduces time resolution and narrow window reduces the frequency resolution as shown in Figure 2.1. Overall quality of the resolution reduces when window size changes and there has to be a compromise between time resolution or frequency resolution.

STFT has been used for detection and diagnosis of AF for the evaluation of ECG [68]. Important time and frequency characteristics of the ECG were extracted using STFT. STFT was also used for the feature extraction and classification of different arrhythmias such as right bundle branch block beat, left bundle branch block beat, premature ventricular contraction beat and atrial premature contraction beat [69]. Dominant frequencies and other features of power spectrum of the intracardiac electrograms were extracted using STFT for the classification of paroxysmal and persistent AF [70]. STFT was used to map out the spatiotemporal organization by analyzing the regions of frequency distribution [71]. STFT was also used to determine the heart rate and heart rate variability using different window functions [72].

STFT is a popular technique, which could be used to compute the energy distribution of the ECG signal [73]. However there is a tradeoff in time and frequency resolution in STFT, limiting authenticity of the features [65]. All four major arrhythmia are time varying signals which often overlap and evolve into one another over time. It is critical to differentiate all four major arrhythmia because treatment is based on the recognition of correct arrhythmia at right time. Especially VA, where timely therapy is critical for survival of the patient. Challenge of time and frequency resolution can be resolved to some extent using wavelet transform. Wavelet transform is a time-scale signal processing tool that can provide higher time and frequency resolution than STFT.

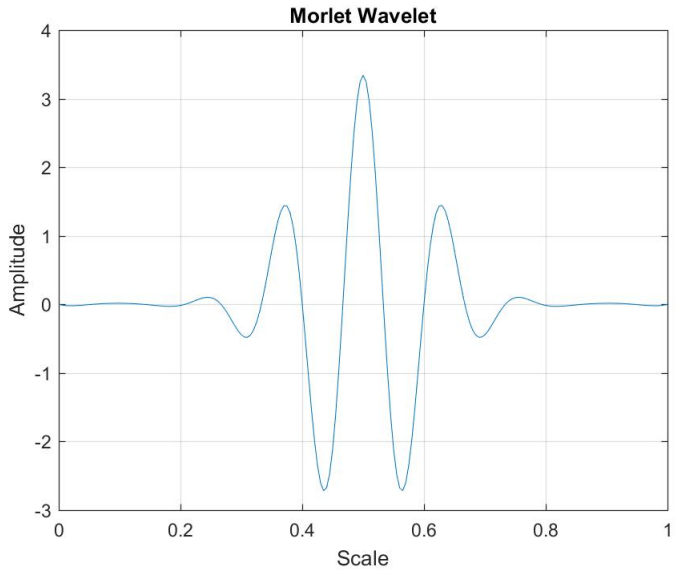

Figure 2.2: Morlet Wavelet

2.3.2 Wavelet Transform

Wavelet transform (WT) is a technique that can overcome the time-frequency resolution challenge. WT is well suited for the analysis of non-stationary signal such as AA and VA.

Wavelet is a small wave or a mathematical function, which is used for analysis of a signal by scaling and translation (shifting) across the signal. Wavelet can also be defined as a building block that represents a 1-dimensional or multi-dimensional signal into 2-dimensional expansion set. Wavelet expansion gives a time-scale representation of a given signal, showcasing its energy distribution over both time and frequency/scale. There exists many wavelets, also known as mother wavelets (MW). These different MW can be used for different task at hand.

Wavelet coefficients can be calculated efficiently using *O(N)*, which means that number of multiplications and additions increase linearly with length of the signal [74]. An example of wavelet (Morlet wavelet) is shown in Figure 2.2.

Signal *f(n)* can be decomposed into a model having all the scales (frequencies) and translation (at all times) with the help of a mother wavelet. This decomposition of the signal and discrete implementation using wavelet transform can be written as [74],

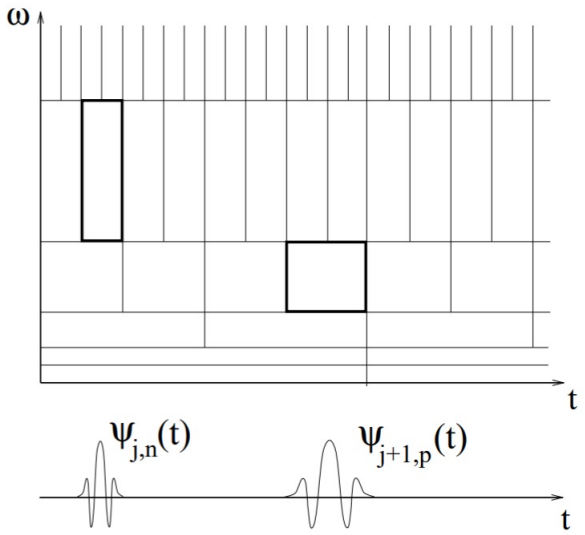

Figure 2.3: The time-frequency boxes of wavelet basis showing scaling and translation. Source: A Wavelet tour of signal Processing, Third Edition, Stephane Mallat, The time-frequency boxes of a wavelet basis define a tiling of the time-frequency plan, Page 20, © Elsevier, 2009

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$$CWT_{s,a} = Wf(s,a) = \frac{1}{\sqrt{s}} \sum_{n=1}^{N} f(n)\, \psi^* \left(\frac{n-a}{s} \right) \tag{2.6}$$

Here $CWT_{s,a}$ and $Wf(s,a)$ represents the coefficients of the CWT, s represents the scale or frequency, a is translation, and ψ represents mother wavelet. Also, ψ^* represents the complex conjugate of mother wavelet. Scale represents the expansion or compression of the mother wavelet as shown in Figure 2.3.

Wavelet Conditions

Generally speaking, a wavelet means a waveform effectively of a limited duration, which is oscillatory in nature. Wavelet has an amplitude that starts at zero, increases and decays quickly back to zero. Some of the basic conditions that a function needs to satisfy in order to qualify as a wavelet are [74]

1. The function should have a zero mean value i.e.,

$$\bar{\psi}(n) = \frac{1}{N} \sum \psi(n) = 0 \tag{2.7}$$

Where $\bar{\psi}(n)$ is the mean value of wavelet basis function $\psi(n)$.

2. It has unit energy,

$$E = \sum_{n} \mid \psi(n) \mid^2 = 1 \tag{2.8}$$

3. It should satisfy *admissibility criterion* [60] [74],

$$C_\Psi = \sum f(n) \left(\frac{\mid \hat{\psi}(\frac{2\pi kn}{N}) \mid^2}{\mid \frac{2\pi kn}{N} \mid} \right) < \infty \tag{2.9}$$

where $\frac{2\pi kn}{N} = \omega$ and $\hat{\psi}(\omega)$ is the Fourier Transform of $\psi(n)$.

For a wavelet to be admissible, it must have the above characteristics and must be localized in both time and frequency space. Orthogonal wavelets are discrete wavelets however non-orthogonal wavelets can either be discrete or continuous wavelet [75]. Wavelet transforms can be complex or real valued, it is considerably faster to do the calculations in Fourier space [76].

2.3.3 Continuous wavelet transform in Fourier domain

By the convolution theorem, wavelet transform is the inverse Fourier transform of the product given as [76],

$$W_n(s) = \sum_{k=0}^{N-1} \hat{F}_k \hat{\psi}^*(s\omega_k) e^{i\omega_k n \delta t} \tag{2.10}$$

Where $\hat{\psi}(s\omega_k)$ is the Fourier transform of the function $\psi(s, a)$ and $*$ denotes the complex conjugate of the wavelet function, also the angular frequency is defined by,

$$\omega_k = \begin{cases} \frac{2\pi k}{N\delta t}, & \text{if } k \leq \frac{N}{2} \\ -\frac{2\pi k}{N\delta t}, & \text{if } k > \frac{N}{2} \end{cases}$$

$\hat{\psi}(s\omega_k)$ can provide continuous wavelet transform for a given s at all n simultaneously and efficiently [76].

Continuous Wavelets in Fourier Space

This section discussed Fourier space mathematical equations of three wavelets namely Morlet [76], Paul [76] and Bump [82], [77].

Morlet Wavelet Transform Equation of the Morlet CW [76],

$$\psi_0(\eta) = \frac{1}{\pi^{\frac{1}{4}}} e^{-i\omega_0\eta} e^{-\frac{\eta^2}{2}} \tag{2.11}$$

Where η is a non-dimensional time parameter. Equation of the Morlet CW in Fourier space [76],

$$\hat{\psi}_0(s\omega) = \frac{1}{\pi^{\frac{1}{4}}} H(\omega) e^{-\frac{(s\omega-\omega_0)^2}{2}} \tag{2.12}$$

where $H(\omega)$ is a Heaviside step function, $H(\omega) = 1$, if $(\omega) > 1$, otherwise $H(\omega) = 0$

Paul Wavelet Transform Equation of the Paul CW [76],

$$\psi_0(\eta) = \frac{2^m i^m m!}{\sqrt{\pi(2m)!}} (1 - i\eta)^{-(m+1)} \tag{2.13}$$

where m is the order of wavelet transform. Equation of the Paul CW in Fourier space [76],

$$\hat{\psi}_0(s\omega) = \frac{2^m}{\sqrt{m(2m-1)!}} H(\omega)(s\omega)^m e^{-s\omega} \tag{2.14}$$

where default order m is 4 and $H(\omega)$ is a Heaviside step function, $H(\omega) = 1$, if $(\omega) > 1$, otherwise $H(\omega) = 0$

Bump Wavelet Transform Equation of the Bump CW for the parameters μ and σ is [77],

$$\psi_0(\eta) = e^{1 - \frac{1}{1-\sigma^2(\eta-\mu)^2}} \chi_{(\mu-\frac{1}{\sigma}, \mu+\frac{1}{\sigma})}(\eta) \tag{2.15}$$

where χ is the indicator function.

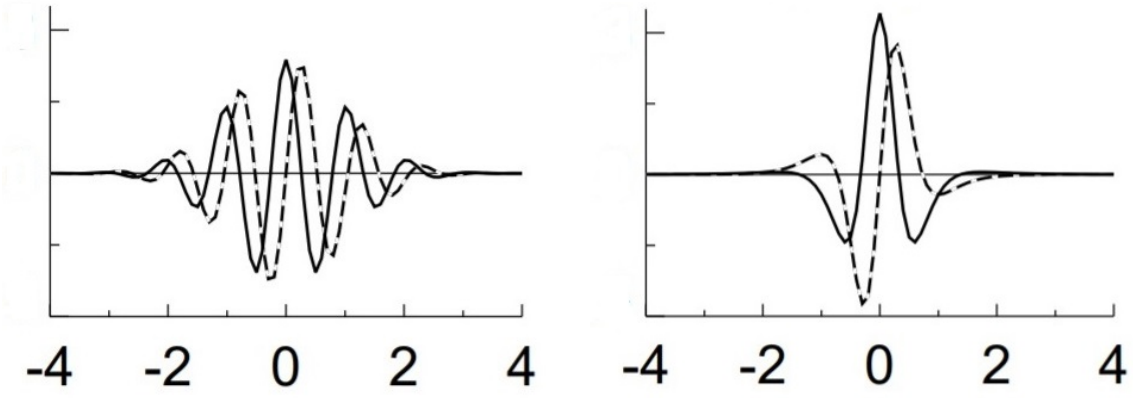
Equation of the Bump CW in Fourier space [82],

$$\hat{\psi}_0(s\omega) = e^{1 - \frac{1}{1-(s\omega-\mu)^2/\sigma^2}} 1_{(\frac{\mu-\sigma}{s}, \frac{\mu+\sigma}{s})} \tag{2.16}$$

where valid values for the parameters μ and σ are between (3 - 6) and (0.1 - 1.2) respectively. Default values for μ and σ are 5 and 0.6 respectively.

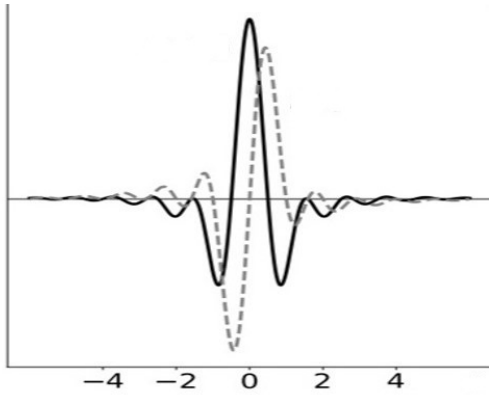
Figure 2.4 shows three different wavelet bases from the above equations. Plots shows real part in solid line and imaginary part in dashed line for the wavelet domain. Plots on the right shows corresponding wavelet in frequency domain.

Past literature showed that Morlet was used for analysis of atrial arrhythmia [84], [38], [39]. Morlet Wavelet was also used for the feature extraction of ECG signal [85], [86] and quantification of ECG late potentials [87]. AF and VF signals are very chaotic signals, and do not have a defined structure. Therefore, these signals require continuous wavelet (CW) which have been used for signal processing of chaotic signals. Paul and Bump wavelets have been used in the past for analyzing chaotic signals [76], [77]. Paul wavelet was also used for noisy signal analysis because it



(a) Morlet Wavelet

(b) Paul Wavelet



(c) Bump Wavelet

Figure 2.4: Plots of Morlet CW, Paul CW and Bump CW. Plots show the wavelet bases function, the real part (solid), imaginary part (dashed). Source for Morlet and Paul wavelet, A Practical Guide to Wavelet Analysis ©American Meteorological Society. Used with permission [76], Source for Bump wavelet, Phase Harmonic Correlations and Convolutional Neural Networks ©IMA Journal of Information and Inference. Used with permission [83].

demonstrates the filtering property [78]. Paul wavelet also has better time localization than Morlet wavelet [79]. Bump wavelet is different from the Gaussian function-based wavelet. It has been used for signal analysis of biomedical signals in the past [80]. However, Morlet wavelet demonstrates good localization in both time and frequency domain [81].

This thesis used two different categories of wavelets. First category was library wavelet, namely Morlet, Paul and Bump. However, second category was adaptive continuous wavelets (ACW), designed based on the signal patterns. Next section will discuss the design of ACW.

2.4 Adaptive Continuous Wavelets

Effectiveness of the WT is dependent on the basis function of the wavelet itself. The choice of wavelet function plays a very important role in determining the resulting WT of the signal. The challenge is to find a wavelet basis that best describes the data rather than arbitrarily choosing a wavelet [88].

A wavelet with a better approximation can be achieved by customizing it to the specific signal at hand. The adaptive continuous wavelet (ACW) basis function is constructed based on the signal pattern and structure to be analyzed. By doing so, the ACW is better able to reveal certain characteristics and markers within the signal to help distinguish it from others [90].

ACW functions have been designed and used for the purpose of QRS complex detection [91], signal classification and compression [88] and analysis of ECG signals [89]. There are several methods to construct pattern based adaptive wavelets such as statistical method [92], Lifting Scheme [93], Bi-orthogonal method [94], Projection based methods [95] and least square optimization for creating an adaptive wavelet [96].

ACW basis must satisfy the conditions of the wavelets in order for it to qualify as a wavelet [91]. Most important properties of wavelets are the admissibility and regularity conditions. Admissibility helps in the reconstruction of the original signal once it has been decomposed and analyzed. Regularity condition states that wavelet function can be localized in both time and frequency space. Design and pattern selection for ACWT for four groups are further discussed in Chapter 4.

Design of ACW in this section is a summary of Wavelet and their applications by Misiti ©Wiley. In short, to construct the approximation in the least squares, using finite linear combination of the form [96],

$$\psi = \sum_{i=1}^{N} \alpha_i \rho_i \tag{2.17}$$

Where vector α contains the constraints and ρ_i are the functions, and where the approximation on ψ satisfies the two conditions. First condition is,

$$\psi \in L^1 \cap L^2 \; and \; t\,\psi \in L^1 \tag{2.18}$$

Second condition is function ψ must integrate to zero. This condition reduces to the following

linear constraint

$$\sum_{i=1}^{N} \alpha_i R_i = 0 \; where \; R_i = \sum_{b}^{c} \rho_i \tag{2.19}$$

The vector $\alpha = \{\alpha_i\}_{i=1}^{N}$ and, consequently, the function $\psi = \sum_{i=1}^{N} \alpha_i \rho_i$ are obtained by solving a linear system:

$$\begin{pmatrix} G & M^t \\ M & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \lambda \end{pmatrix} = \begin{pmatrix} B \\ 0 \end{pmatrix} \tag{2.20}$$

Where,

- G the Gram matrix $(N \times N)$ defined by $G_{ij} = \sum_{b}^{c} \rho_i \, \rho_j$;

- M the matrix of constraints $(1 \times N)$ with $(M_i = R_i)$;

- B the vector $(N \times 1)$ defined by $B_i = \sum_{b}^{c} f_i \, \rho_i$;

where λ is a Lagrange multiplier. λ and vector α are associated with the constraints which are solution of the linear system.

The wavelet designed is based on a pattern of a signal and also bounded by the constraints. The unique wavelet solution is the result of combination of the linear functions and constraints. When constraints are increased, the wavelet solution becomes better.

To improve the solution and approximation of the wavelet, it is necessary to increase the degree of the F polynomials and to add continuity constraint to the ψ function.

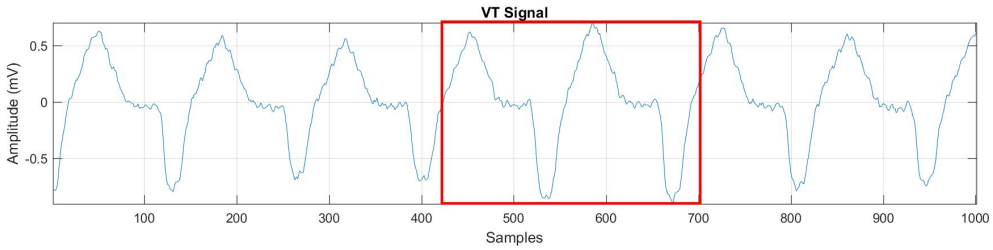
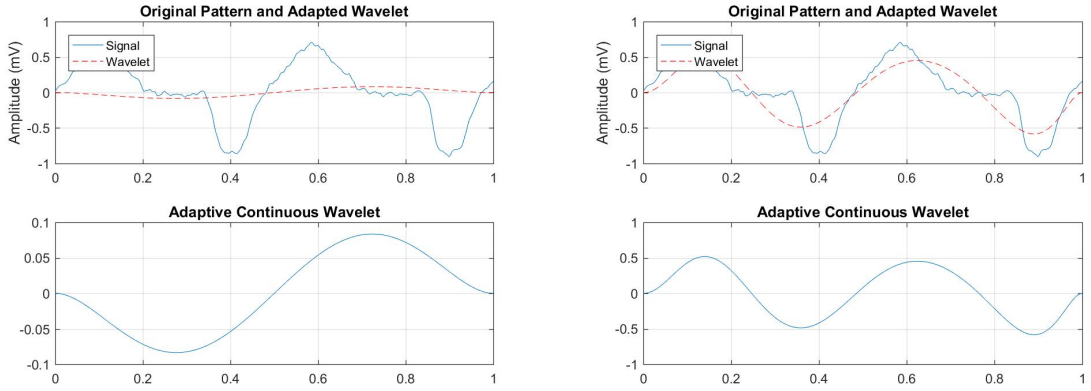


Figure 2.5: VT signal used for designing sample wavelet

Figure 2.5 shows the ECG signal. A small sample of the waveform is used as a pattern, outlined by a red box. Figure 2.6(a) and figure 2.6(b) represents two wavelets, based on this pattern of the signal. It shows by changing a single constraint which was polynomial, wavelet form changed. As mentioned above α is the vector that represents constraint. It can be noticed that the two adaptive wavelets have a different basis function however the pattern for their design is same. This basic



(a) Wavelet Polynomial value as 5

(b) Wavelet Polynomial value as 10

Figure 2.6: Wavelet generation using different values for polynomial

difference between the two wavelets is result of change in constraints. Constraint vector α along with the signal pattern f makes the wavelet unique. It can be noticed that wavelet is dependent on the signal function f and the wavelet bases function will change if this function changed. Secondly, wavelet is dependent on constraint vector α, by changing the polynomial value (part of α) the wavelet bases function will change. ACW was designed based on the constraints and the recurring patterns of the arrhythmias.

Adaptive continuous wavelet transform (ACWT) was used for the signal analysis once the ACW was designed based on signal pattern. CW and ACW were used for the analysis of the four major arrhythmias but analysis of AA is difficult, if ventricular activity is present in the signal. Therefore, it was important to remove ventricular artifact from the ECG signal. Next section will discuss some tools which can help remove ventricular activity from the ECG signal.

2.5 QRS Detection and Cancellation

For the analysis of AA, it is very important to remove the ventricular artifacts, in order to capture atrial signal energy and characteristics. Ventricular activity includes P-wave, QRS complex and T-wave, which are dominant frequencies. These dominating frequencies must be removed for AA signal analysis. There are many different techniques to remove ventricular activity from the signal and some of these techniques are discussed in this section.

Flat, Spline or Linear Method: This method of cancellation of QRS complex from the ECG is called flat, spline or linear method [97]. This method uses atrial electrograms (AEG) and uses surface ECG as reference to identify ventricular activity. Data is collected and sampled simultaneously. Adjusted threshold technique is used for QRS detection and cancellation of QRS complexes and replaced by either flat, linear or spline interpolation before signal analysis.

Spatiotemoral QRST Cancellation: This technique uses average beat subtraction based method for the cancellation of QRS complex and T-wave. The QRS complexes are clustered according to their morphology. Spatial optimization is applied to the QRS template. The fibrillation signal is extracted from the surface ECG by subtracting an aligned average beat [98]. According to this technique an intermediate signal is used for reducing the AF influence in the cancellation process.

Adaptive Ventricular Cancellation: This method is based on the adaptive FIR filtering. This technique uses surface ECG signal and a reference atrial electrogram (AEG) signal to produce an estimate of the interference. This reference signal is subtracted from the main ECG to produce a signal that does not contain QRS, and only the atrial signal is recovered from it [99], [100].

Independent Component Analysis: For ECG signal, it is considered that atrial and ventricular electrical activities can be considered as decoupled electrical signal mixed together. This mixed signal can be separated using a AEG as a reference. This means that the ventricular activity or ventricular artifact is separated from the signal which is atrial signal only [99].

Template Matching Algorithm: The method uses an algorithm that was developed by Dr. Berger, also known as Berger method or QT Variability Algorithm [101], [102]. According to template matching algorithm, a template is created and then, using the interval of this template, interval of other beats are found by the algorithm. This algorithm is used for all other beats to be stretched or compressed in time in order to find best match for a template. Once all the template intervals are found, this template is used to remove R-peaks from the ECG signal.

Once the QRS complex or R-peak was removed from the AA, the AA signal was ready for WT. Features were extracted from CWT and ACWT for the purpose of discrimination and classification of the four major arrhythmias. These features of the arrhythmias are discussed in the next section.

2.6 Feature Extraction

ECG of arrhythmias are very complex signals to analyze, particularly due to their time-varying spectral content. The ECG of the four different arrhythmia groups can be differentiated by quantifying this information within parameters that characterize each group and its behavior. Extraction of different significant features becomes very important for the characterization and classification of the arrhythmia. Features were selected to represent true morphology of all the four arrhythmia.

VT and VF are characterized by continuous bands in the range of 2-5 Hz and 6-8 Hz band respectively [39]. AF and AFL can be characterized by continuous bands in the range of 6-10 Hz and 4-6 Hz respectively [2], [105], [106]. Therefore, four groups of signals were subdivided into three different frequency bands, low frequency band (LFB)(0-3Hz), medium frequency band (MFB) (3-6Hz) and high frequency band (HFB) (6-11Hz) for classification.

Three different categories of features were used to extract appropriate features of the four groups of arrhythmia. Selection of these categories were based on the morphological characteristics of the the four major types of arrhythmia. The three different categories were energy, information theory and statistical features. VT and AFL has a repetitive and uniform waveform structure however VF and AF has non-uniform and random waveform structure. These different waveform structures reflect different energy distribution therefore energy based features were used. There is lack of order or lack of predictability in VF and AF however VT and AFL are more predictable therefore information theory and statistical based features were used.

First category was energy based, out of which relative energy of the signal was used as a feature. Since energy is conserved in each band, it is easier to compare the energy of bands of each CWT as a feature. Admissibility criterion and center frequency is a property of a wavelet, which makes a wavelets unique. Wavelet signal decomposition and time-scale transform is also unique for every wavelet. This signal band energy from the CWT can be compared between different arrhythmias, therefore relative wavelet energy (RWE) was used as a feature for classification four groups [42], [103].

Second category of feature was based on information theory, and feature used was entropy of the signal. Entropy can be defined as lack of order in a system or uncertainty of outcome, where higher uncertainty corresponds to higher entropy. Shannon entropy (SE) and instant Shannon entropy

(ISE) of CWT were used as a feature for classification of four groups and two groups [43], [44], [38]. SE was calculated for the entire band of CWT coefficients [112]. However, for Instant Shannon entropy (ISE), first the coefficients were summed in the scalar space and then entropy was calculated over time space.

Third category of features was based on the statistical functions. Researchers have used statistical features like mean, median, maximum, minimum, variation and standard deviation in order to increase the robustness of the ECG features [104]. CWT mean energy and standard deviation of the signal were used for the classification of the arrhythmia [104], [43], [40]. Four groups of arrhythmia demonstrate very different characteristics in their morphological makeup. For example, AF and VF showed very erratic morphological makeup however AFL and VT showed a uniform morphological makeup. Mean and standard deviation were the features which captured these characteristics and helped classify all four arrhythmia.

2.7 Pattern Classification

Classification of arrhythmia is important since it allows us to identify the proper group that an arrhythmia signal belongs to. Pattern classification allows us to discriminate between the four groups. Once the signal was analyzed using different tools mentioned above, then with the help of classifier algorithm, a given signal could be classified as a specific group. It was very important to classify the signal according to its group specially between VA and AA and further discriminate between VT, VF, AF and AFL.

There are many different classification techniques or algorithms such as Bayes Network, Radial basis function, pruned tree, single conjunctive rule learner and nearest neighbors algorithm [114]. Some other classification techniques are regression tree, random forest, artificial neural network, and K-nearest neighbor [2]. For this thesis, Fisher's Linear Discriminant Analysis is used for classification.

LDA is a supervised learning approach, where classifier is built using a set of training samples which are pre-qualified by experts [3]. A supervised classifier is important because once trained, it will help to discriminate new instances of data based upon the distinguishing characteristics highlighted by the experts in the training data.

2.7.1 Linear Discriminant Analysis

LDA is a supervised machine learning approach. LDA is used when there is a priori knowledge of the groups. The classifier is built using a set of training samples that are pre-classified by experts.

The linear discriminant function will only create linear boundaries, therefore it will perform well if the features exhibit strong discrimination. If the features demonstrate strong discrimination, then the classifier will provide high classification accuracy.

Having multiple groups c in a linear classification system, would require $c - 1$ number of linear discriminant functions. Each function would classify a class from the remainder of classes, and therefore a combination of the discriminant functions would eventually lead to a region, where one class would occupy a region. More information about the linear discriminant function is provided in APPENDIX A.

2.7.2 Cross Validation

Cross validation of data is performed by dividing total number of samples n into equal sized m sets of data. The training is done on all but one of the sets and the remaining set is tested on the classifier. This process is repeated for m times until all m sets of data have been used for training. This method provides a validation error which is an estimate of the accuracy using an unknown number of sets.

One variant of the cross validation method is Leave-One-Out (LOO)method. LOO method is appropriate when the dataset is small. For this thesis number of analyzed signals per arrhythmia was 25 and combined, total number of analyzed signals was 100. These small datasets are typical of biomedical problems and LOO method is recommended for validation [115]. LOO is also appropriate when the accurate estimate of model performance is more important than computational cost of the method. It is also believed to be one of the most optimized validation approaches with least biased estimates [116]. According to this method, the classifier will be trained for n - 1 samples assuming total number of samples as n, and then test the classifier with the remaining sample. LOO method will execute n times to test each sample. The classification accuracy results are based on testing results of each sample [117].

2.7.3 Receiver Operating Characteristics Curve

Receiver Operating Characteristics (ROC) curve defines the trade-off between sensitivity and 1-specificity across a series of points. ROC analyzes the change in sensitivity and 1-specificity with the change in the decision boundary between the groups in a binary classifier i.e. it calculates the sensitivity and specificity pairs for each possible boundary and plots it onto a two dimensional plane. The plane consists of sensitivity on y-axis and 1-specificity on the x-axis [3].

The area under the ROC curve ranges from 0.5 to 1.0 with the larger value indicative of better fit. Higher value of the area under the ROC curve is indicative of higher robustness for the classifier [31].

2.8 Chapter Summary

This chapter presented background information on different signal processing tools such as Fourier transform, time-frequency tools such as STFT and CWT. This chapter covered three analytic continuous wavelets namely Morlet, Paul and Bump along with their Fourier transform. It introduced some of the QRS complex and T-wave cancellation techniques like adaptive cancellation, ICA and template matching. This chapter also introduced least square technique for designing adaptive wavelets. Three different categories of features were discussed, which included features like relative energy, entropy, instant entropy, mean and standard deviation. Linear Discrimination Analysis and cross-validation technique Leave-One-Out method and ROC were also discussed in this chapter.

Chapter 3

Analysis of Arrhythmia Using Standard Wavelets

THIS chapter compares the performance of three different wavelets in classifying four groups of arrhythmia. This chapter also describes the process of pre-processing of the signals, signal processing, features extraction, classification of four major arrhythmia, classification of two group arrhythmia of AA and VA and finally classification accuracy. Figure 3.1 shows the flow of work that will be included in this chapter.

3.1 Database and Pre-processing

The analyzed signals of this study were obtained from a few different publicly available databases, through Physionet and Chapman University and Shaoxing Hospital Zhejiang University School of Medicine. The reason for choosing different databases is lack of unambiguous labeled arrhythmia data. It is a well known fact that due to privacy issues and ethical issues it is extremely difficult to obtain arrhythmia data from the patients. This was also the reason for restricting number of signals of each arrhythmia to only 25 signals.

For VT and VF, two databases were used, first one was MIT-BIH Ventricular Ectopy Database, consisted of 22 recordings, each 24 hour long from 16 patients [119], [121]. The second database was Creighton University Ventricular Tachycardia Database, consisted of 35 recordings from 30 patients [119], [122].

For AF and AFL waveforms, MIT-BIH Atrial Fibrillation Database was used, which consisted of 25 long-term ECG recordings of human subjects [119], [123] and Atrial Fibrillation Termination

Figure 3.1: Block Diagram of the thesis with flow of work that will be covered in this chapter.

Challenge Database, which consisted of 50 patients [119], [124]. Also two recordings from Intercardiac Atrial Fibrillation Database (aif3-afwm, aif5-afwm) were used, consisting of two patients for AFL signals [119]. Rest of the 21 AFL signals obtained from combined database of Chapman University and Shaoxing Hospital Zhejiang University School of Medicine [120]. These 21 signals were 10 seconds long from 21 patients. All the signals acquired from the databases were prelabeled, which were provided by the authors of these databases.

Each of the signals used for analysis were of four second duration and was extracted from an independent patient data (i.e. no more than one signal was extrated for a patient). For each arrhythmia type 25 signals were used, in order to have balance for 4 group classification.

Timely detection of VF and VT is very important because a patient experiencing these arrhythmias can have SCD within minutes, hence any efficient feedback system should be able to detect or segregate arrhythmia in a short duration of time. At the same time we need to have enough information with the short period or time for analysis. Based on the frequency content of four arrhythmias, we choose a four second segment which will have sufficient oscillations for the lowest

dominant frequency content among the arrhythmias.

Every VA signal used in this thesis was of four second duration and each AA signal was six seconds long, but after QRS complex or R-peak removal, only four seconds of signal was retained. All arrhythmia signals were only four seconds in length for CWT signal processing. There were 25 signals each of VT, VF, AF and AFL for a total of 100 signals.

All the signals were sampled to a 250 Hz sampling frequency. A bandpass filter was used with the cutoff frequencies of 0.5 Hz and 50 Hz [101], [102]. AA and VA frequency falls within the range of 1 Hz to 10 Hz therefore, it is safe to filter signals out of this range without loosing any characteristic frequencies of the signals. Signals were filtered using a bandpass filter to retain the characteristic frequencies of the signals but to remove low frequency artifacts like baseline wander and high frequency noise like power line noise.

3.2 Algorithmic Steps

After pre-processing of the signals, next step in the process of arrhythmia classification was R-peak detection and removal for AA signals, as shown in Figure 3.1. A decision was made on the basis of R-peak, to discriminate between AA or VA or NSR. R-peak or QRS complex is present in either an AA or a NSR signal. Algorithmic flow is shown in Figure 3.2.

If the signal was VA, then CWT was used for the time-scale analysis of the signal. With the help of features, the signal was classified as VT or VF. If the signal was not VA, then the signal was discriminated between NS or AA. In case of AA, R-peak was removed using template matching technique and CWT was used for the time-scale analysis of the signal. Signal features were used for the classification of signal as AF or AFL.

3.3 Detection and Removal of QRS complex

Detection of QRS complex or R-peak was very important in order to differentiate between AA or VA. Atrial activity amplitude is very small when compared to the ventricular activity, and for the signal analysis of AA, it is important to remove any ventricular activity.

If QRS was not removed then it was very difficult to analyze AA, due to the fact that atrial activity was masked by the ventricular activity. With R-peak removed, it was much easier to vi-

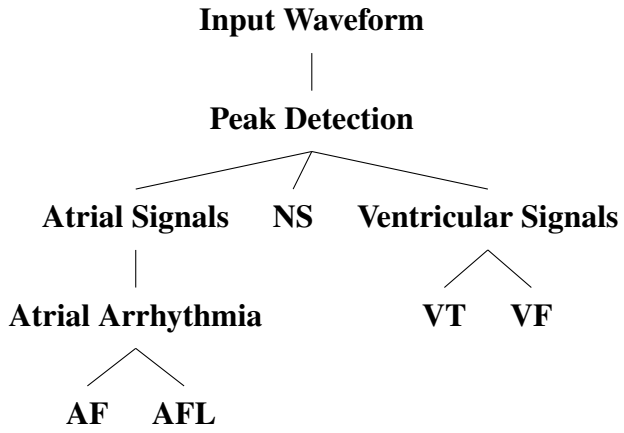


Figure 3.2: Algorithmic Tree: shows the flow of the above Block Diagram

sualize the atrial activities and analyze AA. As mentioned in Chapter 2, there are many different techniques used for removal of R-peak [40], [99]. Most of the techniques require both ECG and AEG signals, except for template matching. Template Matching was the technique that does not require both ECG and AEG and only requires surface ECG for the removal of ventricular artifacts. This technique takes a template of a QRS complex and T-wave and removes rest of the QRST components [102]. This method was modified to remove only R-peak for AA. Steps to analyze the ECG record and removal of QRST are as follows [102]:

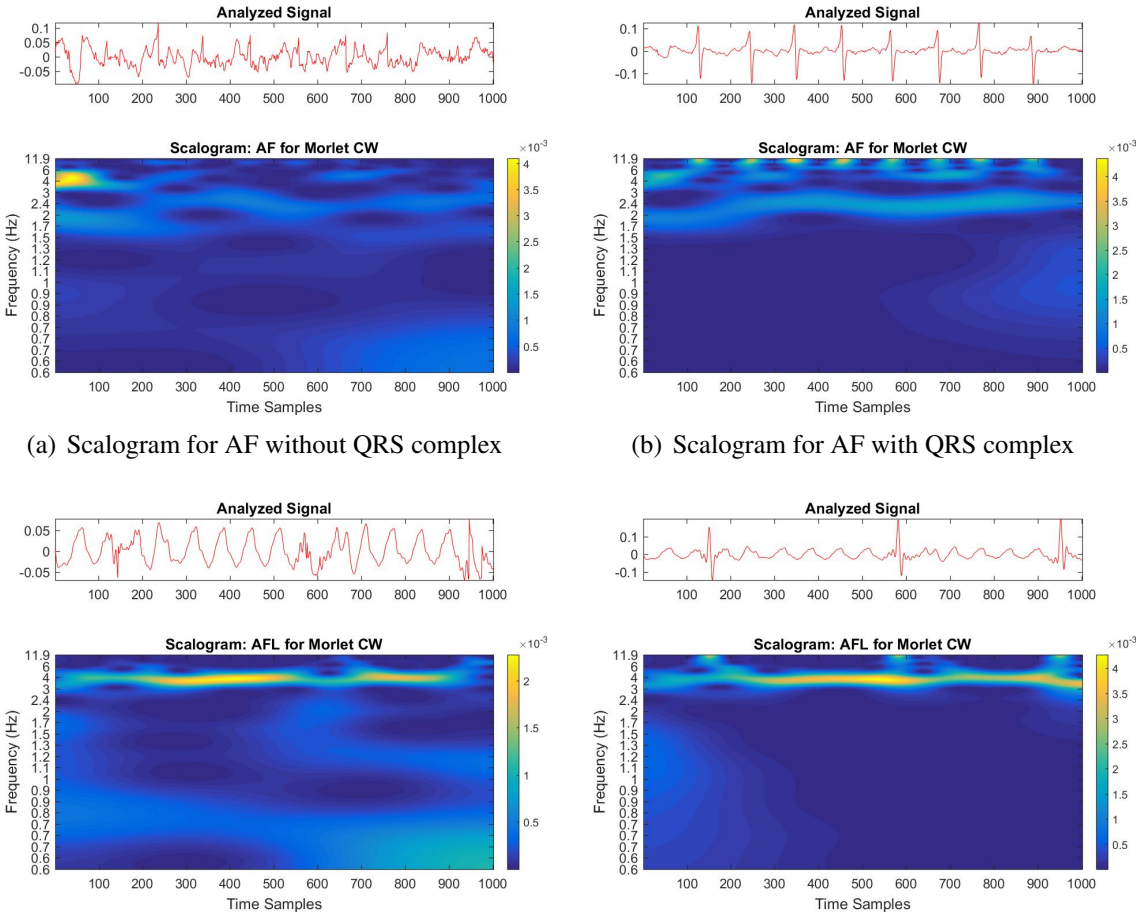
1. An ECG signal was selected, $x(n)$ to analyze. Signal was pre-processed to remove baseline wander and power line noise.

2. R-peak times were determined and noted as T_i.

3. The beginning and end of the R-peak interval template and denoted by k and template denoted as $\varphi(n)$, where n was the sample number. Thus,

$$\varphi(n) = x(n) \text{ from } n = n_0 \text{ to } n_1 \tag{3.1}$$

   where $x(n)$ was the input signal and n_0 and n_1 were the beginning point and end point where R-peak started and R-peak ended, respectively. Therefore the new template was represented by $n = T_k + n_\Delta$ to $n = n_1$ where n_Δ represented a blanking period.



(a) Scalogram for AF without QRS complex

(b) Scalogram for AF with QRS complex

(c) Scalogram for AFL without QRS complex

(d) Scalogram of AFL with QRS complex

Figure 3.3: Scalogram of Morlet wavelets for AF and AFL with and without QRS complex and T wave.

4. For the next step a cost function $\varepsilon_i(\alpha)$ for each beat was defined as,

$$\varepsilon_i(\alpha) = \sum_{j=n_\Delta}^{n_1-T_k} [\varphi(T_k + j) - x(T_i + \alpha j)]^2 \tag{3.2}$$

where α was the time stretching factor, $\varepsilon_i(\alpha)$ was the sum of squared differences between the new template or time stretching factor scaled by α.

5. Once the new template is found, it is matched with other QRS and is used to remove the QRS.

First the algorithm found the R-peak, identified its position and then created an RT interval by finding the next peak in the signal. Using sampling rate, number of sample points that were present

in 52 ms was determined. Then the RT interval was shortened by removing that many sample points from the beginning. The removal of 52 ms ensured that the entire QRS complex was removed. This step reduced error that was calculated later during template scaling due to the QRS complex. The final result was the modified signal without QRS complex.

For the purpose of analyzing AF and AFL, template matching algorithm was used. This method requires only surface ECG signal which can be acquired from public domain unlike other methods that require ECG and AEG. The method was simple and straight forward and could be easily implemented.

Figure 3.3 shows AF and AFL signal with and without QRS complex. Figure 3.3(a) shows the AF signal without QRS complex and Figure 3.3(b) shows AF signal without QRS complex. Figure 3.3(c) shows AFL signal without QRS complex and Figure 3.3(d) shows AFL with QRS complex. Figure 3.3 also shows the scalograms where frequency is on the Y-axis and time samples on X-axis and color represents the energy distribution of CWT coefficients over the time samples and frequency.

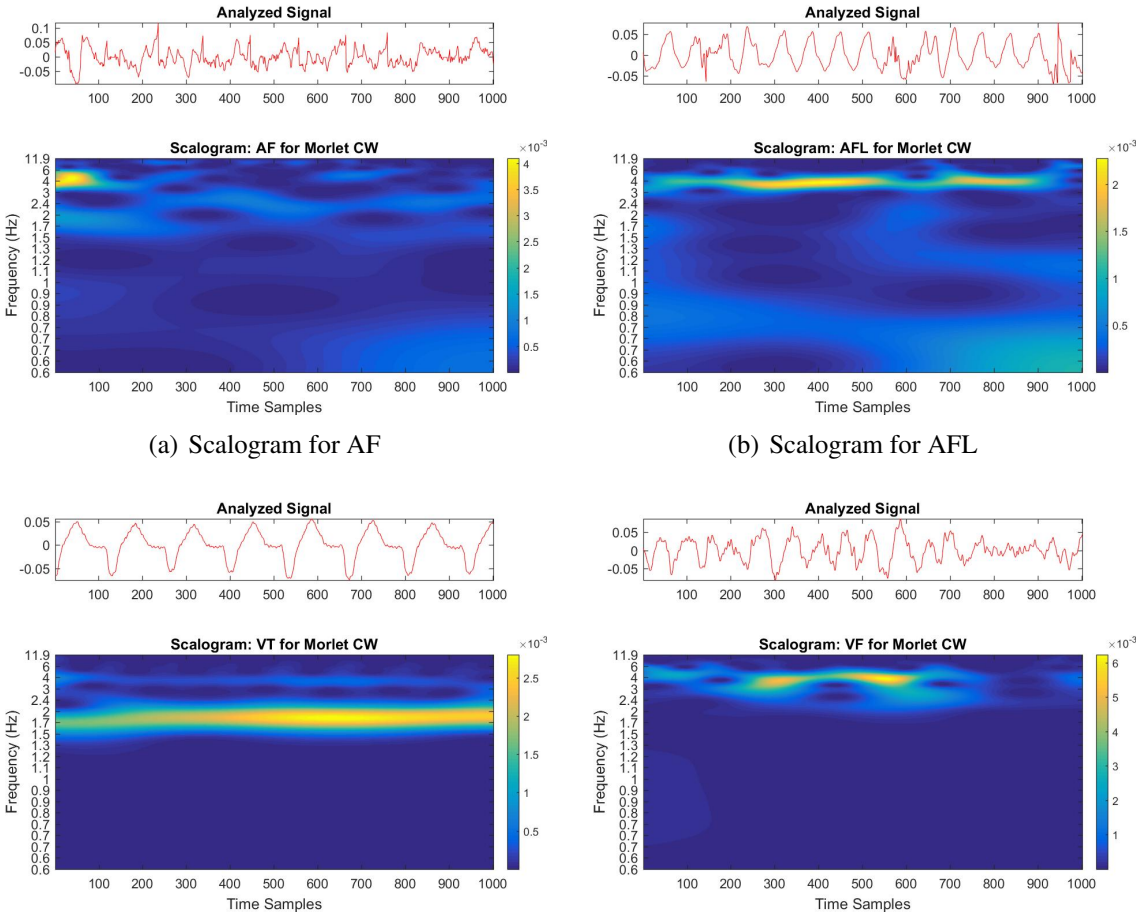
Once the QRS complex was removed, CW were used for the transformation of signal from time series signal to time-scale signal. CWT was performed in the next step to extract features from the signal coefficients.

3.4 Wavelet Transform of ECG

This section covers the CWT of the signals. Every wavelet has unique characteristics, and based on these characteristics, the wavelet transforms of the signal was slightly different. For visual analysis, scalogram of the signal coefficients can be helpful in identifying the type of arrhythmia.

Frequency Bands: One fact that needs attention was the frequency bands where energy components were dominant. For example, in the scalogram of VT, for all three CWT, the energy component was between the range of 1.5 Hz to 4.5 Hz and really concentrated around 2 Hz. Different wavelets decompose the signal in a different way, depending on their shape and center frequency. Figure 3.4 shows the scalograms of Morlet CWT for all four major arrhythmias.

Visual Analysis: Visual analysis of the scalogram plot of the wavelet coefficients can provide an overview of energy distribution of the signal. Figure 3.4 shows the sample scalogram of Morlet CWT for all four arrhythmias. Figure 3.4(a) shows the scalogram of AF, Figure 3.4(b) shows the



(a) Scalogram for AF

(b) Scalogram for AFL

(c) Scalogram for VT

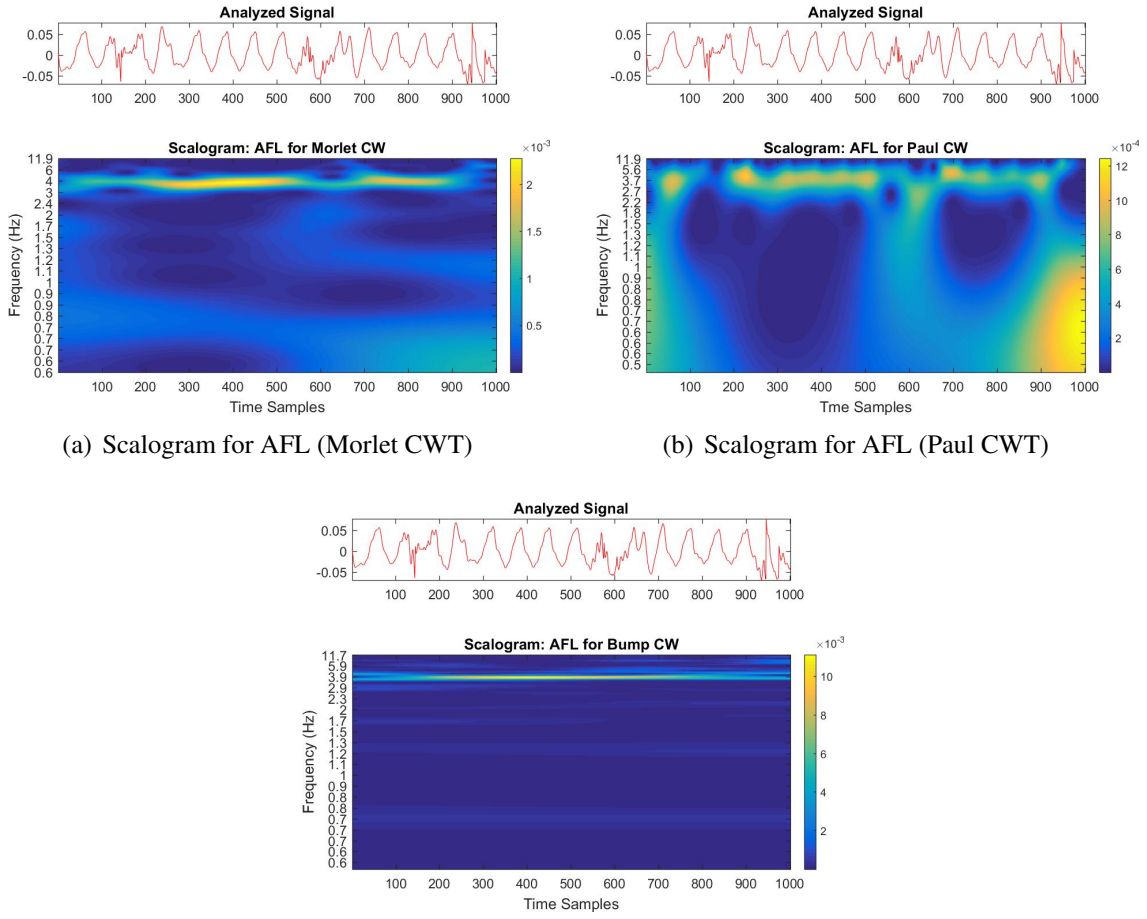
(d) Scalogram of VF

Figure 3.4: Scalograms of Morlet wavelet for four different arrhythmias. These scalograms show frequency on Y-axis and time samples on X-axis. Figure 3.4(a) is the Scalogram of AF, Figure 3.4(b) is the Scalogram of AFL, Figure 3.4(c) is the Scalogram of VT, Figure 3.4(d) is the Scalogram of VF. The signal of each arrhythmia is shown on top of each scalogram respectively.

scalogram of AFL, Figure 3.4(c) shows the scalogram of VT and Figure 3.4(d) shows the scalogram of VF. Scalograms of Paul CWT and Bump CWT are shown in APPENDIX B

Comparison of Wavelets: Visual analysis of different CWT of different arrhythmia showed different energy distribution. For example, when comparing the AFL scalogram plot for all three wavelets, as shown in Figure 3.5, certain differences in energy distribution can be noticed. Scalogram for Morlet CWT in Figure 3.5(a), shows concentration of energy between 4 Hz and 6 Hz. Scalogram for Paul CWT in Figure 3.5(b), also showed presence of frequency component around the same frequency band but more spread out. However, scalogram for Bump CWT in Figure

3.5(c), shows the presence of energy at 3.9 Hz, even more concentrated than Morlet CWT.



(a) Scalogram for AFL (Morlet CWT)

(b) Scalogram for AFL (Paul CWT)

(c) Scalogram for AFL (Bump CWT)

Figure 3.5: Scalograms of all three wavelets for AFL.

These were some of the basic differences that could be seen on scalogram for different wavelets, even for same arrhythmia. This observation also reinforced this fact; that every wavelet has different characteristics.

Comparison of Arrhythmias: It was seen that each arrhythmia type showed a different energy distribution profile on its scalogram. For example, the Morlet wavelet plots in Figure 3.4 showed noticeable differences between each of the four groups. However, in order to better quantify these differences and use them for accurate classification, significant signal features were extracted and used to train the pattern classifier.

3.5 Features Extraction

ECG of arrhythmias are very complex signals to analyze, particularly due to their time-varying spectral content. Every arrhythmia has some features that differentiate one arrhythmia from another. The ECG of the four different arrhythmia groups can be differentiated by quantifying this information within parameters that characterize each group and its behavior. Extraction of different significant features becomes very important for the characterization and classification of the arrhythmia. Different feature categories and features were selected to represent true morphology of all the four arrhythmias. These features are discussed in this section.

3.5.1 Bands of Energy

Energy sub-bands were divided into three main categories depending on the characteristics of the group. The following table shows the group, its beats per second and frequency in Hz [39], [40]. They used different bands for the classification of ventricular arrhythmia and atrial fibrillation.

Table 3.1: Four Groups with Heartbeat/minute and Frequency

Group Name	Heartbeat (per minute)	Frequency (Hz)
Normal Sinus	60-80	1
VT	100-170	1.67-3
VFib	240-420	4-7
AFlut	240-320	4-5.2
AFib	400-600	6.67-10

Energy Bands for the purpose of classification were divided into three sub-bands, Low Frequency band (LFB) (1-3Hz), Mid Frequency band (MFB) (3-6Hz) and High Frequency band (HFB) (6-11Hz). Purpose for subdividing into these 3 major categories is to cover all four groups and also differentiate these four major arrhythmia groups on the bases of their frequency dominance.

3.5.2 Relative Energy

It is very difficult to compare CWT signal energy of one wavelet to the CWT signal energy of another wavelet. Every wavelet's transform is unique and is dependent on wavelet's characteristics.

RE was a good feature for the comparison of energy of a bands, relative to total energy of the signal. RE can be compared between different arrhythmia as a good feature for classification.

(RE_W) of the wavelet coefficients can be defined as

$$RE_W = \frac{E_b}{E_T} \tag{3.3}$$

Where E_b is the energy of a band of frequencies, E_T is the total Energy of the scalogram. E_b can be written as,

$$E_b = \sum_{s=j_1}^{j_2} \sum_{a} |CWT_{s,a}|^2 \tag{3.4}$$

Where E_b represents band energy of coefficients of the wavelet transform $CWT_{s,a}$, and j_1 is the beginning of the scale band and j_2 is the end of the scale band. Band energy E_b represents energy of the LFB, MFB and HFB.

$$E_T = \sum_{s} \sum_{a} |CWT_{s,a}|^2 \tag{3.5}$$

This feature was used for relative wavelet energy of LFB, MFB and HFB. This feature provides a good classification of all four major arrhythmia.

3.5.3 Information Theory: Shannon Entropy

Ent has been used as a feature for ECG arrhythmia classification [43]. The normalized energy $CNE_{s,a}$of the $CWT_{s,a}$ can be written as,

$$CNE_{s,a} = \frac{|CWT_{s,a}|^2}{E_T} \tag{3.6}$$

And the entropy S_{en} for wavelet coefficients can be written as [109], [110]

$$S_{en} = \sum_{s} \sum_{a} p(CNE_{s,a}) \log \frac{1}{p(CNE_{s,a})} = -\sum_{s} \sum_{a} p(CNE_{s,a}) \log p(CNE_{s,a}) \tag{3.7}$$

IEnt refers to the entropy of a signal at an instance rather than total Ent of a band of frequencies [112], [113]. The result of wavelet measures or wavelet coefficients consists of a matrix. Rows of the matrix stretch on the whole time space and the column stretches on the whole frequency space. $SCNE_a$ is the summation over scales and can be written as,

$$SCNE_a = \sum_{s} (CNE_{s,a}) \tag{3.8}$$

where $SCNE_a$ is the summation of scales of the normalized coefficient matrix. Then the IS_{en} can be written as,

$$IS_{en} = \sum_{a} p(SCNE_a) \log \frac{1}{p(SCNE_a)} = -\sum_{a} p(SCNE_a) \log p(SCNE_a) \tag{3.9}$$

Ent and IEnt are good features for classification because of the inherent characteristics of four major arrhythmia. AF and VF are more random than AFL and VT. When an event is highly uncertain, Ent is high, as in the case of AF and VF. But, when the event is certain, entropy is low as in the case of AFL and VT. Due to this, Ent as a feature was chosen to provide discrimination between the four major arrhythmia.

3.5.4 Statistical Features: Energy Mean and Standard Deviation

Mean and STD of the energy has been used as a feature to classify arrhythmia in the past [43], [40]. Mean and STD have been used in order to provide a more robust understanding of the ECG features, and understand the distribution of energy of wavelet coefficients. For the wavelet coefficients $CWT_{s,a}$, mean energy can be described as:

$$\bar{E} = \frac{1}{N \times M} \sum_{s} \sum_{a} |CNE_{s,a}| \tag{3.10}$$

Where $\bar{E}$ is the mean energy, CWT are the wavelet coefficients, N refers to length of the signal. SD of the arrhythmia signal coefficients can be described as,

$$\sigma = \sqrt{\frac{\sum_s \sum_a (|CNE_{s,a}| - \bar{E})^2}{N \times M}} \tag{3.11}$$

Where σ is the STD and N and M are the dimensions of the matrix. Mean and STD are very good candidates as feature for classification because of the arrhythmia characteristics. Since VT and AFL have concentrated energy around one frequency band and VF and AF have more dispersed energy over different energy bands, therefore mean and STD are desirable features.

3.5.5 Feature Short Form

Features and their short forms are shown in Table 3.2 below. These feature short form can be used to reference the features on the figures in the following section.

Table 3.2: Features and Their Short Form

Feature	HFB	MFB	LFB
Relative Energy (RE)	REH	REM	REL
Shannon Entropy (Ent)	EntH	EntM	EntL
Instant Shannon Entropy (IEnt)	IEntH	IEntM	IEntL
Mean	MeanH	MeanM	MeanL
Standard Deviation (STD)	StdH	StdM	StdL

3.6 Differentiating Normal Sinus Rhythm From Atrial Arrhythmia

NSR has pattern of segments where P-wave is followed by QRS complex which is followed by T-wave. This pattern defines the normal functioning of the heart, but AA does not have all the segments but has a R-peaks. Presence of R-peak makes it difficult to differentiate between NSR and AA. A medical device must know the difference between NSR and AA.

Different signal processing techniques can be used to differentiate between NSR and AA. Morphologically, VT and VF differentiate themselves from NSR or AA. It is the morphological similarities between AA and NSR that requires careful consideration. The presence of the dominant QRS structure within AA and NSR make it challenging to identify subtle differences between these two groups.

One of the time domain methods for differentiating NSR and AA is the use of late potential [57]. This method, analyzes the high gain amplification and then uses signal averaging. Basically, the terminals collecting QRS complex and ST segment of ECG were analyzed for the presence of late potential. The signal which had the deflection after QRS complex was considered ST segment. The signal was gathered from three different leads and then they were averaged. Root-mean-square and the duration of QRS complex is used for classification. This method was used for differentiating NSR and AA, however the focus of this thesis is not in separating NSR from AA but to classify 4 major arrhythmias. The next section of this report discusses the results of the classification of four major arrhythmias.

3.7 Linear Discriminant Analysis

The main idea behind LDA is to project dataset onto a lower-dimensional space with good class-separability in order to avoid over-fitting. The linear discriminant function will only create linear boundaries. There are many different methods used to calculate the classification of boundary and accuracy. Fisher's linear discriminant analysis is one of these methods. This technique was used for the classification of four groups or arrhythmia.

The signals were pre-qualified and differentiated between four groups, labeled one to four. Cross-validation was performed using Leave-one-out (LOO) method. The final results of the LOO cross-validation method were the final classification accuracy results reported.

3.7.1 Linear Discriminant Analysis Results For Four Groups

Four group classification confusion matrix is shown in Table 3.3. Paul CWT attained the best classification accuracy out of three wavelets for four group classification. Classification accuracy of Paul CWT for four group was **77%**. It required seven feature to attain the classification accuracy. The four groups classification accuracy for Morlet CWT and Bump CWT was **65%** and **72%** respectively. To attain said classification accuracy for Morlet CWT and Paul CWT, it required total of four and eight features respectively. Features that provided best classification accuracy for Paul CW were REM, EntH, EntM, IEntH, IEntM IEntL and StdM.

Most of the activity captured by the features of all four arrhythmias was in MFB and HFB. Only IEnt captured activity in LFB. All three feature categories were used to attain the classification results. Std, Ent and RE were the features that helped attain classification accuracy for all four major arrhythmia. Ent and IEnt were dominant features. AFL and VT are more organized, therefore the entropy values should be lower depending on the frequency band. However, AF and VF are more random therefore their entropy values should be higher depending on the frequency band.

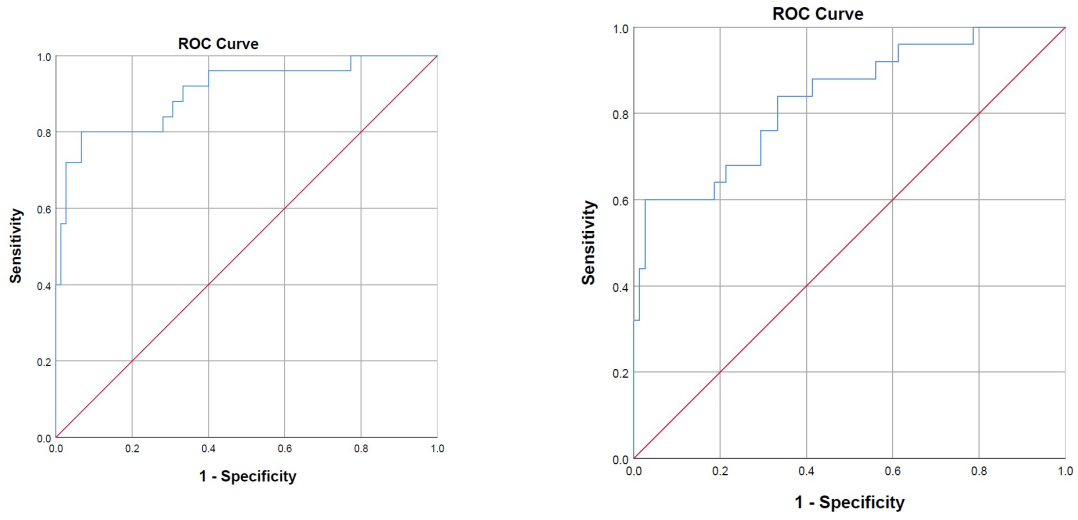
Frequency bands also play a role in differentiating the arrhythmias. Even AFL and VT are more organized feature, but they both belong to different frequency band. AFL frequency activity was within HFB and MFB, on the other hand, VT frequency activity was in LFB. AF and VF frequency activity also belong to different frequency bands. AF frequency activity was in HFB and VF frequency activity was within MFB and HFB.

Table 3.3: Four Group Classification Results using Paul CWT.

Method	Group	AF	AFL	VT	VF	Total
Cross-Validation with LOO Method	AF	**16**	9	0	0	25
	AFL	5	**20**	0	0	25
	VT	1	0	**22**	2	25
	VF	0	2	4	**19**	25
Percentage of Classification	AF	**64**	36	0	0	100
	AFL	20	**80**	0	0	100
	VT	4	0	**88**	8	100
	VF	0	8	16	**76**	100

3.7.2 ROC Curves For Four Group Classification Features

The ROC for the dominant features were obtained that provided the best accuracy for the four group classification. ROC curve for the two dominating features are shown in Figure 3.6. The area under the curve for IEntM and EntM was **90.5%** and **83.3**%, respectively. The dominating ROC curves show good discriminant strength.



(a) ROC Curve for IEntM

(b) ROC Curve for EntM

Figure 3.6: ROC Curves for dominant features for four group classification using Paul CWT

3.7.3 Linear Discriminant Analysis Results For Two Group For AA

The classification accuracy results for AA are shown Table 3.4 below. Paul CWT performed best among all other CWTs for the two groups classification of atrial arrhythmia. Classification accuracy for Paul CWT for two group classification was **76%**. One feature that provided best classification accuracy for the discrimination of AF and AFL for Paul CWT was IEntH. Two group classification accuracy for Morlet CWT and Bump CWT was **72%** and **70%** respectively. Morlet CWT required 2 features and Bump CWT required only one feature for their classification results.

AFL energy was more concentrated around small band of frequencies, whereas AF energy was more dispersed. IEnt was able to provide classification accuracy because the AFL signal was more organized compared to AF which was more disorganized. This was reflected on the classification of AA. Also, AFL has more activity in MFB and some in HFB, however AF has more activity in HFB and some in MFB.

Table 3.4: Two Group Classification Results of AA using Paul CWT.

Method	Group	AF	AFL	Total
Cross-Validation LOO Method	AF	**18**	7	25
	AFL	5	**20**	25
Percentage of Classification	AF	**72**	28	100
	AFL	20	**80**	100

Boxplot was generated to examine the discriminating ability of the features. Boxplot was generated for the IEntH that provided best classification accuracy for two group arrhythmia classification. Figure 3.7 shows the boxplot for IEntH, which provided the best results for Paul CWT. It is evident from the boxplot of the features, that they demonstrate discrimination of AF and AFL.

3.7.4 ROC Curves For Two Group Classification Features for AA

The ROC curve was obtained for the feature that provided the best accuracy for the two group classification. ROC curves for IEntH is shown in Figure 3.8. The area under the curve was found to be **87.4%**. ROC curve of the feature shows good discriminant strength. ROC curve result indicate the robustness of the feature for the discrimination of AA.

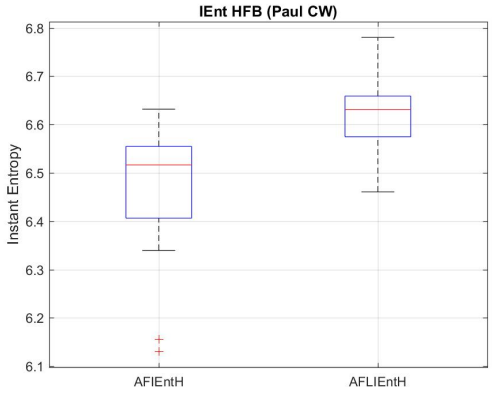


Figure 3.7: Boxplot of the feature for two group classification of AA using Paul CWT

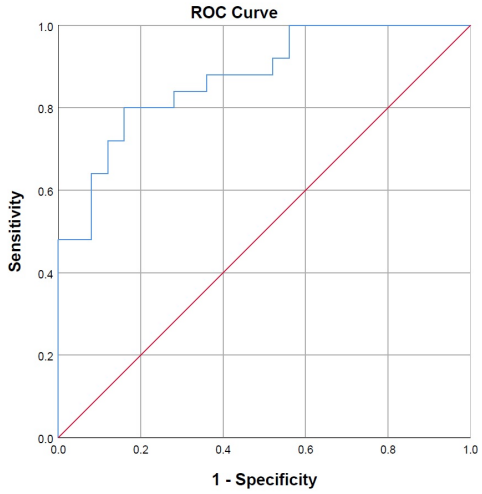


Figure 3.8: ROC Curve for IEntH two group Classification AA using Paul CWT.

3.7.5 Linear Discriminant Analysis Results For Two Group For VA

Linear discriminant analysis (LDA) for two group classification accuracy results for VA are shown in Table 3.5. Bump CWT performed best for two group classification of VA with accuracy of **92%**. Bump CWT required three features for attaining this classification accuracy and these features were REM, EntM and IEntM. Morlet CWT and Paul CWT also performed very well for the two group classification of ventricular arrhythmia. Their classification result were **88%** and both required three features as well for the classification of VA.

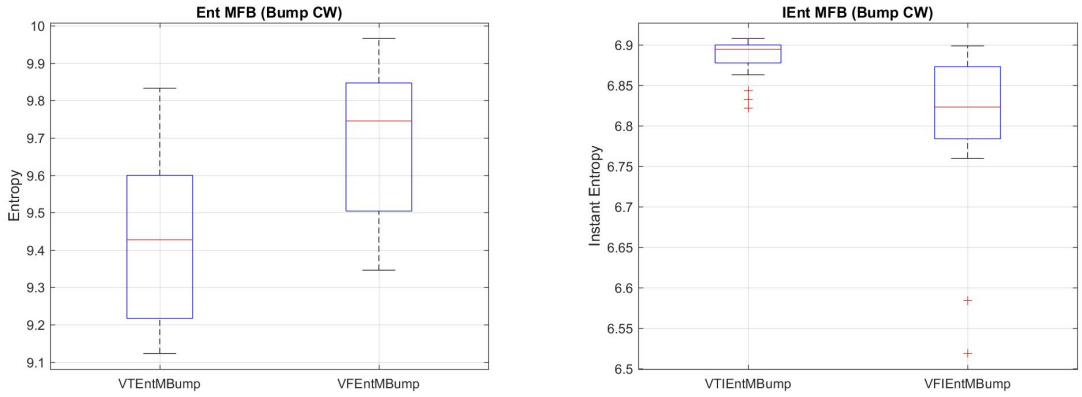
EntM and IEntM provided better classification of VA because VF was a more disorganized sig-

nal compared to VT. A uniform or an organized signal would have a lower value of Ent. However a more random signal would have a higher values of Ent. This was reflected in the two group classification results of VA. RE was a weak feature and provided only a little difference for classification accuracy.

Table 3.5: Two Group Classification Results of VA using Bump CWT.

Method	Group	VT	VF	Total
Cross-Validation LOO Method	VT	**23**	2	25
	VF	2	**23**	25
Percentage of Classification	VT	**92**	8	100
	VF	8	**92**	100

Boxplot was generated for the most dominating feature that provided best classification accuracy for VA. Figure 3.9 shows the boxplot for EntM and IEntM, that provided best classification accuracy for Paul CWT.



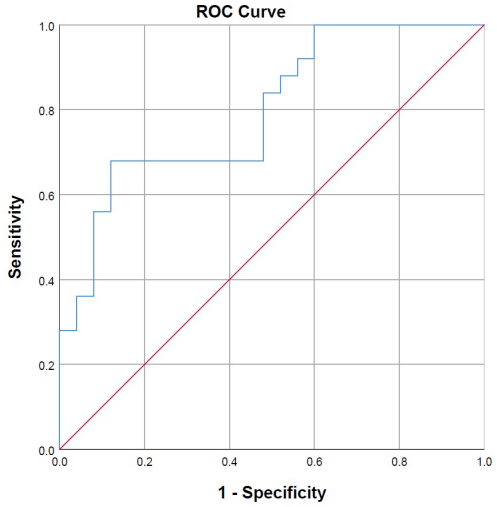
(a) Boxplot for EntM

(b) Boxplot for IEntM

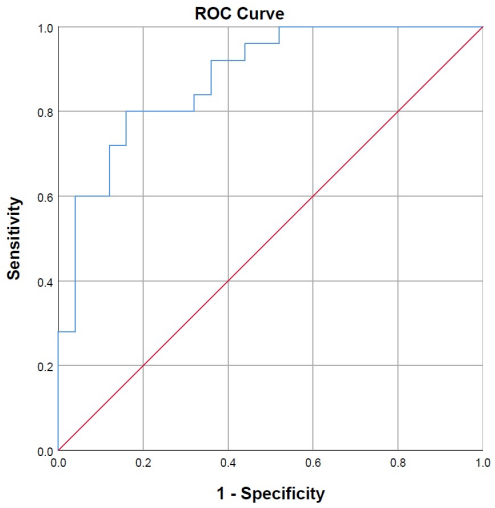
Figure 3.9: Boxplots of dominating features for two group classification of VA using Bump CWT.

3.7.6 ROC Curves For Two Group Classification Features for VA

The ROC curve was obtained for the dominating features that provided best accuracy for the two group classification. ROC curves for the EntM and IEntM are shown in Figure 3.10. The area under the curve for EntM and IEntM were **79.8%** and **88.0%**, respectively. Both features show very strong discriminatory power.



(a) ROC curve for EntM



(b) ROC curve for IEntM

Figure 3.10: ROC Curves for dominant feature for two group Classification of VA using Bump CWT.

3.8 Comparative Results

Our group has been working extensively on the VA for number of years and has been researching for different techniques for the classification of VT and VF. In the past our group members have used different mathematical and signal processing techniques for classification of VA. They have also used CW for their research for the classification of VA. Different feature extraction techniques were used for the classification of VA such as Blind Source Seperation, KSVD and SVD. Results of this study were also comparable with our research group members. There classification results varied anywhere between and **70%** to **95%**. This was the first attempt to classify VA and AA. All four major arrhythmias were classified in this chapter using library wavelets.

For the purpose of comparative analysis two well known features namely dominant frequency (DF) and entropy (Ent) were used. As discussed in Chapter 2, these two features were previously used for arrhythmia classification. Using the same database, classification accuracies were computed for these two features. For four group classification, DF and Ent individually performed poorly and when combined they attained **58%** accuracy.

For the two group classification of AA, DF attained **78%** accuracy and Ent performed poorly. For the two group classification of AA, combined DF and Ent attained the accuracy of **78%**. For

the two group classification of VA, individually, DF attained **68%** accuracy and Ent attained **82%** accuracy. For the two group classification of VA, combined DF and Ent, attained **78%** accuracy and their combination did not improve the maximum classification accuracy achieved by Ent.

As it could be observed from the results that our wavelet based scheme of discriminating arrhythmia outperforms time based or frequency based or combined time and frequency based features.

3.9 Chapter Summary

This chapter introduced the block diagram of flow of work for the classification of four groups of signals namely AF, AFL, VT and VF. This chapter also presented scalogram plots of four different groups of signals for three different CWT. Moreover, it also covered features that were extracted from the wavelet coefficients. LDA results of all four groups and all wavelets were discussed in this chapter. Classification results showed that for four group classification, Paul CWT had the best results among the three wavelets. Classification accuracy for four group was **77%**. LDA results for two groups, VA and AA were also presented in this chapter. For AA, Paul CWT attained best classification accuracy, which was **76%**. For VA, Bump CWT attained best classification accuracy, which was **92%**. Boxplots and ROC curve were generated for strong features that performed best four group and two group classification.

Results of the classification showed some overlap between features for four group and two group classification. It is understandable since the signal frequencies overlap. AFL and AF can overlap each other, therefore, it was very difficult to discriminate AA. VT can lead to VF, and therefore their frequencies can overlap. There were overlapping of frequencies for all four major arrhythmia and for that reason, it proved difficult to classify the four groups of arrhythmias.

Chapter 4

Analysis of Arrhythmia Using Adaptive Wavelets

THIS chapter includes the design of adaptive continuous wavelets (ACW) based on the signal patterns present in VA and AA. This chapter also included the comparison of three different ACW and features extracted from coefficients of adaptive continuous wavelet transform (ACWT) that best classify four groups of arrhythmias and two group arrhythmias. This chapter feature extraction from the ACWT coefficients and four group classification of four major arrhythmias and two group classification of VA and AA.

Adaptive Wavelets were designed based on the signal patterns. Signal patterns were selected for the design of ACW. In total three adaptive wavelets were designed, A-pattern ACW, M-pattern ACW and W-pattern ACW. These ACWs were combination of recurring signals, extracted from arrhythmia.

4.1 Algorithmic Steps

The block diagram in Figure 4.1 shows the steps included for classification of four major arrhythmia using ACW. These algorithmic steps were similar to the algorithmic steps followed in Chapter 3, with a difference that ACW was designed before signal processing. After pre-processing of input signals, signals were discriminated between NSR, AA and VA. If the signal was VA, ACW was designed based on the signal patterns. Signals were transformed, using ACWT, features were extracted and VA signal was discriminated and classified either as VT or VF. If the signal did not belong to VA group, then it was discriminated as NSR or AA. QRS complex or R-peak was

Figure 4.1: Block Diagram of the Adaptive Wavelet Transform with flow of work that will be covered in this chapter.

removed using template matching technique. ACW was designed based on signal patterns. Signals were transformed and using ACWT, features were extracted and AA signal was discriminated and classified either as AF or AFL.

4.2 Recurring ECG Pattern during Arrhythmia

Adaptive wavelets were designed based on the recurring patterns of the arrhythmia signals. Recurring signal patterns can provide information about the signal. Sometimes a wavelet out of the library may not decompose the signal and provide all the necessary information about the signal. ACW can help transform the signal and can provide the information that a library wavelet may not provide. An optimally matched wavelet for the ECG signal can provide useful information about four major types of arrhythmia and provide customized treatment options for ICDs, ICVs

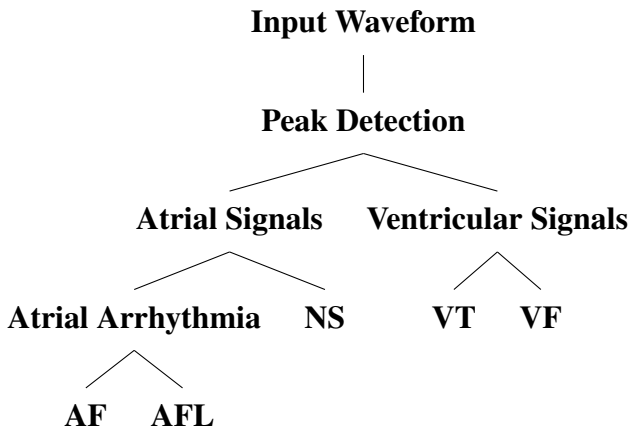


Figure 4.2: Algorithmic Tree: shows the flow of the above Block Diagram of Adaptive Wavelet Transform

and ECVs.

Past literature suggests different methodologies to detect and classify VA. But, there is a lack of studies which explore the recurring morphological patterns of arrhythmia signal that can be fatal [125]. Some of these patterns can be detected and become predictor signals for early warning. Early detection of VT/VF is also very essential for wearable devices for timely delivery of electric shock therapy [126]. Not only essential for timely delivery but also essential for accurately distinguishing between shockable and non-shockable arrhythmia [127].

It was also very critical to differentiate between paroxysmal AF (PAF) and long term AF (LTAF). There were recurring patterns in LTAF which are non existent in PAF [128]. It was also important to differentiate between PAF and LTAF because LTAF requires treatment however PAF may pass without symptoms and may not require treatment. AF morphological patterns can also provide information about the source of AA [129]. This can help target the shock therapy rather than providing a blind shock for any arrhythmia detected. AF mapping was employed to help determine potentially effective targets [130]. AF has highly complex wave dynamics, which makes it difficult to understand underlying frequencies of AF. AF mapping could resolve the challenging problem of AF faced by many cardiologists.

The search for these patterns was unbiased and there was no previous knowledge of the physiological condition. Extra care was used to find similar patterns that were present in VA and AA. These recurring patterns were used in designing the adaptive wavelets. In total there were 3 different

complex recurring patterns emerging from the arrhythmia signals. These patterns were A-pattern signal, M-pattern signal and W-pattern signal.

Figure 4.3 - Figure 4.5 shows three different recurring patterns which were used for designing four different ACW. Figure 4.3 was the signal pattern for A-pattern signal, Figure 4.4 was the signal pattern for M-pattern, Figure 4.5 was the signal pattern for W-pattern, and all three patterns are localized wave patterns. There might be other patterns which exist in ECG signals but were not recognized. These above mentioned patterns also coexist with each other in the same signal.

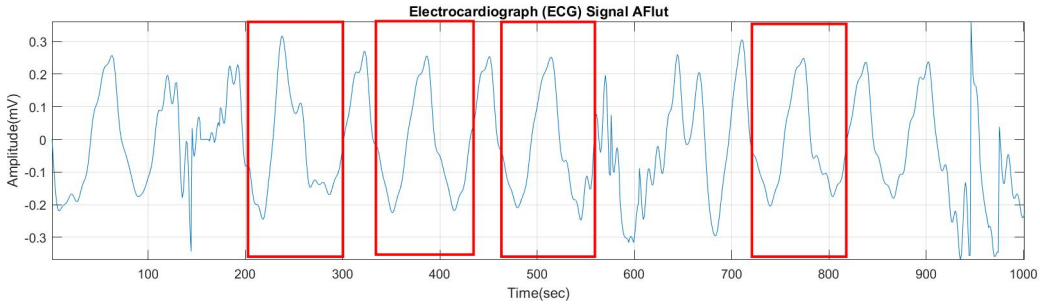


Figure 4.3: Recurring A-patterns in Arrhythmia.

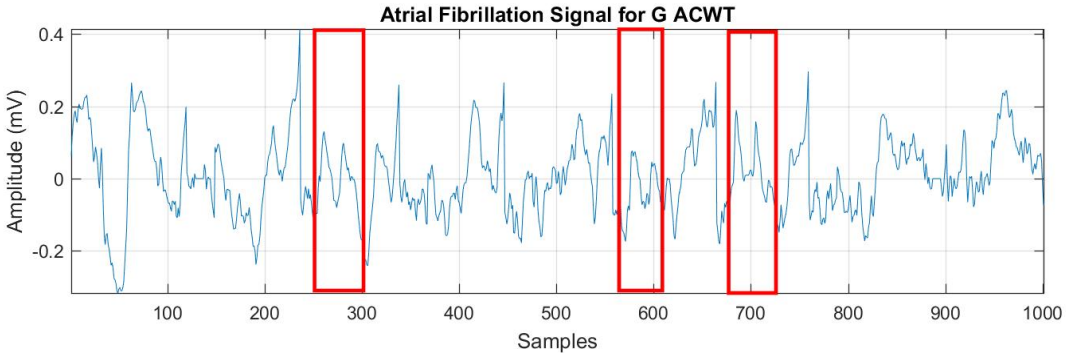


Figure 4.4: Recurring M-patterns in Arrhythmias.

It was very important to understand the relevance for selecting these following patterns. The M-pattern was an indicative of a conduction block in the heart [131]. Sinoatrial (SA) block is a conduction disorder in which impulses generated in the sinus node are intermittently conducted or not conducted to the atrial myocardium. Atrioventrical (AV) conduction block is divided into first-degree, second-degree and third degree block [133]. Third degree block is most dangerous and characterized by no conduction between the atrium and ventricles. Proper detection of AV conduc-

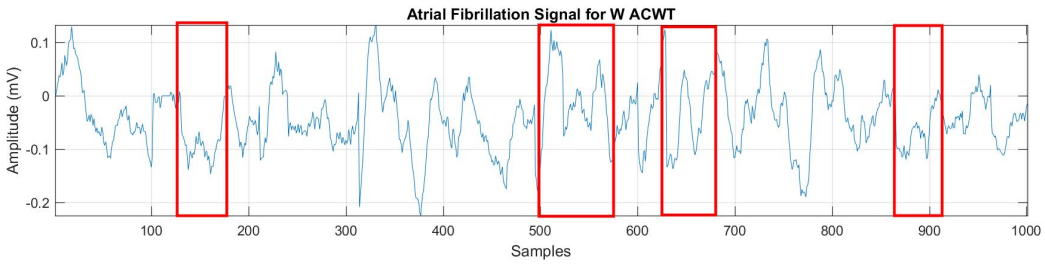


Figure 4.5: Recurring W-patterns in Arrhythmias.

tion is very important because these patients require emergency treatment. Patients undergoing heart surgery are at risk of immediate or delayed AV conduction block [134].

The W-pattern was inverted M-pattern, which was also a recurring pattern in ECG. A-pattern was the resemblance of re-entry electrical pulse within atria or monomorphic VT (MVT) [132]. MVT signal resembles NSR but without the P-wave and T-wave with heart rate higher than 100 beats per minute [135].

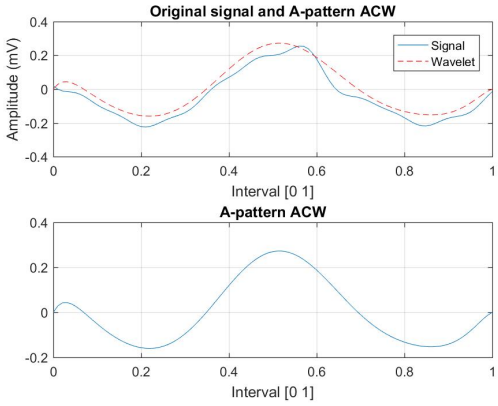
4.3 Design of Adaptive Wavelet

Design of the pattern based ACW was generated using least square fitting method. The method used was the Least square fitting, using a polynomial with an interval of [0 1]. This wavelet also satisfies the definition of wavelet because it integrates to zero and that its L_2 norm was equal to 1, as discussed in Chapter 2.

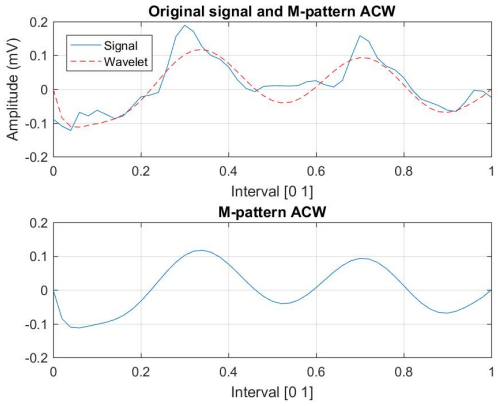
ACW generated can be used for the ACWT and analysis of the ECG signal. Three ACW which were generated for the analysis of VA and AA are shown in Figure 4.6. It shows the pattern which was used to design the ACW. These adaptive wavelets were used for the analysis of ECG signal, feature extraction and classification of four groups of arrhythmias.

4.4 Adaptive Wavelet Transform of Arrhythmia

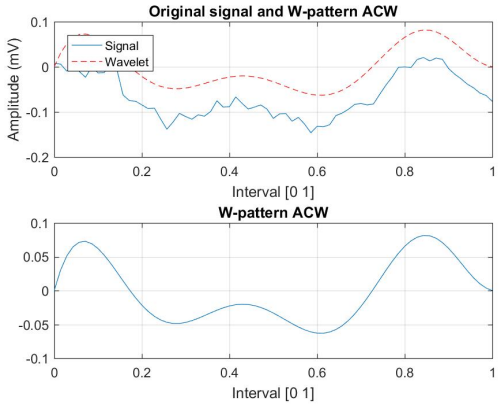
Information related to QRST cancellation has been shared in Chapter 3. Template matching QRST cancellation has been used for analysis of AA; please refer to Chapter 3 for more information. Similar to the CWT analysis of four arrhythmia, this chapter performed the analysis of all four arrhythmias using three pattern based ACWT. Visual analysis of four arrhythmias with the help of



(a) A-pattern ACW



(b) M-pattern ACW



(c) W-pattern ACW

Figure 4.6: Three ACW based on the patterns extracted from different Arrhythmia signals.

scalogram, using features similar to ones referred in Chapter 3 will be discussed. Scalogram of the three ACWT are shown in APPENDIX B. Finally classification of arrhythmia using LDA similar to what was discussed in Chapter 3.

We will compare all three ACWT classification accuracy of four major arrhythmia with the help of the extracted features from ACWT coefficients. Features used for classification of all four major arrhythmia were the same as discussed in Chapter 2 and Chapter 3. Features used for classification were RE, Ent, IEnt, Mean and STD. Fisher's linear discriminant analysis was used as a classifier and Leave-one-out method for cross-validation.

4.5 Linear Discriminant Analysis

As discussed in Chapter 2, Fisher's linear discriminant analysis based classifier was used for the classification of all four major types of arrhythmia. Cross-validation was performed using LOO method. The final classification accuracy results were the results of LOO cross-validation classification.

4.5.1 Linear Discriminant Analysis For Four Groups

Four group classification results are shown in Table 4.1. The best result attained for the four group classification was using A-pattern ACWT. The classification accuracy of **81%** was attained using Five features. Features that provided best classification accuracy for A-pattern ACWT were EntM, EntL, IEntM, StdM, and StdL. Results for M-pattern ACWT and W-pattern ACWT for the four groups classification were **72%** and **69%** respectively. For classification accuracy M-pattern ACWT and W-pattern ACWT required five and four features respectively.

The result achieved by the pattern-based ACWT improved upon that obtained with CW from library, with the help of fewer features. Surprisingly, A-pattern ACWT did not require energy based feature for classification, and only used MFB and LFB. Similar to CWT result, A-pattern ACWT also used Ent, IEnt and Std, due to the fact VT and AFL are more organized compared to VF and AF. StdL and StdM were strong features for A-pattern ACWT for four group classification. Since, standard deviation is a measure of variation of values, it captured variation in energy of the arrhythmias. There is more variation of energy in AF and VF as compared to AFL and VT. Ent values are lower for VT and AFL, which are more organized. On the other hand, VF and AF are disorganized or unpredictable, Ent values should be higher. Std, Ent and IEnt combined provided good classification results.

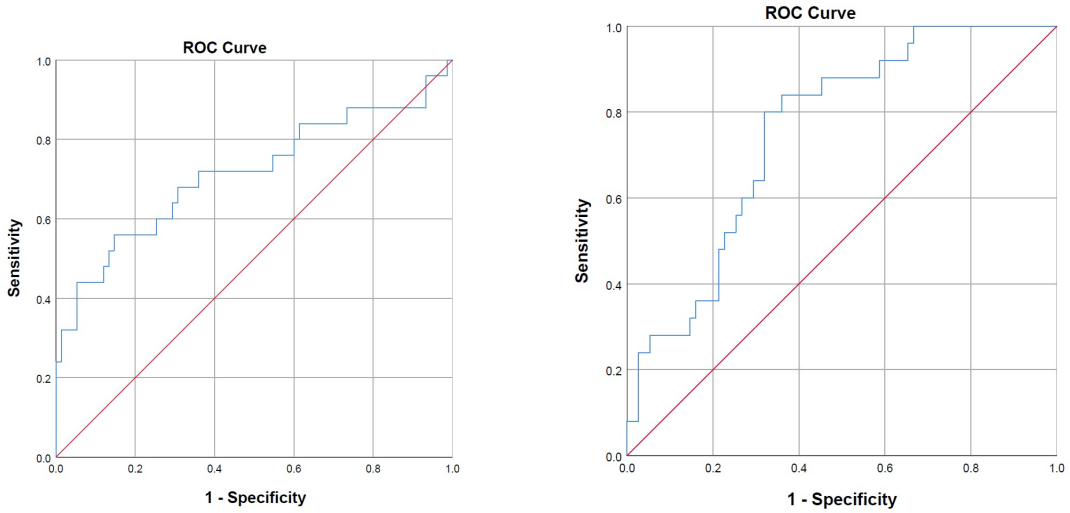
4.5.2 ROC Curves For Four Group Classification Features

The ROC curve for two dominating features for A-pattern ACWT were also obtained for the four group classification. ROC curve for the these features are shown in Figure 4.7.

The area under the ROC curve for StdM and StdL was **75.4%** and **71.4%**, respectively. The combined features from A-pattern ACWT showed promising discriminatory power.

Table 4.1: Four Group Classification Results for ACWT using A-pattern ACWT.

Method	Group	AF	AFL	VT	VF	Total
Cross-Validation with LOO Method	AF	**20**	5	0	0	25
	AFL	4	**20**	0	1	25
	VT	2	0	**23**	0	25
	VF	2	3	2	**18**	25
Percentage of Classification	AF	**80**	20	0	0	100
	AFL	16	**80**	0	4	100
	VT	8	0	**92**	0	100
	VF	8	12	8	**72**	100



(a) ROC curve for StdL

(b) Roc curve for StdM

Figure 4.7: ROC Curves for dominating features for four group classification using A-pattern ACWT.

4.5.3 Linear Discriminant Analysis For Two Group For AA

Two group classification accuracy results shown Table 4.2. For the two group classification of AA, M-pattern ACWT attained best classification accuracy. Classification accuracy for M-pattern ACWT for two group classification was **86%** using only three features. Features that provided best classification accuracy for the discrimination of AF and AFL for M-pattern ACWT were EntH, IEntH and StdL. Two group classification accuracy for A-pattern ACWT and W-pattern ACWT was **78%** for both ACWT. A-pattern ACWT and W-pattern ACWT required two features for classification accuracy.

Table 4.2: Two Group Classification Results for AA using M-pattern ACWT.

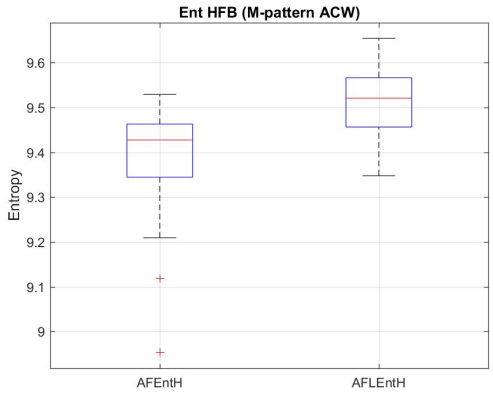
Method	Group	AF	AFL	Total
Cross-Validation LOO Method	AF	**20**	5	25
	AFL	2	**23**	25
Percentage of Classification	AF	**80**	20	100
	AFL	8	**92**	100

Boxplot was generated for the dominating features that provided best classification accuracy for two group arrhythmia classification. Figure 4.8 shows the box plot for EntH and IEntH, which were the dominating features and provided the best results for M-pattern ACWT. It is evident from the boxplot, that the feature demonstrated overlapping of AF and AFL, however the combined features attained strong classification accuracy.

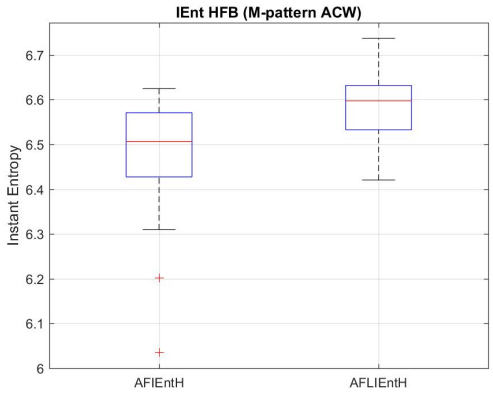
4.5.4 ROC Curves For Two Group Classification Features for AA

The ROC curve was obtained for the dominating feature that provided the best accuracy for the two group classification. Figure 4.9 shows ROC curves EntH and IEntH for M-pattern ACWT.

Area under the curve for EntH and IEntH was found to be **82.4%** and **77.1%**. Both the curves were above diagonal line demonstrating that the feature had good discriminatory power. It was observed that the combination of weak feature and strong feature provided **86%** classification accuracy.

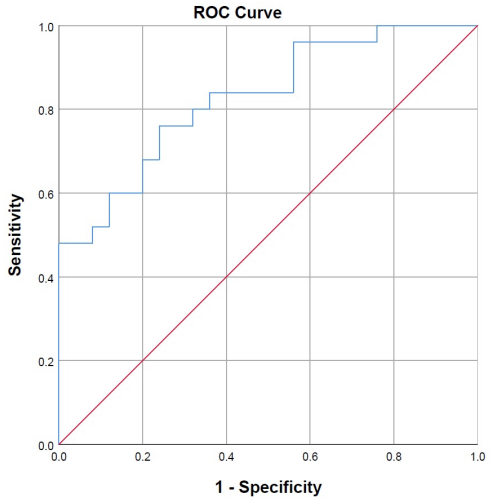


(a) Boxplot for EntH

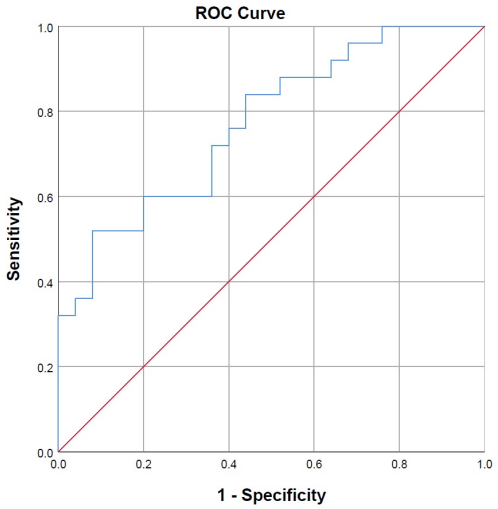


(b) Boxplot for IEntH

Figure 4.8: Boxplots of dominating features for two group classification of AA using M-pattern ACWT.



(a) ROC curve for EntH



(b) ROC curve for IEntH

Figure 4.9: ROC Curves for dominant feature for two group Classification for AA using M-pattern ACWT.

4.5.5 Linear Discriminant Analysis For Two Group For VA

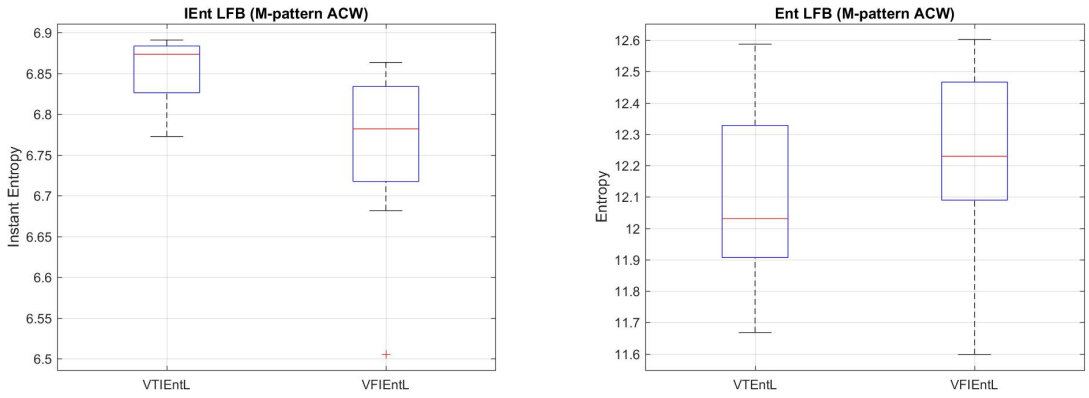
Two group classification accuracy results for VA are shown in Table 4.3. M-pattern ACWT performed best for two group classification of VA and attained the accuracy of **94%**. M-pattern ACWT required four features for Classification of VA. Features that performed best were EntL, IEntL,

MeanM, and StdL.

Table 4.3: Two Group Classification Results for VA using M-pattern ACWT.

Method	Group	VT	VF	Total
Cross-Validation LOO Method	VT	**23**	2	25
	VF	1	**24**	25
Percentage of Classification	VT	**92**	8	100
	VF	4	**96**	100

Boxplots were also generated for the dominant features that provided best classification accuracy for VA. Figure 4.10 shows the boxplots for IEntL and EntL, the dominating feature, combined with other features provided best accuracy results for M-pattern ACWT for the two group classification of VA.



(a) Boxplot for IEntL

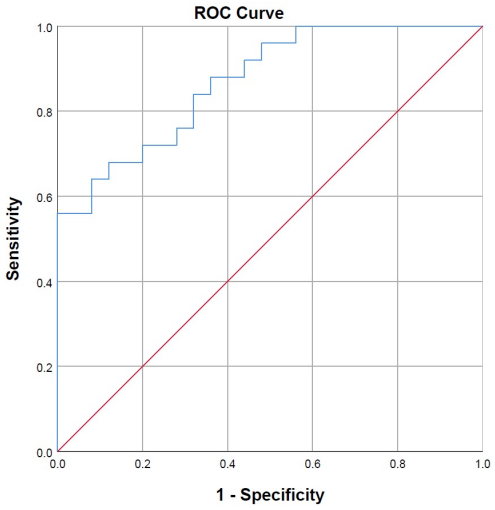
(b) Boxplot for EntL

Figure 4.10: Boxplots of dominant features for two Group classification for VA using M-pattern ACWT.

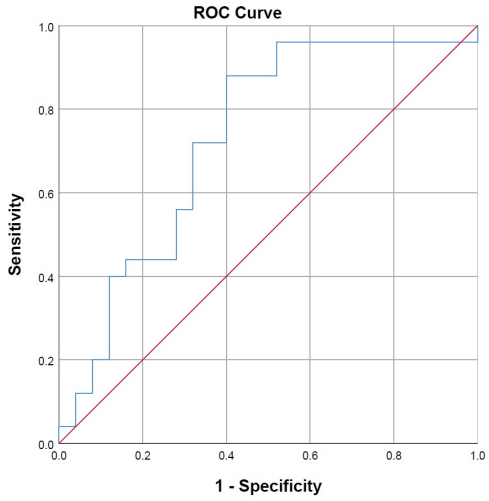
For two group classification of VA results from A-pattern ACWT and W-pattern ACWT were **90%** and **84%** respectively. Both the ACWT required two feature for the classification of VA.

4.5.6 ROC Curves For Two Group Classification Features for VA

The ROC curve for the dominant feature was obtained that provided the best accuracy for the two group classification of VA. ROC curves for the IEntL and EntL are shown in Figure 4.11.



(a) ROC curve for IEntL



(b) ROC curve for EntL

Figure 4.11: ROC Curves for the dominant features for two group Classification of VA using M-pattern ACWT.

The area under the curve for the IEntL and EntL was **87%** and **74%**. ROC of the combined features demonstrated very strong discriminatory power.

4.6 Chapter Summary

This chapter introduced the flow of work for the classification of four groups of signals namely AF, AFL, VT and VF in the form of block diagram. Some of the basic differences of wavelets were also discussed in this chapter. It also covered different energy, information theory and statistical features that were extracted from the wavelet coefficients. For four group classification, A-pattern ACWT attained best classification accuracy of **81%**. M-pattern ACWT attained best classification for both AA and VA. M-pattern ACWT attained best classification accuracy of **86%** and **94%** for AA and VA, respectively.

Comparison between the results of ACWT and CWT for four group major arrhythmia classification, and two group arrhythmia classification showed that ACWT performed better than library CWT in all the three categories. Although there was not much of a difference in the results for four group classification and VA, but the difference in results for AA was considerable.

Chapter 5

Conclusion and Future Work

5.1 Discussion

HEART is an organ, size of a human fist and its major responsibility is to pump blood to the entire body. Any abnormalities to rhythmic contracting of the heart is known as arrhythmia. There are four major types of arrhythmia that require attention, especially VF and VT, if not treated swiftly can lead to deadly condition like SCD. AF and AFL also need to be treated promptly, and if not treated can lead to VT and VF. AA is also a source of embolic strokes. Survivors of cardiac arrest or arrhythmia patients face lifestyle changes and require long term treatment. Treatment options are anti-arrhythmic drugs, controlled pacing of the heart, defibrillators and ablation of the underlying tissue substrate.

ICDs are useful in preventing SCA in patients with known VT and VF. Studies in the past have confirmed that patients who receive ICD have deteriorating heart condition because of inappropriate shock treatment by ICDs. Inappropriate shocks in patients causes more trips to the hospitals or clinics, extra strain on healthcare system and poor quality of life. Patients with ICD develop AA and it requires further treatment to control heart rate. Treatment for AA includes medication, pacemaker implantation with rate control therapy, cardioversion and some patients require combination of medication and pacemaker. External or internal EVC are used by patients suffering from AA for heart rate control.

It is a major task of the medical professionals to provide proper treatment to patients with known cardiac diseases. Another major task of medical professionals is to monitor the cardiac health of patients. Holter is used for long-term recording of ECG, which provides detailed information about the condition of heart. Holters are used to monitor heartbeat and records ECG of high risk

postinfarction patients, patients on antiarrhythmic medication and patients who had inconclusive results of traditional ECG test. These ECG readings are one day or two days long. Analysis of long ECG recordings from the patient's holter is also a very challenging task. Analysis of these readings, especially discriminating and classification of four major arrhythmias is time consuming and tedious.

A major challenge is the difficulty in discriminating four major types of arrhythmia. The proposed study introduced automated methods of discriminating four major arrhythmia using surface ECG. Different wavelets from a library and pattern based adaptive wavelet were explored for the classification of four major types of arrhythmia. Being able to more accurately and dependably classify between the four major types of arrhythmias can help improve health outcomes for patients with ICDs or EVCs, and select appropriate treatment options to improve survival rates among VA sufferers and better quality of life for AA sufferers.

In this work, two different categories of wavelets were explored for classification of four major types of arrhythmia. First category was library based CW and second category was pattern based ACW. These two categories of wavelet show slight difference for classification accuracy of four major arrhythmia. ACW performed better than CW for the classification of AA and VA. A major reason for better performance could be use of recurring patterns represented in the arrhythmia signals.

5.1.1 Results

Three different categories of features were selected for discrimination of four arrhythmia. These categories were energy based, information based and statistics based. Features used were relative energy, Shannon entropy, instant Shannon entropy, mean and standard deviation. These select features with the help of LDA provided high classification accuracy. From the library wavelets, Paul CWT attained best classification accuracy of **77%** for four major group of arrhythmia. Paul CWT also attained best classification accuracy of **76%** for AA and Bump CWT attained best classification accuracy of **92%** for VA.

In the pattern based ACW category, A-pattern ACWT attained best classification accuracy of **81%** for four major group of arrhythmia. M-pattern ACWT attained classification accuracy of **86%** and **94%** for AA and VA, respectively.

For the four group classification, Paul CW and A-pattern ACW attained the highest accuracy. Both the wavelets have a very similar structure which is a very closet representation of AFL and VT. It is also a very close representation of oscillations and signal structure of AF and VF. Paul CW also attained highest classification accuracy for AA. Although Paul CW did not attain highest accuracy for the classification of VA but performed very well and attained **88%** classification accuracy. Similarly, A-pattern ACW also performed well for the classification accuracy of AA and VA. A-pattern ACW attained classification accuracy of **78%** and **90%** for AA and VA respectively.

5.1.2 Application

An automated system for differentiating and classifying four major arrhythmias can improve the treatment options. Different CW or ACW which were discussed in this thesis can improve the treatment methods for the patients by accurately discriminating the type and severity of arrhythmia. Better treatment options can be designed specifically for each arrhythmia. This will also reduce the number of trips to hospitals and health clinics and improve quality of life.

Automated system of differentiating and classifying four major arrhythmia can help medical community to simplify the strenuous task of analyzing long ECG recordings. An automated system can greatly simplify this task of analyzing data recorded by Holter or any device that records long ECG information.

5.1.3 Future Work

Discrimination of four major types of arrhythmia is a challenging task. More research in different signal patterns can provide further information about activity of heart and ACW can help recognize these patterns. Since the ACW is pattern based, it can be designed to predict the arrhythmia based on the type of pattern detected. Recognized patterns can warn the devices of heart activity or alarm the patients and help them change their activity or mood. Automated arrhythmia detection can help medical professionals in more than one way. Their tasks can be simplified and health of the patients can be improved.

Appendix A

Fisher's Linear Discriminant Analysis

Fisher's linear discriminant function can be seen as projecting the d dimensional samples x onto a corresponding set of samples y using weight vector matrix w. A generalized matrix W can be seen as matrix that incorporates all the class' w_i weigh vector with a dimension of $d \times c - 1$. Equation 2.38 [117] shows the projection of x onto y with weight vector W^t.

$$y = W^t x \tag{A.1}$$

where W can be seen as a *generalized matrix* equation containing all class' *weight vector* w_i and whose *dimensions* are $d \times c - 1$. Weight vector W can be obtained from the criterion function $J(W)$ such that,

$$J(W) = \frac{|W^t S_B W|}{|W^t S_W W|} \tag{A.2}$$

Where S_B indicates the *between class scatter matrix* and S_W indicates *within class scatter matrix*. Scatter matrices provide information about the distribution of the classes themselves and the distribution of samples within each class respectively. Within class scatter matrix S_W and S_i can also be written as,

$$S_i = \sum_{i=1}^{c} \sum_{x \epsilon D_i} n_i (x - m_i)(x - m_i)^t \tag{A.3}$$

where n_i is the number of samples and m_i is the mean for each class i, and

$$S_W = S_1 + S_2 \tag{A.4}$$

Between class scatter matrix S_B can be written as,

$$S_B = (m_1 - m_2)(m_1 - m_2)^t \tag{A.5}$$

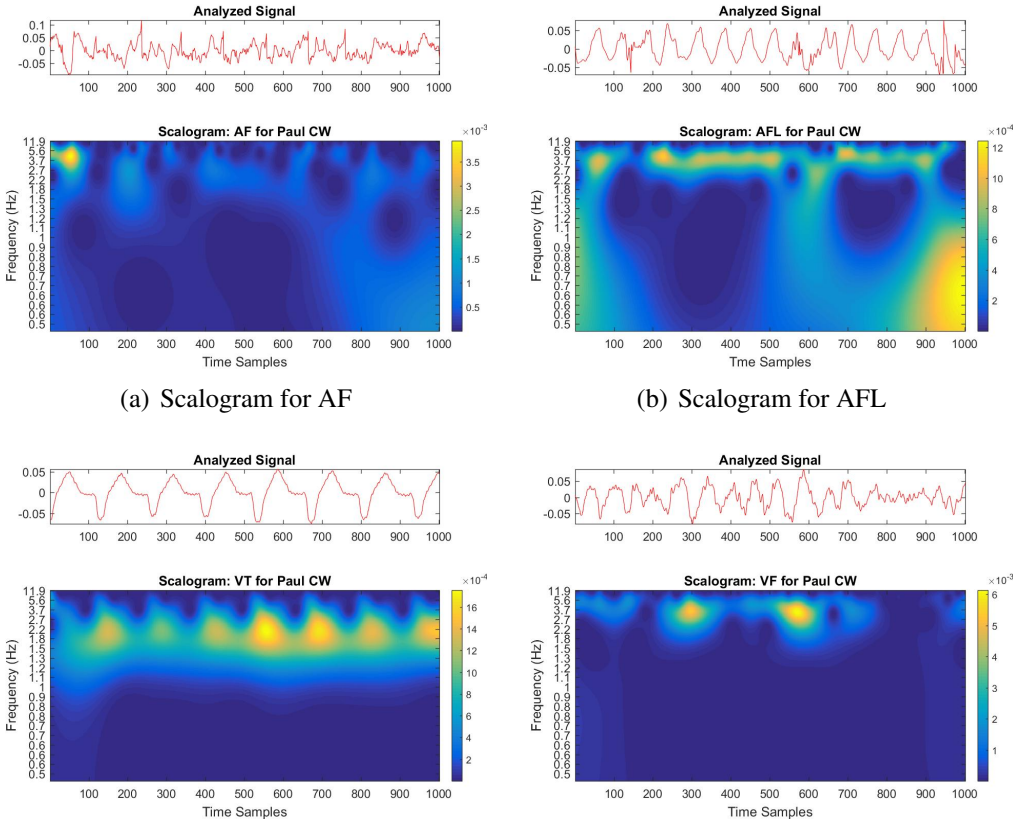
Within class scatter matrix S_w is the summation of the variance from each class and the optimal weight vectors is calculated by maximizing criterion function $J(W)$. Class separation increases and/or the variance of each group decreases by maximizing the criterion function. The weight vectors are calculated to provide the best projection of x onto y therefore forms the basis of the linear boundaries.

The classifier uses supervised learning, which means the data is pre-qualified. In obtaining the weight vectors W, the classifier is trained with pre-classified set of training samples. These training samples create an optimal boundary for the given training set.

Appendix B

Scalograms

B.1 Scalograms of Paul wavelet for four different arrhythmias



(a) Scalogram for AF

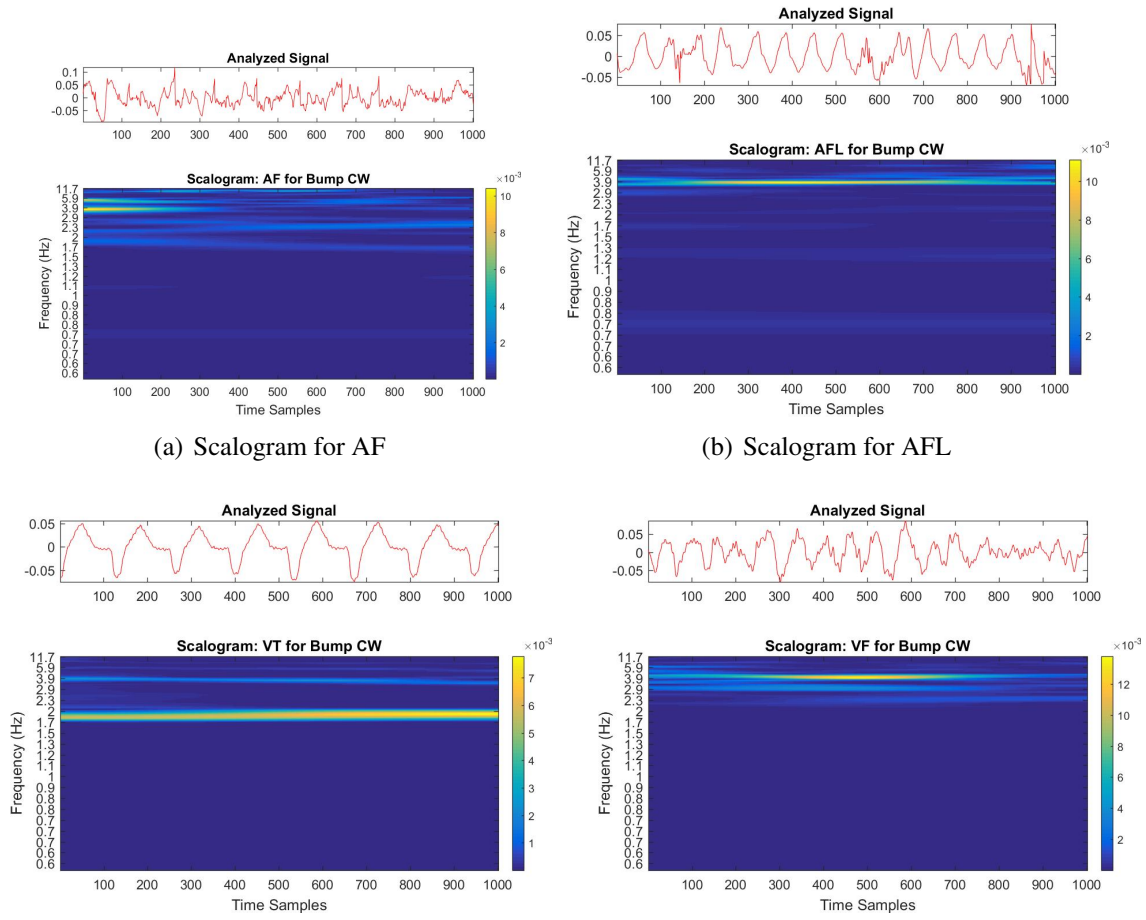
(b) Scalogram for AFL

(c) Scalogram for VT

(d) Scalogram of VF

Figure B.1: Scalograms of Paul wavelet for four different arrhythmias. These scalograms show frequency on Y-axis and time samples on X-axis. Figure B.1(a) is the Scalogram of AF, Figure B.1(b) is the Scalogram of AFL, Figure B.1(c) is the Scalogram of VT, Figure B.1(d) is the Scalogram of VF. The signal of each arrhythmia is shown on top of each scalogram respectively.

B.2 Scalograms of Bump wavelet for four different arrhythmias



(a) Scalogram for AF

(b) Scalogram for AFL

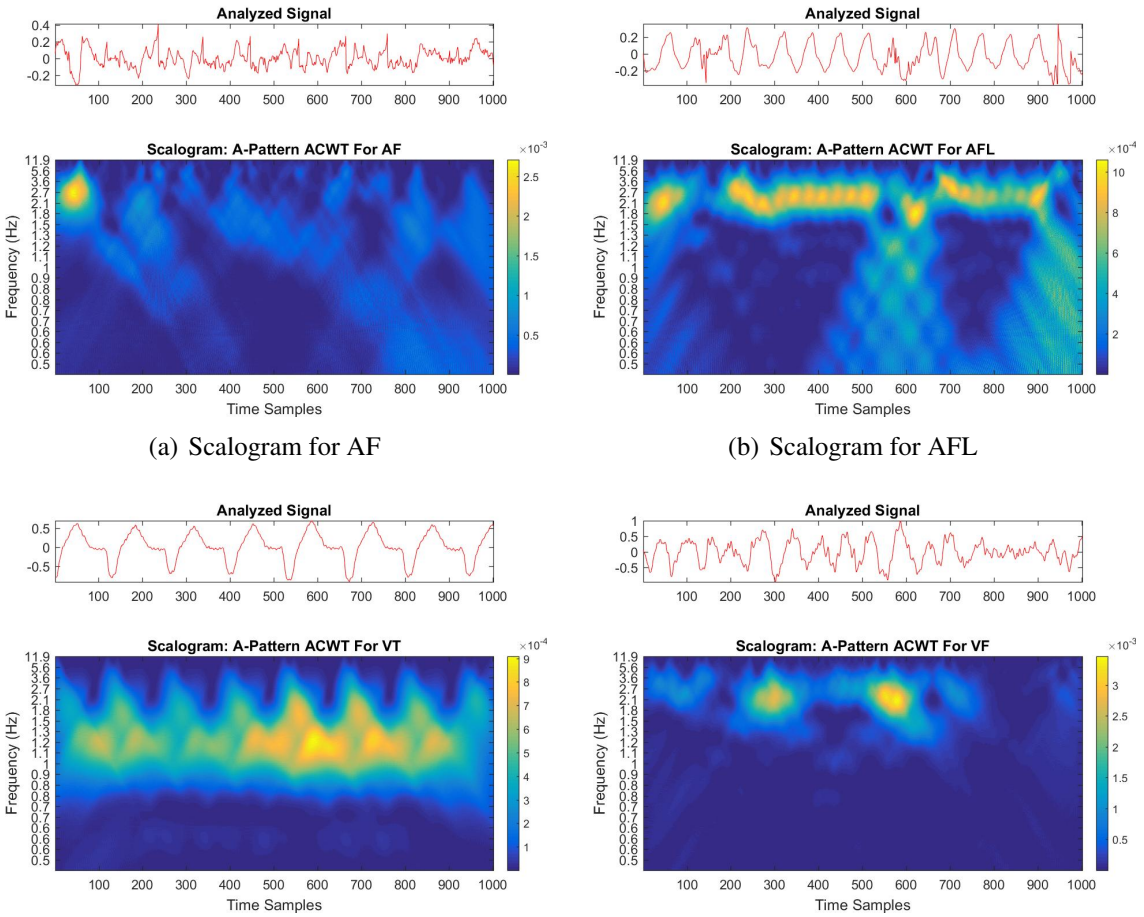
(c) Scalogram for VT

(d) Scalogram of VF

Figure B.2: Scalograms of Bump wavelet for four different arrhythmias. These scalogram show frequency on Y-axis and time samples on X-axis. Figure B.2(a) is the Scalogram of AF, Figure B.2(b) is the Scalogram of AFL, Figure B.2(c) is the Scalogram of VT, Figure B.2(d) is the Scalogram of VF. The signal of each arrhythmia is shown on top of each scalogram respectively.

Scalogram of four major arrhythmia for A-pattern ACWT, M-pattern ACWT and W-pattern ACWT are shown in Figure B.3, Figure B.4 and Figure B.5. Scalograms of different wavelets can help us to visually analyze and discriminate the different types of arrhythmia.

B.3 Scalograms of A-pattern ACWT for four different arrhythmias



(a) Scalogram for AF

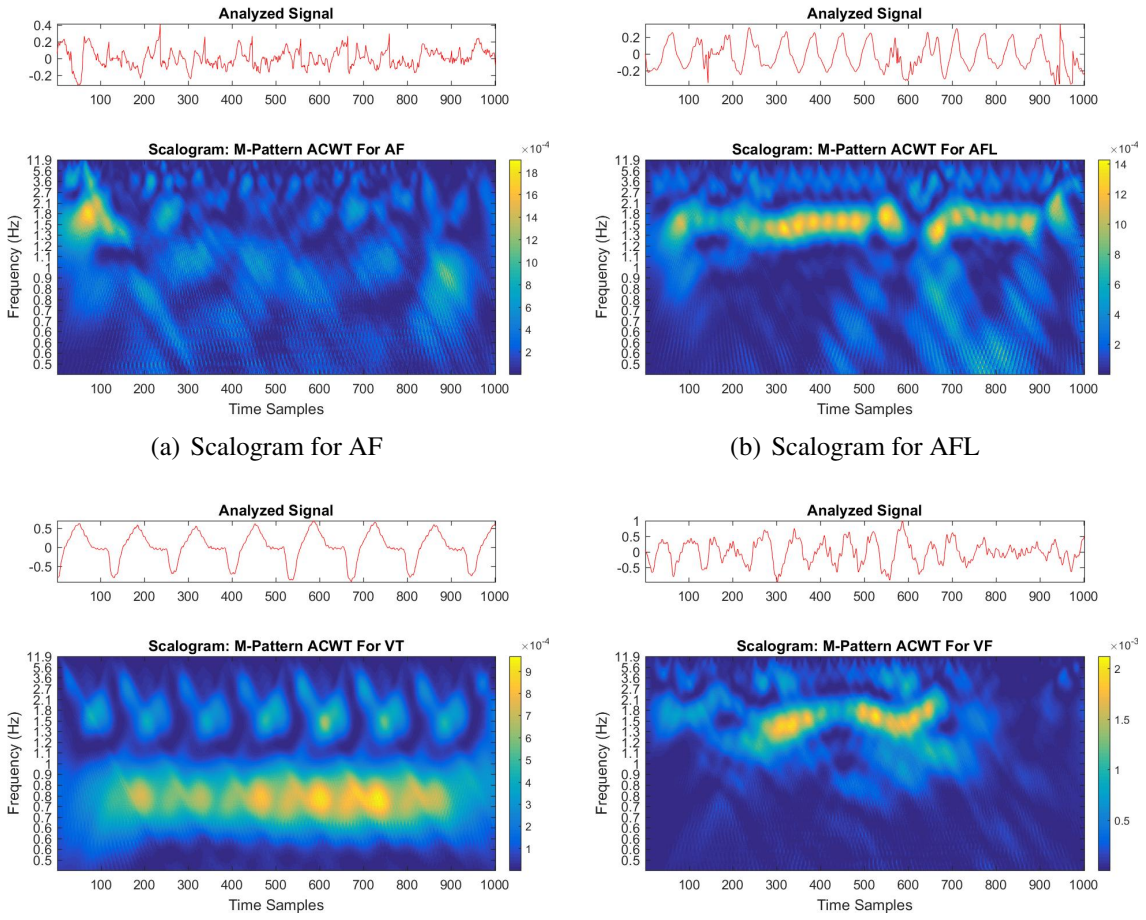
(b) Scalogram for AFL

(c) Scalogram for VT

(d) Scalogram of VF

Figure B.3: Scalogram of A-pattern ACWT for four different arrhythmias. These scalogram show frequency on Y-axis and time samples on X-axis. Figure B.3(a) is the Scalogram of AF, Figure B.3(b) is the Scalogram of AFL, Figure B.3(c) is the Scalogram of VT, Figure B.3(d) is the Scalogram of VF. The signal of each arrhythmia is shown on top of each scalogram respectively.

B.4 Scalograms of M-pattern ACWT for four different arrhythmias



(a) Scalogram for AF

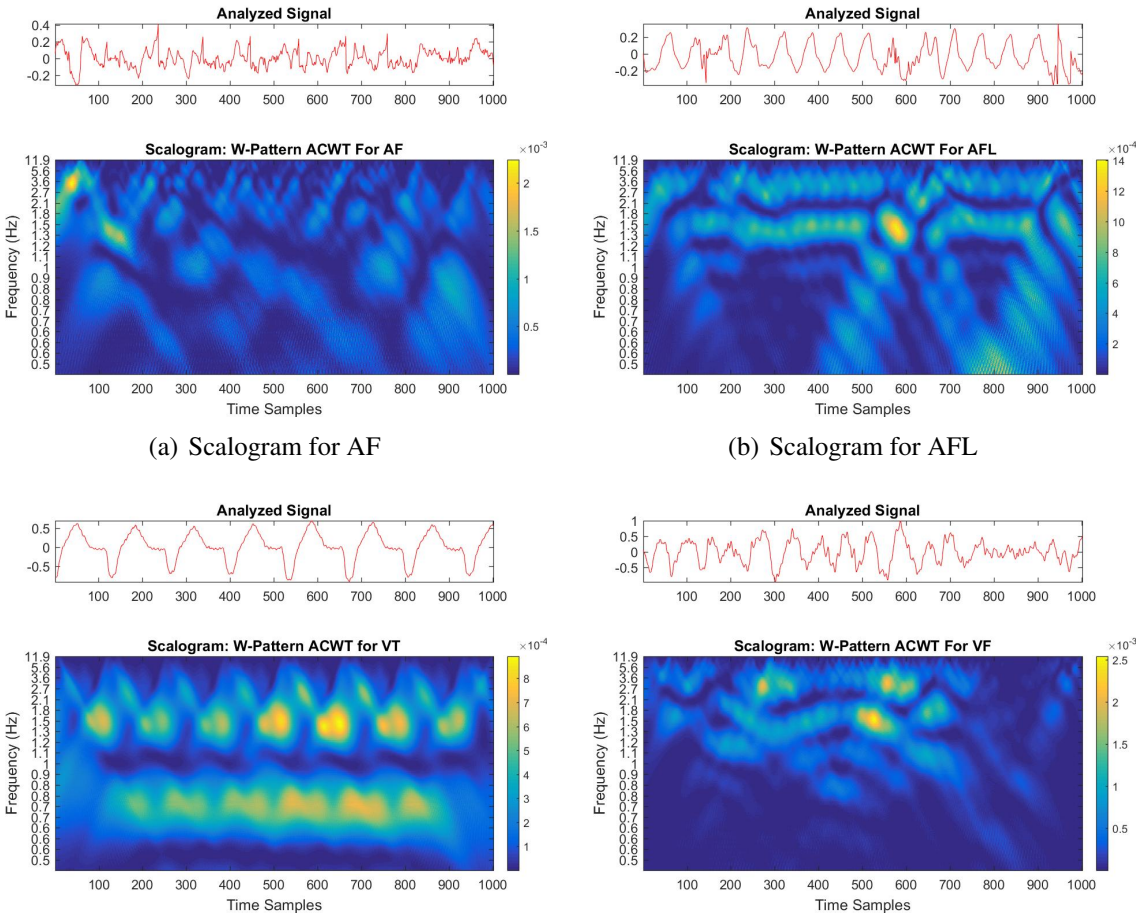
(b) Scalogram for AFL

(c) Scalogram for VT

(d) Scalogram of VF

Figure B.4: Scalogram of M-pattern ACWT for four different arrhythmias. These scalograms show frequency on Y-axis and time samples on X-axis. Figure B.4(a) is the Scalogram of AF, Figure B.4(b) is the Scalogram of AFL, Figure B.4(c) is the Scalogram of VT, Figure B.4(d) is the Scalogram of VF. The signal of each arrhythmia is shown on top of each scalogram respectively.

B.5 Scalograms of W-pattern ACWT for four different arrhythmias



(a) Scalogram for AF

(b) Scalogram for AFL

(c) Scalogram for VT

(d) Scalogram of VF

Figure B.5: Scalogram of W-pattern ACWT for four different arrhythmias. These scalograms show frequency on Y-axis and time samples on X-axis. Figure B.5(a) is the Scalogram of AF, Figure B.5(b) is the Scalogram of AFL, Figure B.5(c) is the Scalogram of VT, Figure B.5(d) is the Scalogram of VF. The signal of each arrhythmia is shown on top of each scalogram respectively.

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