Wavelet

Wavelet can be defined as a wave-like oscillation with an amplitude, starting at zero, increases and decreases and finally reaches zero [1]. Wavelet is a small wave or a mathematical function, which is used for analysis of a signal by scaling and translation (shifting) across the signal. Wavelet can also be defined as a building block that represents a 1-dimensional or multidimensional signal into a 2-dimensional expansion set. Wavelet expansion gives a time-scale representation of a given signal, showcasing its energy distribution over both time and frequency/scale. There exist many wavelets known as mother wavelets (MW). Some of the MW are Morlet wavelet, Daubechies wavelet, Haar wavelet, Symlet wavelet and Coiflet wavelet, to name a few. Figure 1 shows Morlet wavelet.

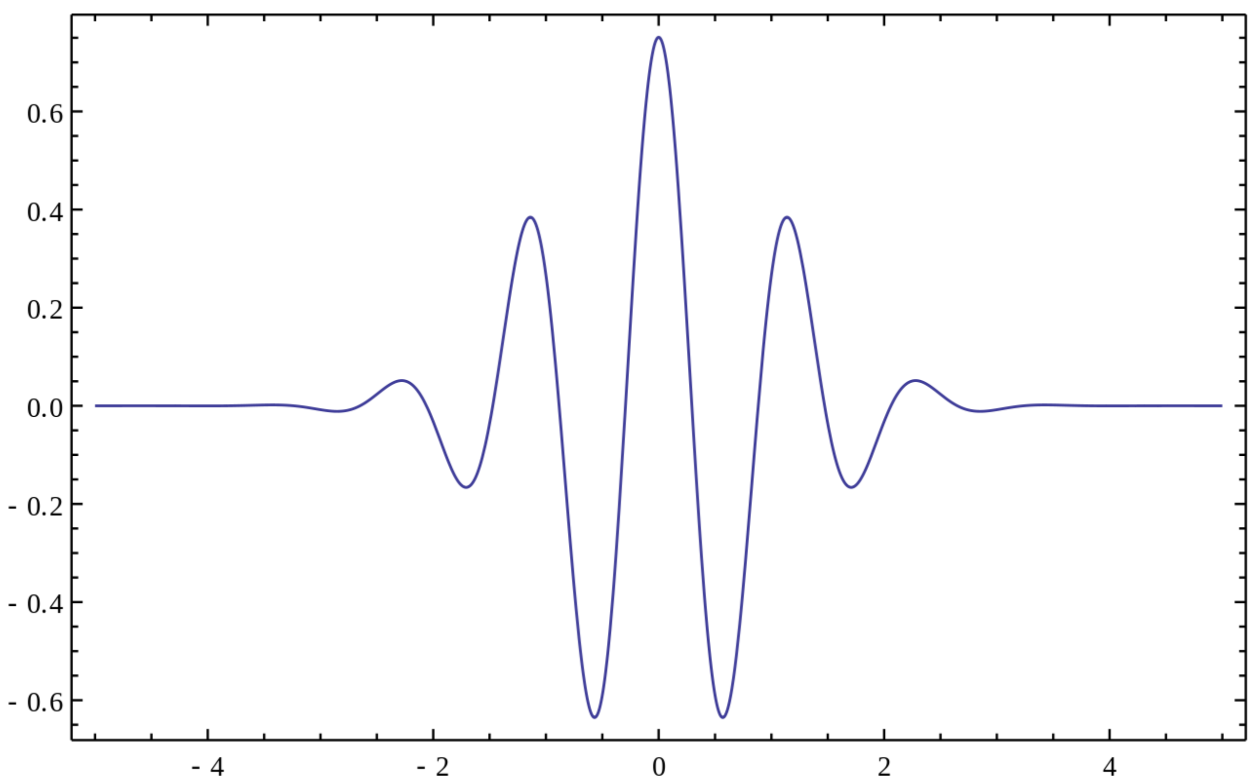

Figure 1: Morlet wavelet. Source:en.wikipedia.org/wiki/Morlet_wavelet.

Wavelets are used when temporal resolution is of importance. Time series analysis signal analysis provides temporal information, frequency signal analysis provides frequency information, but fails to provide time stamp. However, wavelet transformation can provide time and frequence/scale information of the signal [2].

$$CWT_{s,a} = Wf(s,a) = \frac{1}{\sqrt{s}} \sum_{n=1}^{N} f(n)\psi(\frac{n-a}{s})$$

Where $CWT_{s,a}$ and $Wf(s,a)$ represents the coefficients of the CWT, ***s*** represents the scale or frequency, ***a*** represents translation and ψ represents mother wavelet.

Wavelet Conditions

Generally speaking, wavelet represents a waveform effectively of a limited duration, which is oscillatory in nature, has an amplitude that starts at zero, increases and decays quickly back to zero. Wavelet needs to satisfy some basic conditions in order for it to be qualified as a wavelet. These conditions are:

1. Wavelet function should have a zero mean value,

$$\overline{\psi}(n) = \frac{1}{N}\sum \psi(n) = 0$$

   Where $\overline{\psi}(n)$ is the mean value of wavelet basis function $\psi(n)$.

2. Wavelet has unit energy,

$$E = \sum_{n} |\psi(n)|^2 = 1$$

   Where E is the signal energy.

3. It should satisfy admissibility criterion,

$$C_{\psi} = \sum f(n)\left(\frac{\widehat{\psi}(\frac{2\pi kn}{N})|^2}{|\frac{2\pi kn}{N}|}\right) \leq \infty$$

Where $\frac{2\pi kn}{N} = \omega$ and $\widehat{\psi}(\omega)$ is the Fourier Transform of $\psi(n)$.

Difference between Discrete Wavelets Transform (DWT) and Continuous Wavelets Transform (CWT)

CWT is a tool that provides complete representation of a signal by letting the translation and scale parameter of the wavelet vary continuously. CWT is a convolution of the input data sequence with a set of function generated by MW. The convolution can be computed by using a fast Fourier transform (FFT) algorithm. Applications include feature extraction of signals, acoustics processing and pattern recognition.

DWT is wavelet transform for which the wavelets are discretely sampled. DWT also provides temporal resolution in other words, it provides time and frequency/scale information. DWT is used for many different applications but most notably for image compression i.e. to represent a discrete signal in a more redundant form [1].

References

1. https://en.wikipedia.org/wiki/Wavelet
2. ecb.torontomu.ca/~nmalhotr/Thesis_Nripendra_Final_Submission_11April2021.pdf