RYERSON UNIVERSITY

Department of Electrical and Computer Engineering

ELE 635
Communication Systems

Frequency Modulation − Part II

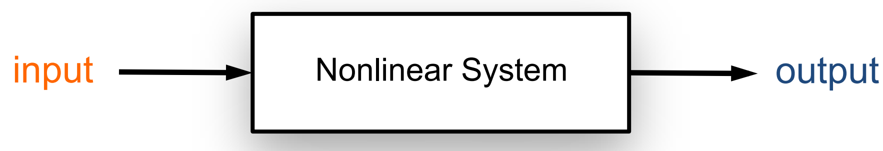
Winter 2015

- **Time- and frequency-domain description of angle modulated signals**
  - Phase Modulated (PM) signals
  - Frequency Modulated (FM) signals
  - Bandwidth of FM signals
- **Effects of nonlinearities on modulated signals**
  - Amplitude Modulated (AM) signals
  - Frequency Modulated (FM) signals
- **Generation of FM signals**
  - Indirect Method
- **FM Stereo Broadcasting**
  - Stereo signal multiplexing
  - Stereo signal demodulation
  - Tips, tricks, standards …

Effects of nonlinearities on modulated signals

Immunity to Nonlinearities

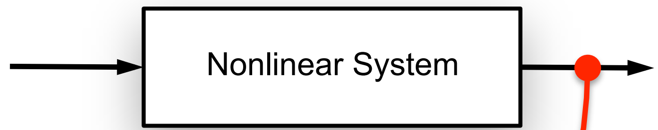
To observe the effects of system nonlinearities on modulated waveforms consider the following model:



$$[\text{ output }] = a\,[\text{ input }] + b\,[\text{ input }]^2 + c\,[\text{ input }]^3 + \ldots$$

Immunity to Nonlinearities

modulated waveform
$\varphi(t)$



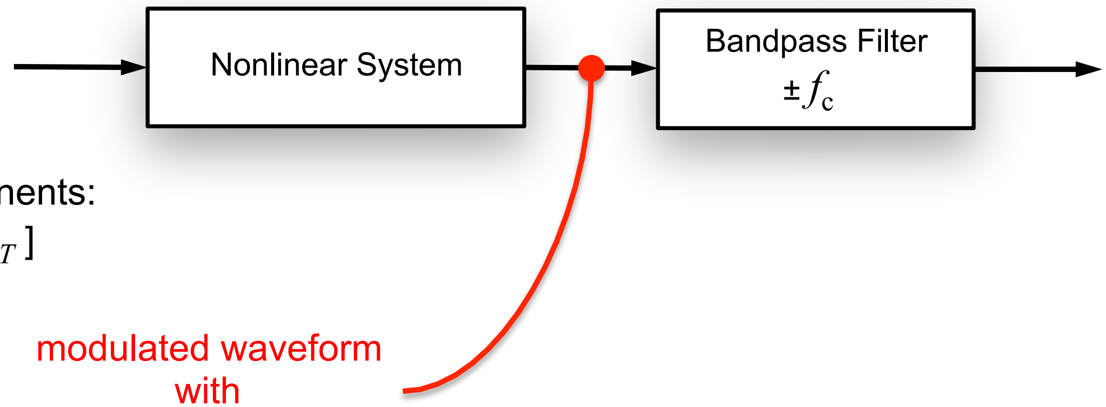
Frequency components:
$[f_c - B_T, f_c + B_T]$

modulated waveform with nonlinear distortion

Frequency components:
outside of $[f_c - B_T, f_c + B_T]$

Immunity to Nonlinearities

modulated waveform
$\varphi(t)$



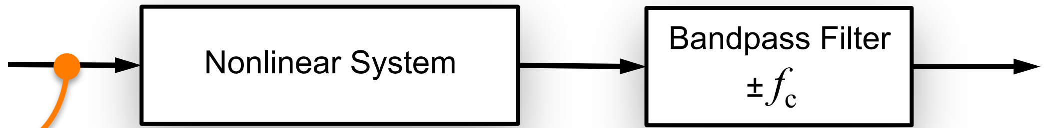
"cleaned" output

Frequency components:
$[f_c - B_T, f_c + B_T]$

modulated waveform
with
nonlinear distortion

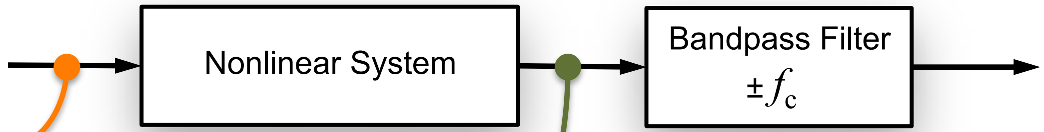
Frequency components:
outside of $[f_c - B_T, f_c + B_T]$

Immunity to Nonlinearities: AM



$$\varphi_{\text{AM}}(t) = m(t)\cos\omega_c t$$

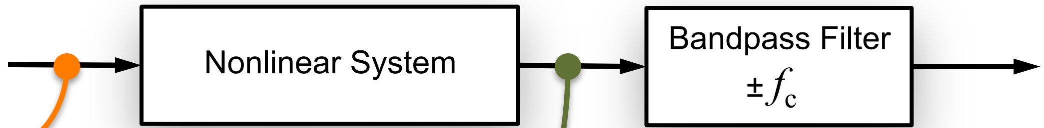
Immunity to Nonlinearities: AM



$$\varphi_{\mathrm{AM}}(t) = m(t)\cos\omega_c t$$

$$\begin{aligned}
\text{output} &= a\,m(t)\cos\omega_c t + b\,m^2(t)\cos^2\omega_c t + c\,m^3(t)\cos^3\omega_c t + \cdots \\
&= a\,m(t)\cos\omega_c t + \frac{b}{2}m^2(t) + \frac{b}{2}m^2(t)\cos 2\omega_c t + \frac{3}{4}c\,m^3(t)\cos\omega_c t + \frac{1}{4}c\,m^3(t)\cos 3\omega_c t + \cdots \\
&= \left[\frac{b}{2}m^2(t)\right] + \left[a\,m(t) + \frac{3}{4}c\,m^3(t)\right]\cos\omega_c t + \left[\frac{b}{2}m^2(t)\right]\cos 2\omega_c t + \left[\frac{c}{4}m^3(t)\right]\cos 3\omega_c t + \cdots
\end{aligned}$$

Immunity to Nonlinearities: AM



$$\varphi_{\mathrm{AM}}(t) = m(t)\cos\omega_c t$$

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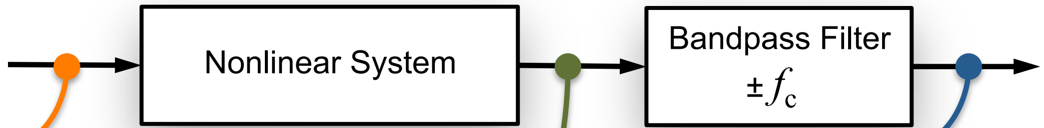
\text{output} &= a\,m(t)\cos\omega_c t + b\,m^2(t)\cos^2\omega_c t + c\,m^3(t)\cos^3\omega_c t + \cdots \\
&= a\,m(t)\cos\omega_c t + \frac{b}{2}m^2(t) + \frac{b}{2}m^2(t)\cos 2\omega_c t + \frac{3}{4}c\,m^3(t)\cos\omega_c t + \frac{1}{4}c\,m^3(t)\cos 3\omega_c t + \cdots \\
&= \underbrace{\boxed{\left[\frac{b}{2}m^2(t)\right]}}_{\text{baseband}} + \underbrace{\boxed{\left[a\,m(t) + \frac{3}{4}c\,m^3(t)\right]\cos\omega_c t}}_{\text{bandpass centered at } \pm f_c} + \underbrace{\boxed{\left[\frac{b}{2}m^2(t)\right]\cos 2\omega_c t}}_{\text{bandpass centered at } \pm 2f_c} + \underbrace{\boxed{\left[\frac{c}{4}m^3(t)\right]\cos 3\omega_c t}}_{\text{bandpass centered at } \pm 3f_c} + \cdots
\end{aligned}$$

Immunity to Nonlinearities: AM



$$\varphi_{\mathrm{AM}}(t) = m(t)\cos\omega_c t$$

$$\left[\, a\, m(t) + \frac{3}{4} c\, m^3(t) \,\right] \cos\omega_c t$$

$$\begin{aligned}
\text{output} &= a\, m(t)\cos\omega_c t + b\, m^2(t)\cos^2\omega_c t + c\, m^3(t)\cos^3\omega_c t + \cdots \\
&= a\, m(t)\cos\omega_c t + \frac{b}{2} m^2(t) + \frac{b}{2} m^2(t)\cos 2\omega_c t + \frac{3}{4} c\, m^3(t)\cos\omega_c t + \frac{1}{4} c\, m^3(t)\cos 3\omega_c t + \cdots \\
&= \boxed{\cancel{\left[\frac{b}{2} m^2(t)\right]}} + \boxed{\left[a\, m(t) + \frac{3}{4} c\, m^3(t)\right]\cos\omega_c t} + \boxed{\cancel{\left[\frac{b}{2} m^2(t)\right]\cos 2\omega_c t}} + \boxed{\cancel{\left[\frac{c}{4} m^3(t)\right]\cos 3\omega_c t}} + \cdots
\end{aligned}$$

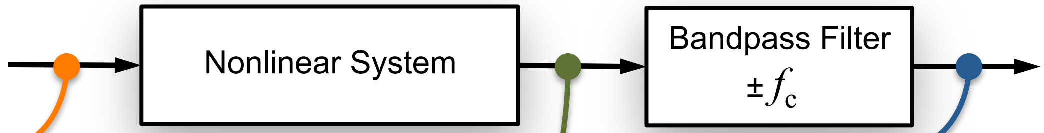
baseband

bandpass centered at $\pm f_c$

bandpass centered at $\pm 2f_c$

bandpass centered at $\pm 3f_c$

Immunity to Nonlinearities: AM



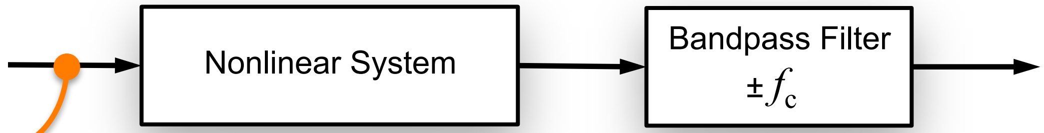
$$\varphi_{\mathrm{AM}}(t) = m(t)\cos\omega_c t$$

$$\underbrace{\left[\, a\, m(t) + \frac{3}{4} c\, m^3(t) \,\right]}_{\neq\, m(t)} \cos\omega_c t$$

Distortion! …

$$\begin{aligned}
\text{output} &= a\, m(t)\cos\omega_c t + b\, m^2(t)\cos^2\omega_c t + c\, m^3(t)\cos^3\omega_c t + \cdots \\
&= a\, m(t)\cos\omega_c t + \frac{b}{2}m^2(t) + \frac{b}{2}m^2(t)\cos 2\omega_c t + \frac{3}{4}c\, m^3(t)\cos\omega_c t + \frac{1}{4}c\, m^3(t)\cos 3\omega_c t + \cdots \\
&= \left[\frac{b}{2}m^2(t)\right] + \left[a\, m(t) + \frac{3}{4}c\, m^3(t)\right]\cos\omega_c t + \left[\frac{b}{2}m^2(t)\right]\cos 2\omega_c t + \left[\frac{c}{4}m^3(t)\right]\cos 3\omega_c t + \cdots
\end{aligned}$$

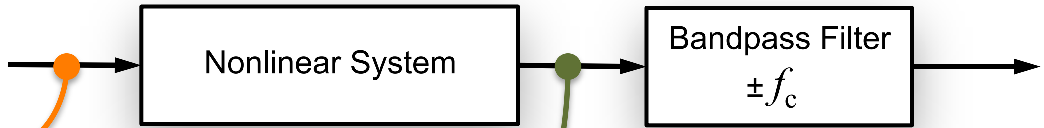
Immunity to Nonlinearities: FM



$$\varphi_{\mathrm{FM}}(t) = \cos\big(\omega_c t + \psi(t)\big)$$

with $\psi(t) = K_f \int_0^t m(\lambda) d\lambda$

Immunity to Nonlinearities: FM



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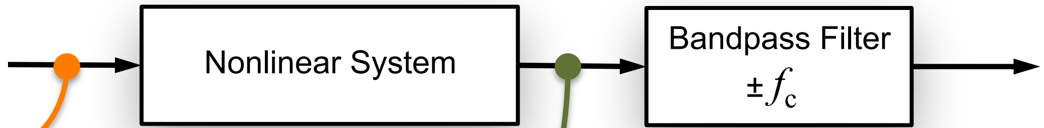
$$\begin{aligned}
\text{output} &= a\ \cos(\ ) + b\ \cos^2(\ ) + c\ \cos^3(\ ) + \cdots \\
&= a\ \cos(\ ) + \frac{b}{2} + \frac{b}{2}\cos\big(2\omega_c t + 2\psi(t)\big) + \frac{3}{4}c\ \cos(\ ) + \frac{1}{4}c\ \cos\big(3\omega_c t + 3\psi(t)\big) + \cdots \\
&= \Big[\frac{b}{2}\Big] + \Big[a + \frac{3}{4}c\Big]\cos\big(\omega_c t + \psi(t)\big) + \Big[\frac{b}{2}\Big]\cos\big(2\omega_c t + 2\psi(t)\big) + \Big[\frac{c}{4}\Big]\cos\big(3\omega_c t + 3\psi(t)\big) + \cdots
\end{aligned}$$

Immunity to Nonlinearities: FM



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$$= a\ \cos(\ ) + \frac{b}{2} + \frac{b}{2}\cos\big(2\omega_c t + 2\psi(t)\big) + \frac{3}{4}c\ \cos(\ ) + \frac{1}{4}c\ \cos\big(3\omega_c t + 3\psi(t)\big) + \cdots$$

$$= \boxed{\left[\frac{b}{2}\right]} + \boxed{\left[a + \frac{3}{4}c\right]\cos\big(\omega_c t + \psi(t)\big)} + \boxed{\left[\frac{b}{2}\right]\cos\big(2\omega_c t + 2\psi(t)\big)} + \boxed{\left[\frac{c}{4}\right]\cos\big(3\omega_c t + 3\psi(t)\big)} + \cdots$$

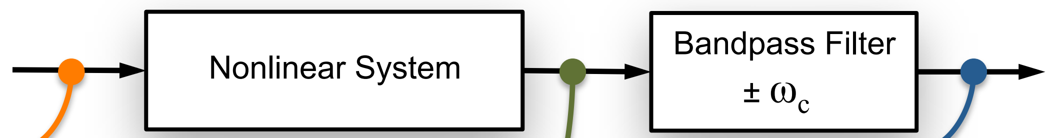
d.c.

bandpass centered at $\pm f_c$

bandpass centered at $\pm 2f_c$

bandpass centered at $\pm 3f_c$

Immunity to Nonlinearities: FM



$$\varphi_{\mathrm{FM}}(t) = \cos\big(\omega_c t + \psi(t)\big)$$

with $\psi(t) = K_f \int_0^t m(\lambda)d\lambda$

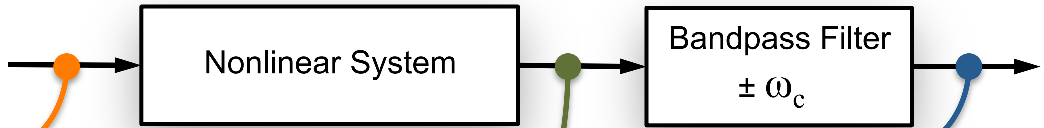
$$\Big[a + \frac{3}{4}c\Big] \cos\big(\omega_c t + \psi(t)\big)$$

$$\text{output} = a\,\cos(\ ) + b\,\cos^2(\ ) + c\,\cos^3(\ ) + \cdots$$

$$= a\,\cos(\ ) + \frac{b}{2} + \frac{b}{2}\cos\big(2\omega_c t + 2\psi(t)\big) + \frac{3}{4}c\,\cos(\ ) + \frac{1}{4}c\,\cos\big(3\omega_c t + 3\psi(t)\big) + \cdots$$

$$= \underbrace{\boxed{\cancel{\Big[\frac{b}{2}\Big]}}}_{\text{d.c.}} + \underbrace{\boxed{\Big[a + \frac{3}{4}c\Big]\cos\big(\omega_c t + \psi(t)\big)}}_{\text{bandpass centered at } \pm f_c} + \underbrace{\boxed{\cancel{\Big[\frac{b}{2}\Big]\cos\big(2\omega_c t + 2\psi(t)\big)}}}_{\text{bandpass centered at } \pm 2f_c} + \underbrace{\boxed{\cancel{\Big[\frac{c}{4}\Big]\cos\big(3\omega_c t + 3\psi(t)\big)}}}_{\text{bandpass centered at } \pm 3f_c} + \cdots$$

Immunity to Nonlinearities: FM



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with $\psi(t) = K_f \int_0^t m(\lambda)d\lambda$

$$[a + \frac{3}{4}c]\,\underbrace{\cos\big(\omega_c t + \psi(t)\big)}_{f_i \sim m(t)}$$

$$\text{output} = a\ \cos(\ ) + b\ \cos^2(\ ) + c\ \cos^3(\ ) + \cdots$$

$$= a\ \cos(\ ) + \frac{b}{2} + \frac{b}{2}\cos\big(2\omega_c t + 2\psi(t)\big) + \frac{3}{4}c\ \cos(\ ) + \frac{1}{4}c\ \cos\big(3\omega_c t + 3\psi(t)\big) + \cdots$$

$$= \boxed{\cancel{[\frac{b}{2}]}} + \boxed{[a + \frac{3}{4}c]\cos\big(\omega_c t + \psi(t)\big)} + \boxed{\cancel{[\frac{b}{2}]\cos\big(2\omega_c t + 2\psi(t)\big)}} + \boxed{\cancel{[\frac{c}{4}]\cos\big(3\omega_c t + 3\psi(t)\big)}} + \cdots$$

d.c. | bandpass centered at ± f_c | bandpass centered at ± $2f_c$ | bandpass centered at ± $3f_c$

- **Time- and frequency-domain description of angle modulated signals**
  - ❖ Phase Modulated (PM) signals
  - ❖ Frequency Modulated (FM) signals
  - ❖ Bandwidth of FM signals

- **Effects of nonlinearities on modulated signals**
  - ❖ Amplitude Modulated (AM) signals
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- **Generation of FM signals**
  - ❖ Indirect Method

- **FM Stereo Broadcasting**
  - ❖ Stereo signal multiplexing
  - ❖ Stereo signal demodulation
  - ❖ Tips, tricks, standards …

Generation of FM signals

Two main techniques:

1. Direct Method (**read from course reference text**)

Two main techniques:

1. Direct Method (**read from course reference text**)
2. Indirect Method (**will discuss now …**)

Indirect method

$m(t)$ → NBFM → WBFM

Indirect method

Step 1 $m(t)$ → NBFM → WBFM

Indirect method

Step 1

$m(t)$ → NBFM → WBFM

$$\varphi_{\text{NBFM}}(t) = A_c \cos \omega_c t - A_c \left[K_f \int_0^t m(\lambda) d\lambda \right] \sin \omega_c t$$

Indirect method

Step 1 $m(t)$ → NBFM → WBFM

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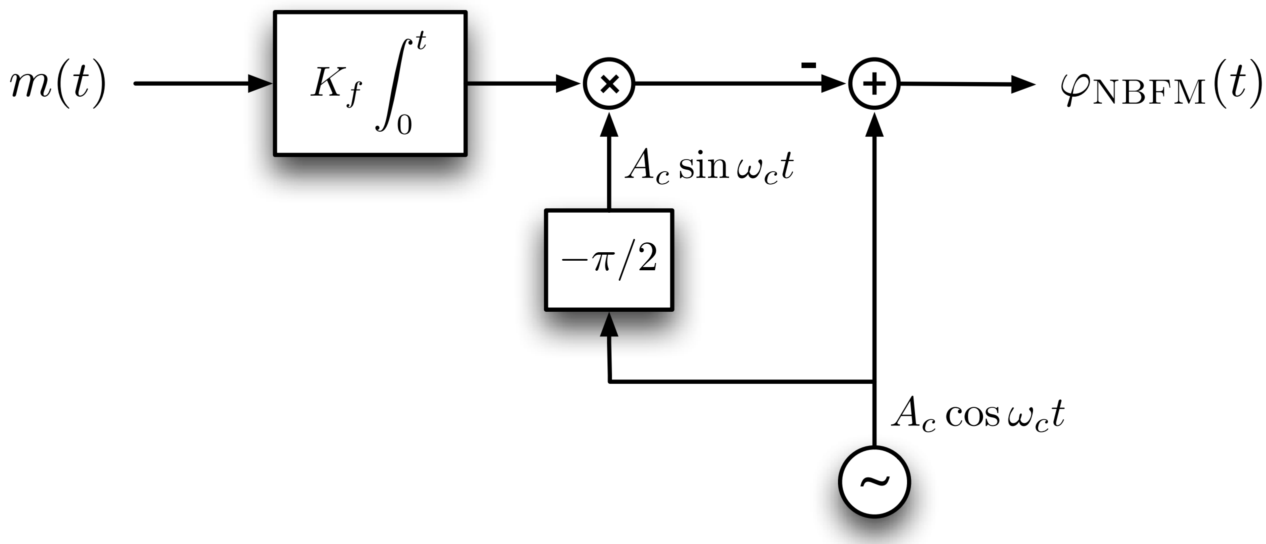


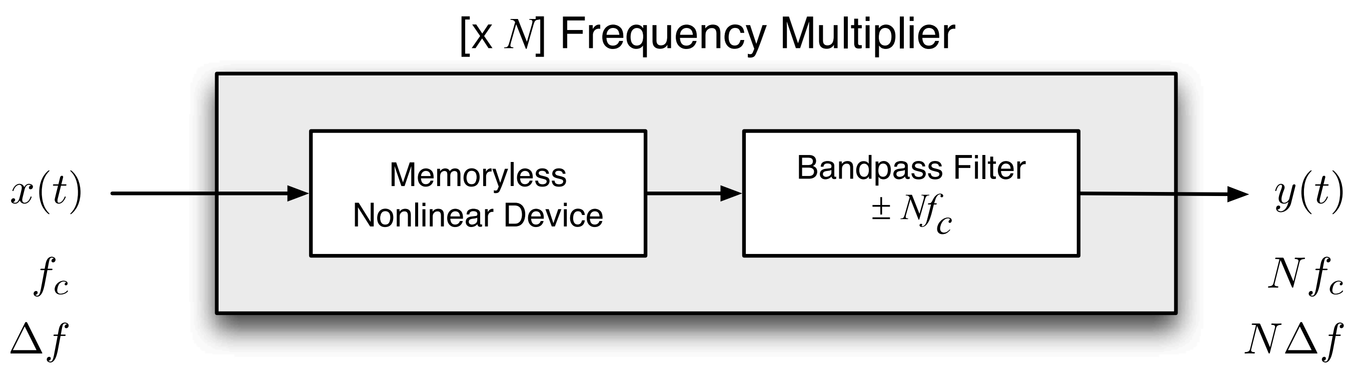
Indirect method

$m(t)$ → NBFM → WBFM **Step 2**

Step 2 requires the use **Frequency Multiplier** units.

Indirect method

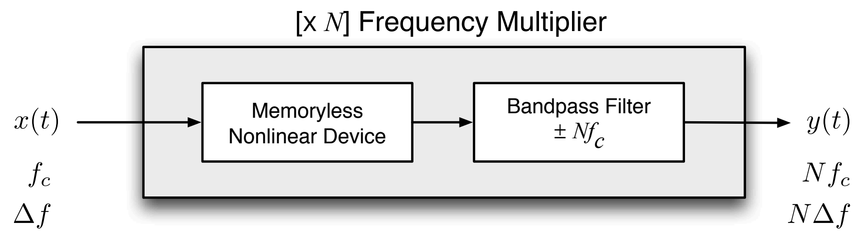
Frequency Multiplier



$$y(t) = \left[\, a_1\, x(t) + a_2\, x^2(t) + a_3\, x^3(t) + \ldots \,\right] * h_{\mathrm{BPF}}(t)$$

Indirect method

Frequency Multiplier



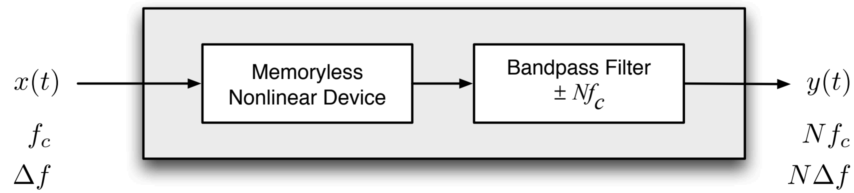
Consider the case with a **2nd Order Nonlinearity:**

$$a_2\, x^2(t)$$

Indirect method

Frequency Multiplier

[x N] Frequency Multiplier



Consider the case with a **2nd Order Nonlinearity:**

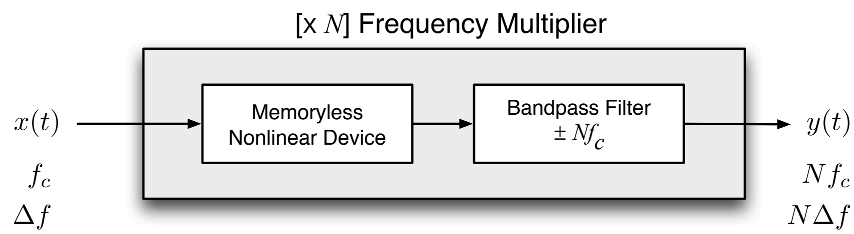
BPF tuned to ± $2f_c$

$$\boxed{a_2\, x^2(t)}$$

$$
\begin{aligned}
y(t) &= a_2\, \cos^2\big(\omega_c t + \psi(t)\big) * h_{\mathrm{BPF}}(t) \\
&= \left[\frac{a_2}{2} + \frac{a_2}{2}\cos\big(2\omega_c t + 2\psi(t)\big)\right] * h_{\mathrm{BPF}}(t) \\
&= \frac{a_2}{2}\cos\big(2\omega_c t + 2\psi(t)\big)
\end{aligned}
$$

Indirect method

Frequency Multiplier



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$$
\begin{aligned}
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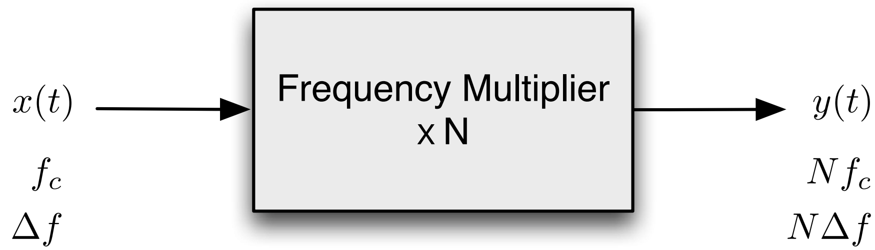
&= \left[\frac{a_2}{2} + \frac{a_2}{2}\cos\big(2\omega_c t + 2\psi(t)\big)\right] * h_{\mathrm{BPF}}(t) \\
&= \frac{a_2}{2}\cos\big(\boxed{2\omega_c t + 2\psi(t)}\big)
\end{aligned}
$$

f_c and Δf doubled

Indirect method

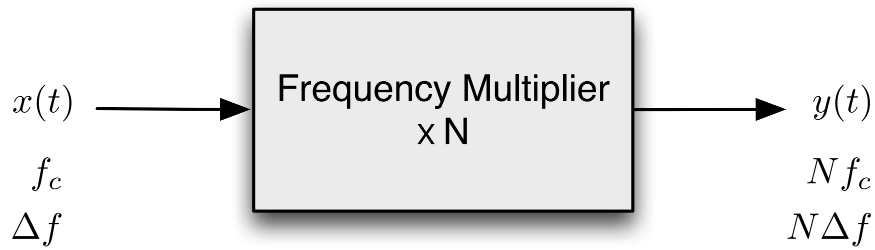
Frequency Multiplier



Single-Tone Modulation $\beta = \dfrac{\Delta f}{f_m} \longrightarrow N\beta = \dfrac{N\Delta f}{f_m}$

Indirect method

Frequency Multiplier



Single-Tone Modulation $\beta = \dfrac{\Delta f}{f_m} \Rightarrow N\beta = \dfrac{N\Delta f}{f_m}$

Baseband Modulation $\mathcal{D} = \dfrac{\Delta f}{B_m} \Rightarrow N\mathcal{D} = \dfrac{N\Delta f}{B_m}$

Indirect method

Frequency Multiplier

$x(t)$ → **Frequency Multiplier x N** → $y(t)$

f_c		Nf_c
Δf		$N\Delta f$

Single-Tone Modulation $\beta = \dfrac{\Delta f}{f_m}$ ⟹ $N\beta = \dfrac{N\Delta f}{f_m}$

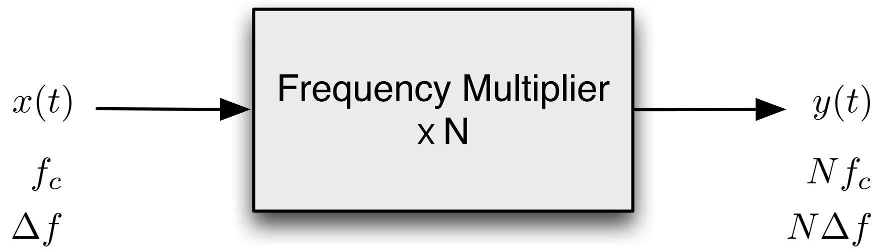
Baseband Modulation $\mathcal{D} = \dfrac{\Delta f}{B_m}$ ⟹ $N\mathcal{D} = \dfrac{N\Delta f}{B_m}$

Increasing β or $\mathcal{D}$ is exactly what we need for

NBFM → WBFM.

Indirect method

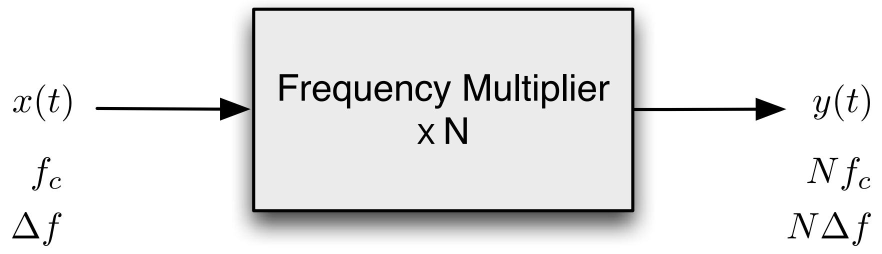
Frequency Multiplier



- In practice very abrupt nonlinearities can be generated … N ~ 1000.
- Nth order Frequency Multiplier **increases both** f_c and Δf by N.

Indirect method

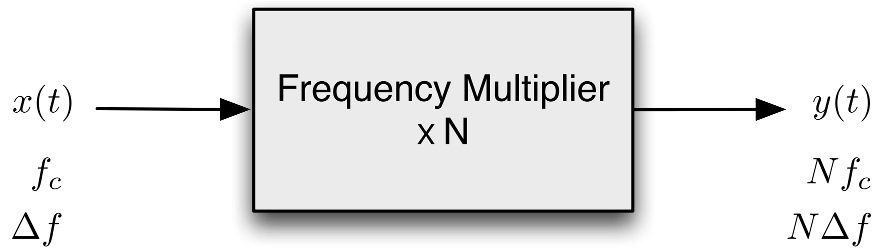
Frequency Multiplier



- In practice very abrupt nonlinearities can be generated … N ~ 1000.
- Nth order Frequency Multiplier **increases both f_c** and **Δf** by N.
- Need another mechanism to **control f_c only**

Indirect method

Frequency Multiplier

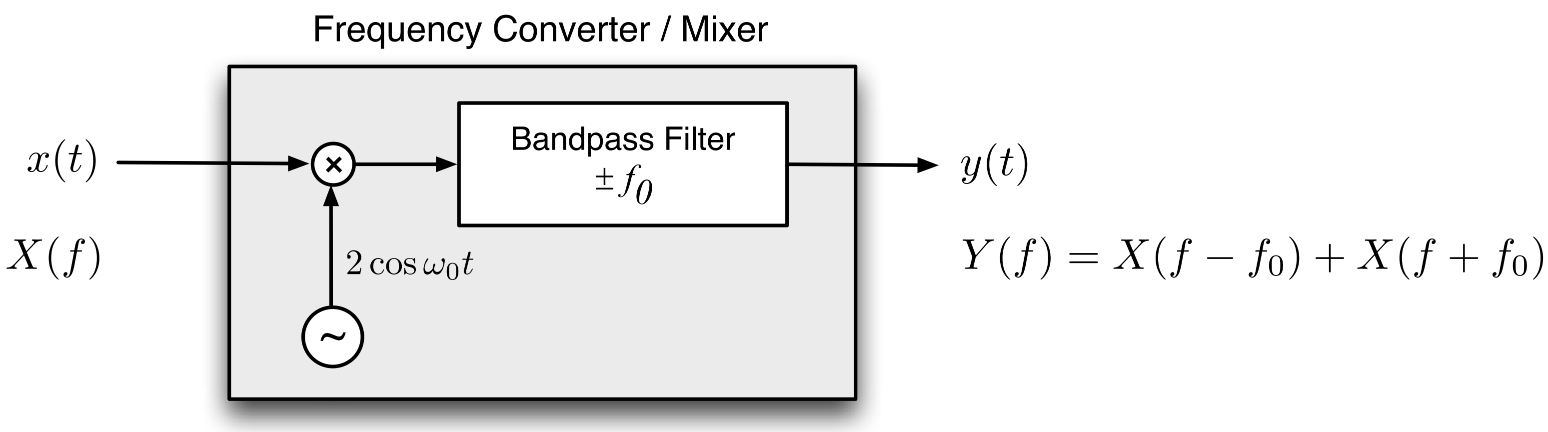


- In practice very abrupt nonlinearities can be generated … N ~ 1000.
- Nth order Frequency Multiplier **increases both f_c** and **Δf** by N.
- Need another mechanism to **control f_c only**

→ **Frequency Converter / Mixer**

Indirect method

Frequency Converter / Mixer

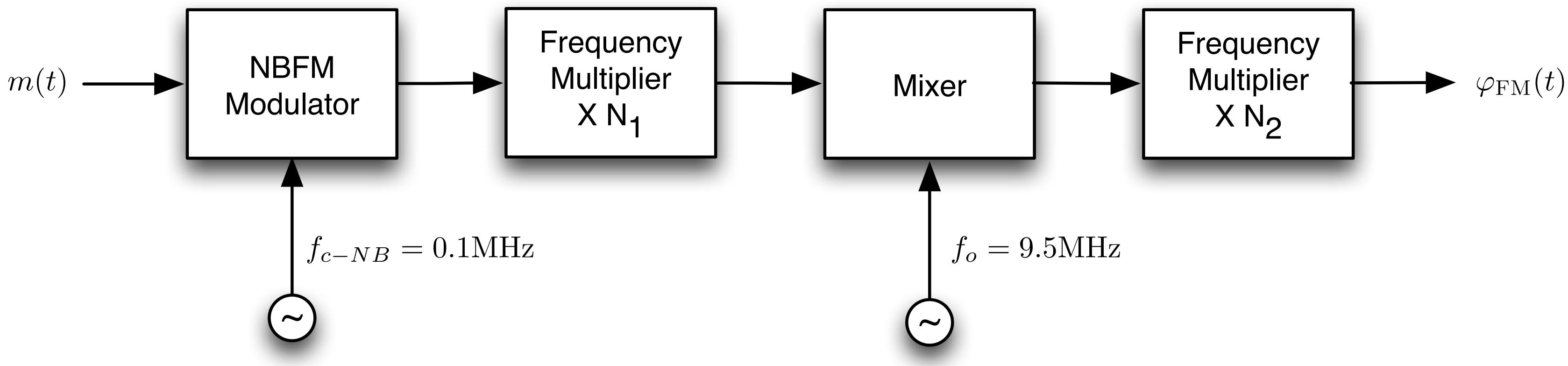


Indirect method

$m(t)$ → NBFM → WBFM **Step 2**

- **Frequency Multiplier** units …………………… increase **both** f_c and Δf
- Increasing Δf ………………………………… increases β or $\mathcal{D}$
- **Frequency Converter / Mixer** units ………… increase / decrease f_c **only**
- Complete NBFM to WBFM conversion requires the use of multiple **Frequency Multiplier** and **Frequency Mixer** units.

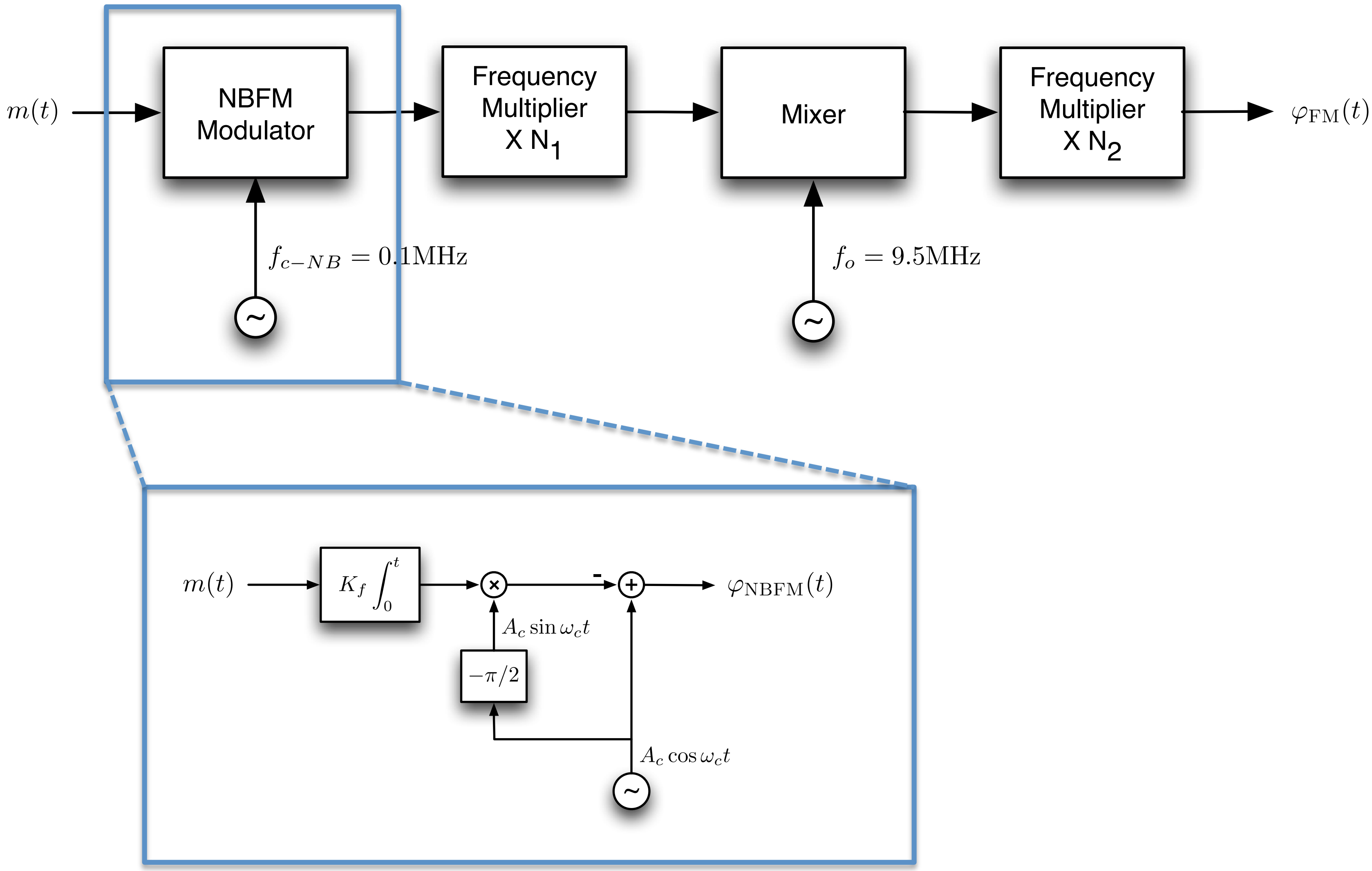
Indirect method – An Example



Given:

- $m(t)$ ………………… bandlimited to [100 Hz, 15 kHz]
- NBFM ……………. $f_{\text{c-NB}}$ = 0.1 MHz and Δf_{NB} = 20 Hz
- WBFM …………… $f_{\text{c-WB}}$ = 100 MHz and Δf_{WB} = 75 kHz
- Mixer …………….. down-mixer

Indirect method – An Example



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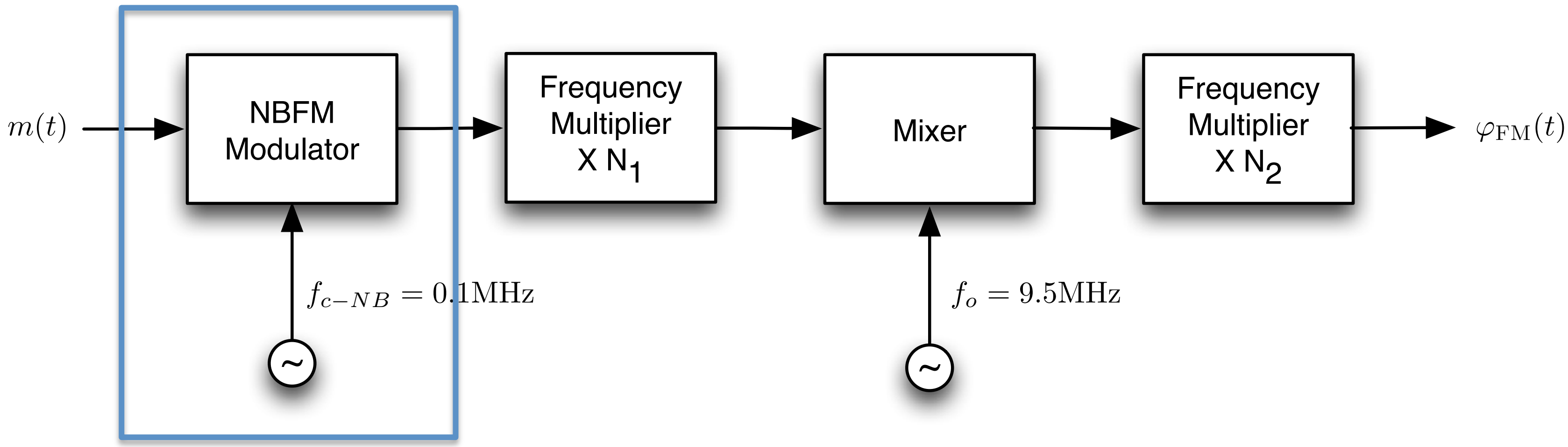
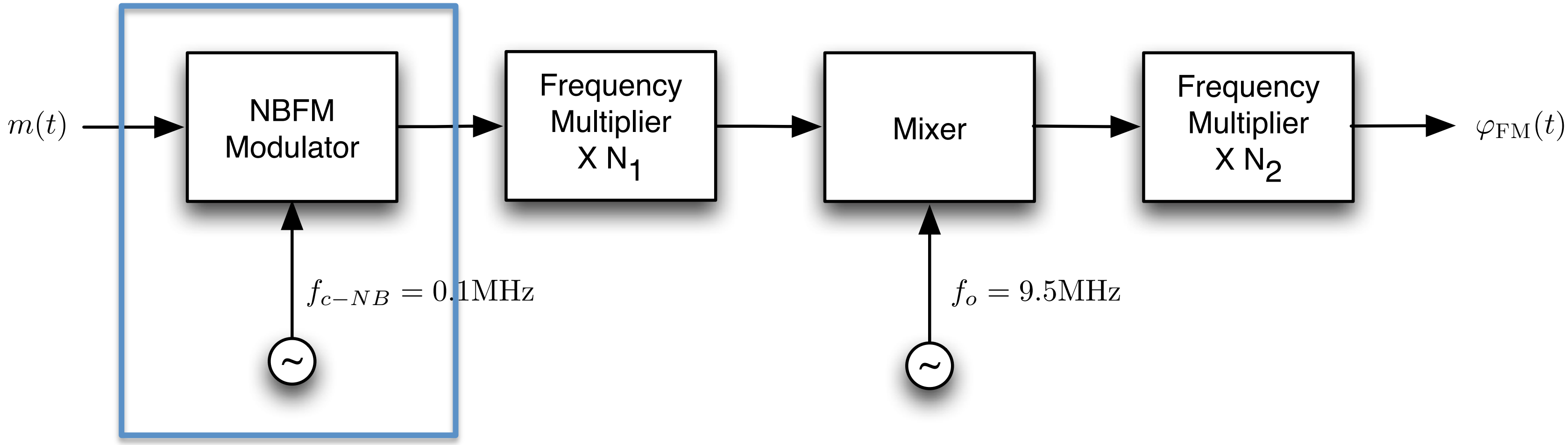


Check if NBFM condition is satisfied ($\beta < 0.3$)

Indirect method – An Example



Check if NBFM condition is satisfied ($\beta < 0.3$)

As $m(t)$ is bandlimited to [100 Hz, 15 kHz], modulation/deviation index β ranges from:

$$\beta_{\min} = \Delta f_{\text{NB}} / 15000 = 20/15000 \approx \text{very small}$$

$$\beta_{\max} = \Delta f_{\text{NB}} / 100 = 20/100 = 0.2$$

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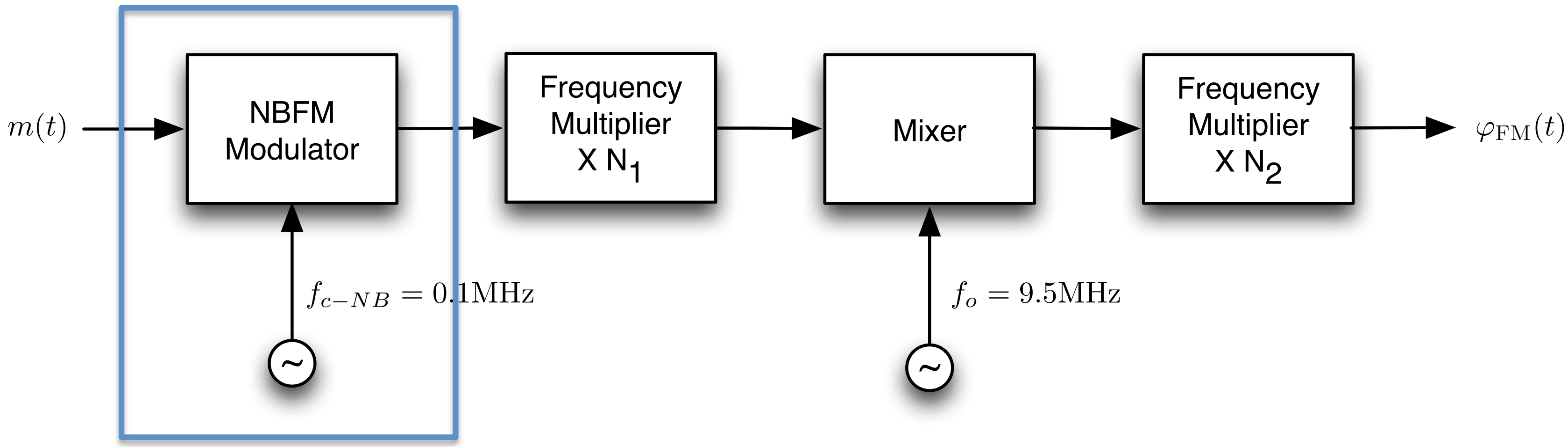

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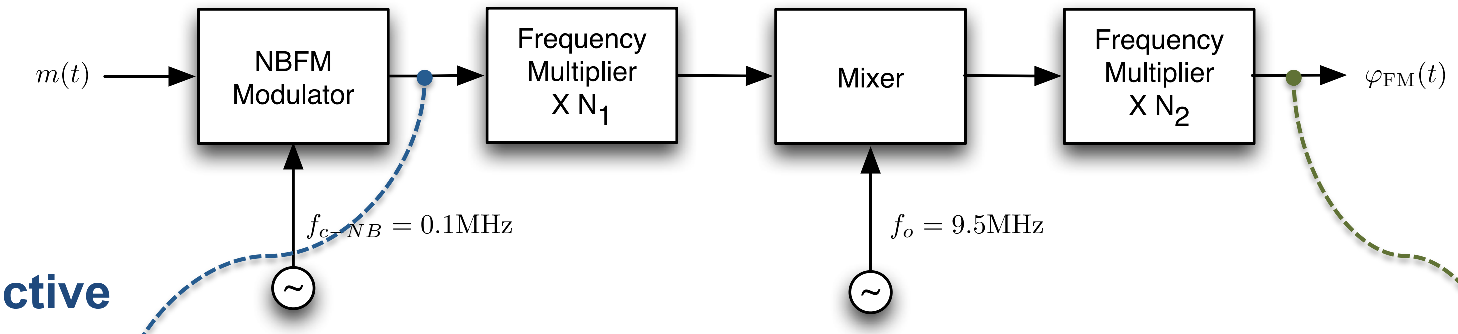
Worst Case Scenario
NBFM condition is satisfied

Indirect method − An Example

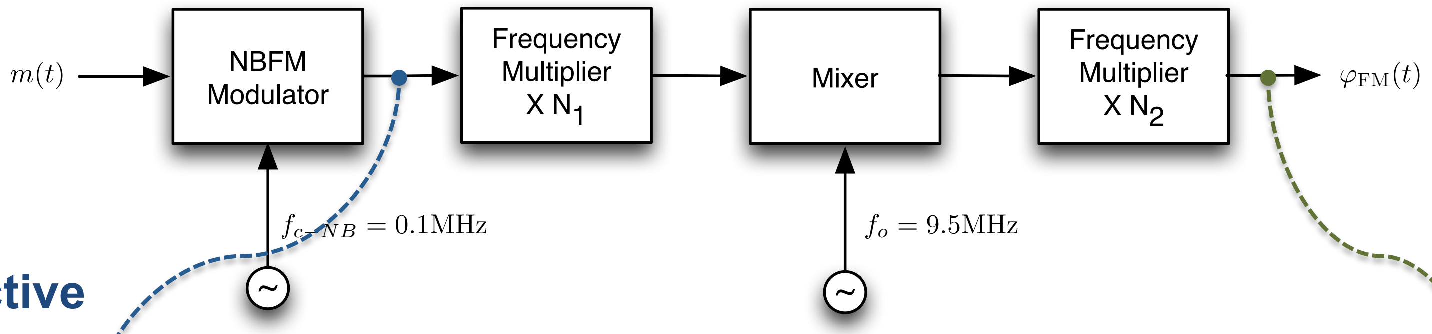


Objective

	NBFB
Δf	Δf_{NB}
f_c	$f_{c-\mathrm{NB}}$

WBFM
Δf_{WB}
$f_{c-\mathrm{WB}}$

Indirect method – An Example



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Δf_{WB}
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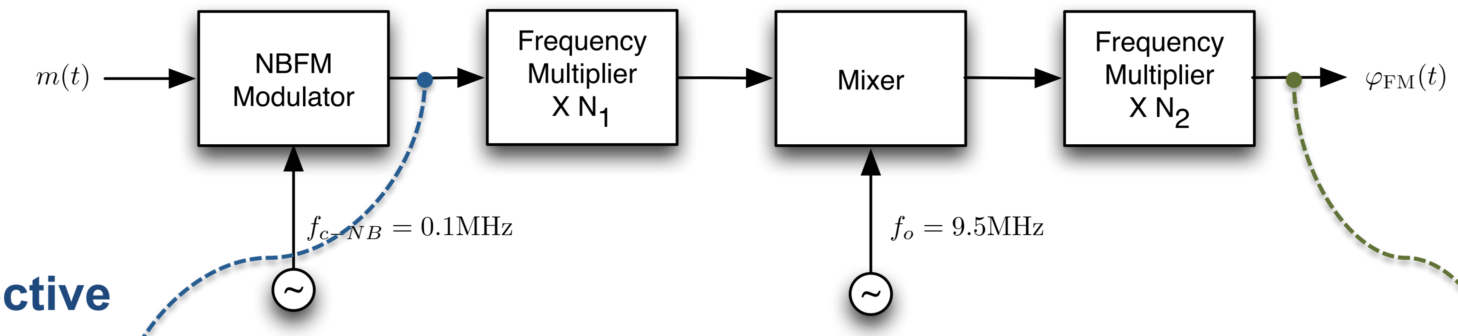
- Frequency Mixing effects f_c only.

Indirect method – An Example



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	NBFB
Δf	Δf_{NB}
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WBFM
Δf_{WB}
$f_{c-\text{WB}}$

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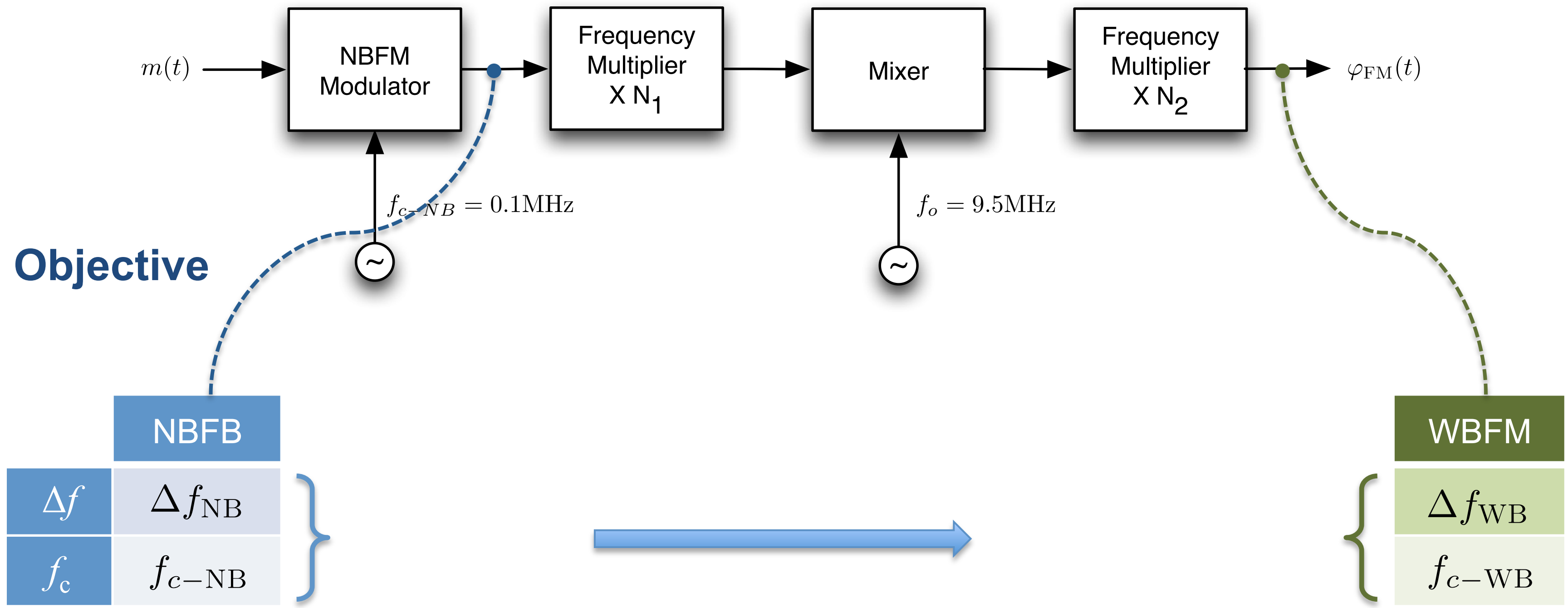
- Overall Frequency Multiplication factor $N_1 N_2$ is determined by $\Delta f_{WB} / \Delta f_{NB}$

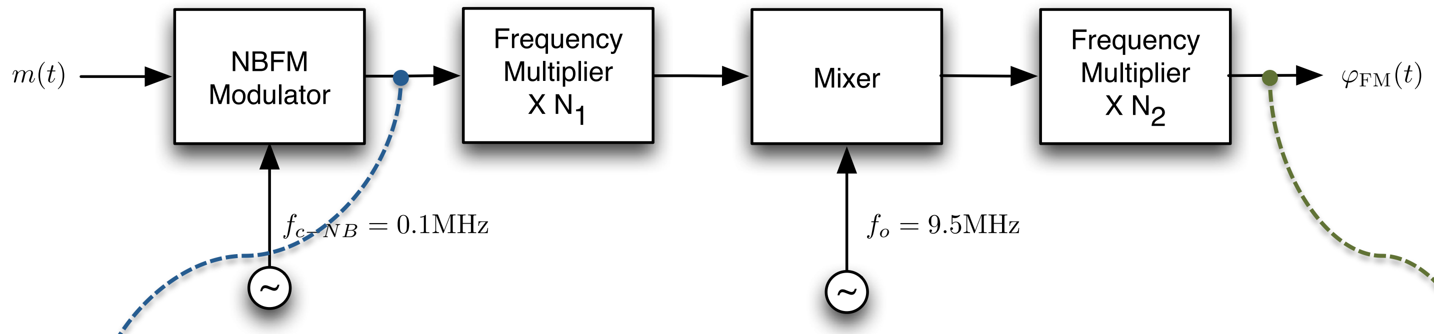
Indirect method – An Example



- Frequency Mixing effects f_c only.
- Overall Frequency Multiplication factor N_1N_2 is determined by $\Delta f_{WB} / \Delta f_{NB}$

$$N_1 N_2 = \frac{[\Delta f]_{\max}}{[\Delta f]_{\min}} = \frac{\Delta f_{\mathrm{WB}}}{\Delta f_{\mathrm{NB}}} = \frac{75000}{20} = 3750$$

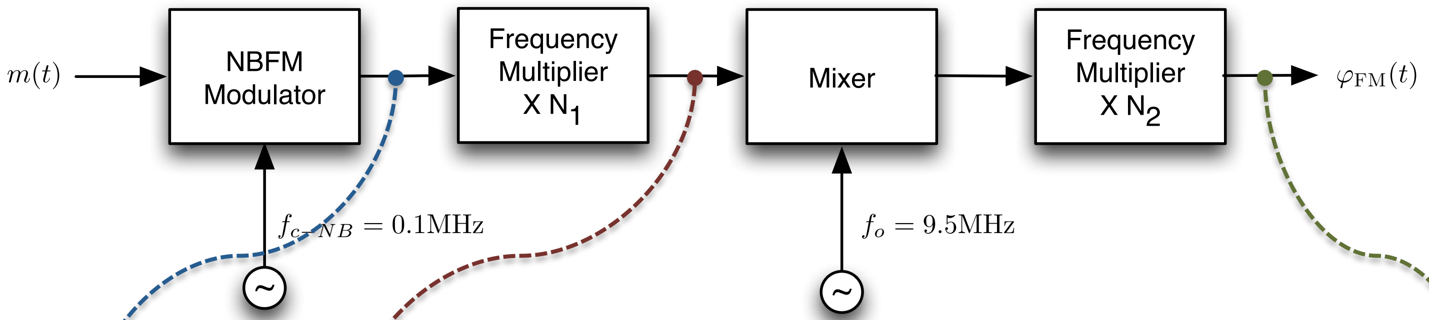
Indirect method – An Example



	NBFB
Δf	Δf_{NB}
f_c	$f_{c-\text{NB}}$

WBFM
Δf_{WB}
$f_{c-\text{WB}}$

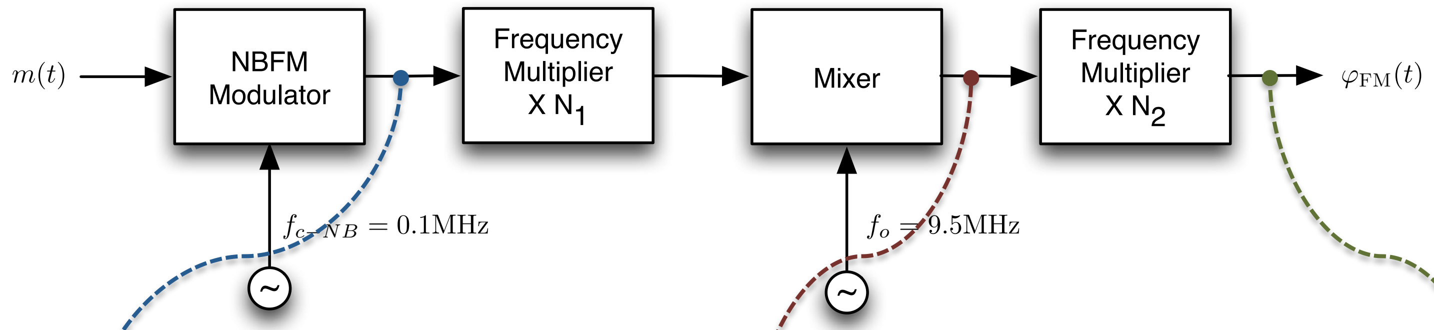
Indirect method – An Example



	NBFB	After Multiplier-1
Δf	Δf_{NB}	$N_1 \Delta f_{\mathrm{NB}}$
f_c	$f_{c-\mathrm{NB}}$	$N_1 f_{c-\mathrm{NB}}$

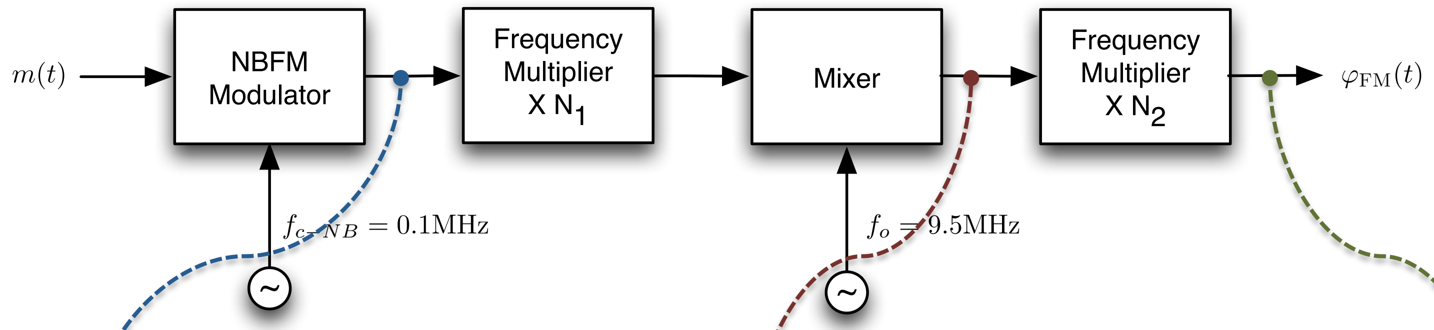
WBFM
Δf_{WB}
$f_{c-\mathrm{WB}}$

Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\text{NB}}$	$N_1 \Delta f_{\text{NB}}$	Δf_{WB}
f_c	$f_{c-\text{NB}}$	$N_1 f_{c-\text{NB}}$	$f_o - N_1 f_{c-\text{NB}}$	$f_{c-\text{WB}}$

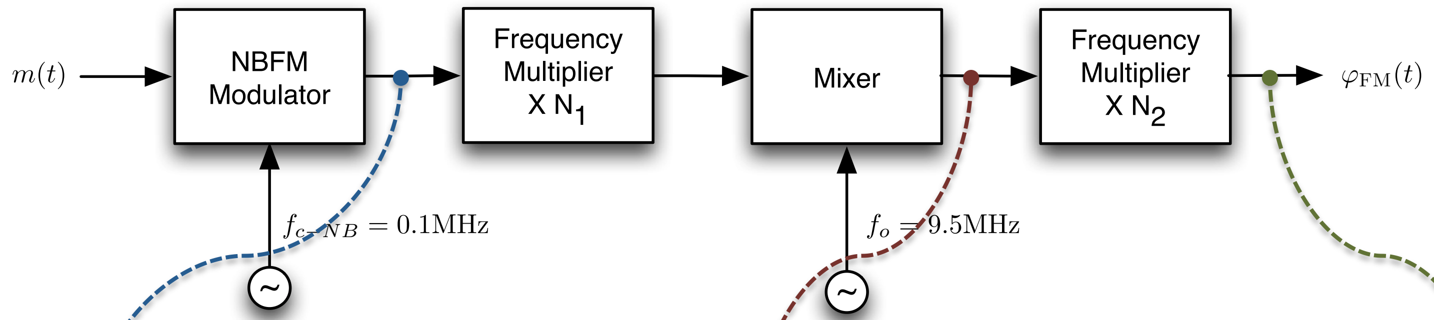
Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\mathrm{NB}}$	$N_1 \Delta f_{\mathrm{NB}}$	Δf_{WB}
f_c	$f_{c-\mathrm{NB}}$	$N_1 f_{c-\mathrm{NB}}$	$f_o - N_1 f_{c-\mathrm{NB}}$	$f_{c-\mathrm{WB}}$

Mixer does NOT alter $\boldsymbol{\Delta f}$

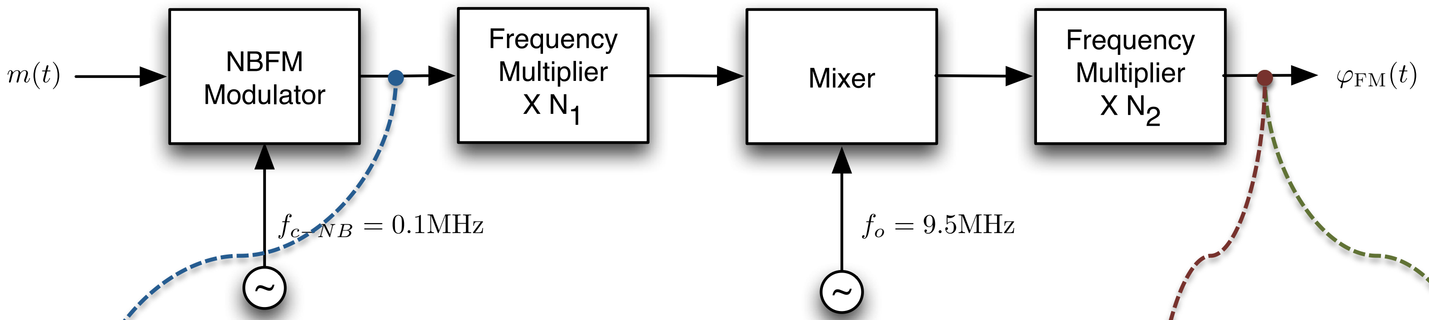
Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\mathrm{NB}}$	$N_1 \Delta f_{\mathrm{NB}}$	Δf_{WB}
f_c	$f_{c-\mathrm{NB}}$	$N_1 f_{c-\mathrm{NB}}$	$f_o - N_1 f_{c-\mathrm{NB}}$	$f_{c-\mathrm{WB}}$

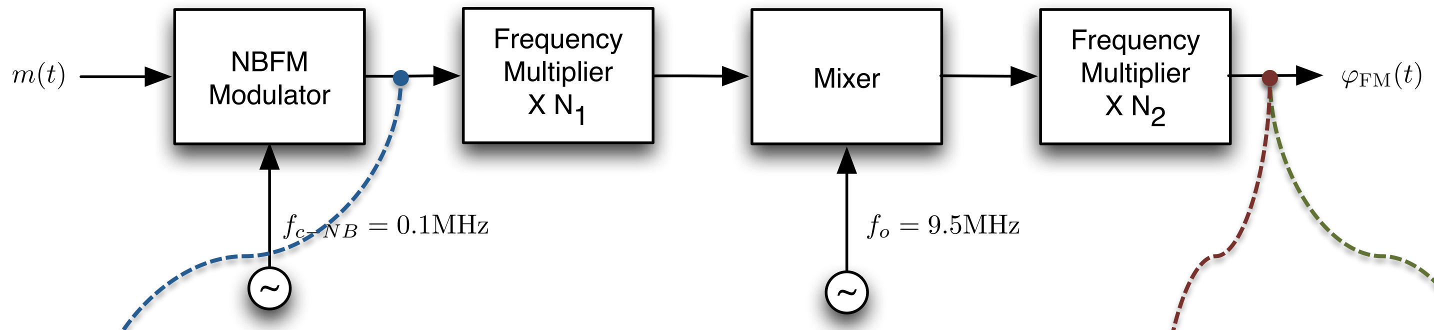
down-mixer!

Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\text{NB}}$	$N_1 \Delta f_{\text{NB}}$	$N_2 N_1 \Delta f_{\text{NB}}$	Δf_{WB}
f_c	$f_{c-\text{NB}}$	$N_1 f_{c-\text{NB}}$	$f_o - N_1 f_{c-\text{NB}}$	$N_2 f_0 - N_2 N_1 f_{c-\text{NB}}$	$f_{c-\text{WB}}$

Indirect method – An Example



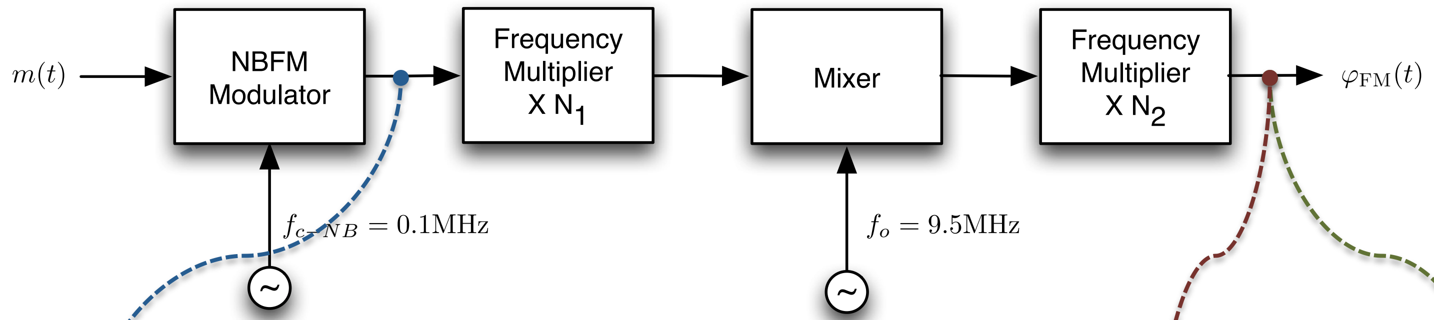
	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\text{NB}}$	$N_1 \Delta f_{\text{NB}}$	$N_2 N_1 \Delta f_{\text{NB}}$	Δf_{WB}
f_c	$f_{c-\text{NB}}$	$N_1 f_{c-\text{NB}}$	$f_o - N_1 f_{c-\text{NB}}$	$N_2 f_0 - N_2 N_1 f_{c-\text{NB}}$	$f_{c-\text{WB}}$

$$N_2 N_1 \Delta f_{\text{NB}} = \Delta f_{\text{WB}}$$

$$N_2 f_o - N_2 N_1 f_{c-\text{NB}} = f_{c-\text{WB}}$$

Design Equations

Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\mathrm{NB}}$	$N_1 \Delta f_{\mathrm{NB}}$	$N_2 N_1 \Delta f_{\mathrm{NB}}$	Δf_{WB}
f_c	$f_{c-\mathrm{NB}}$	$N_1 f_{c-\mathrm{NB}}$	$f_o - N_1 f_{c-\mathrm{NB}}$	$N_2 f_0 - N_2 N_1 f_{c-\mathrm{NB}}$	$f_{c-\mathrm{WB}}$

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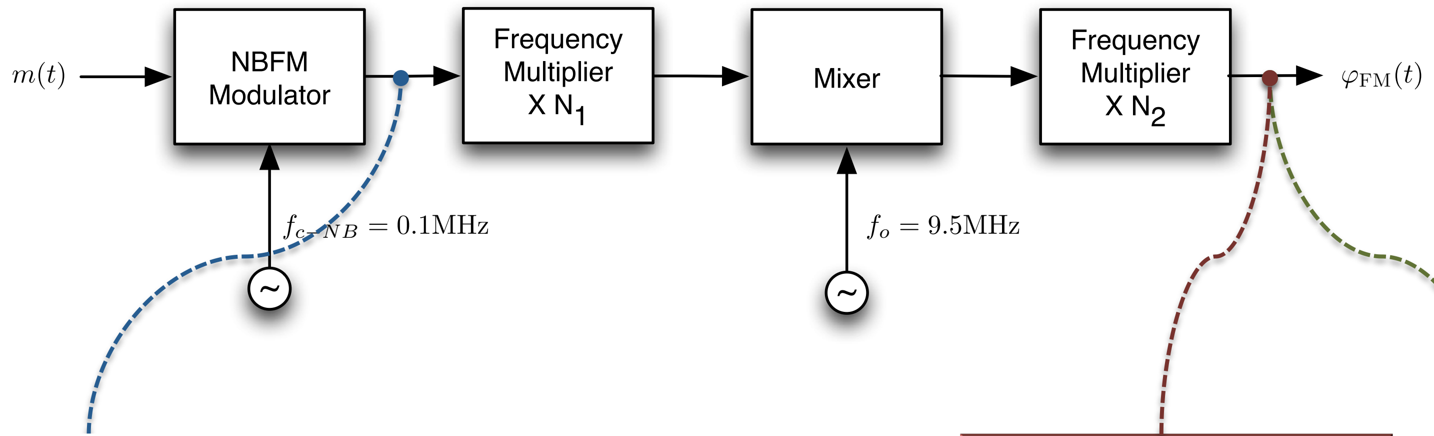
Design Equations

Indirect method – An Example

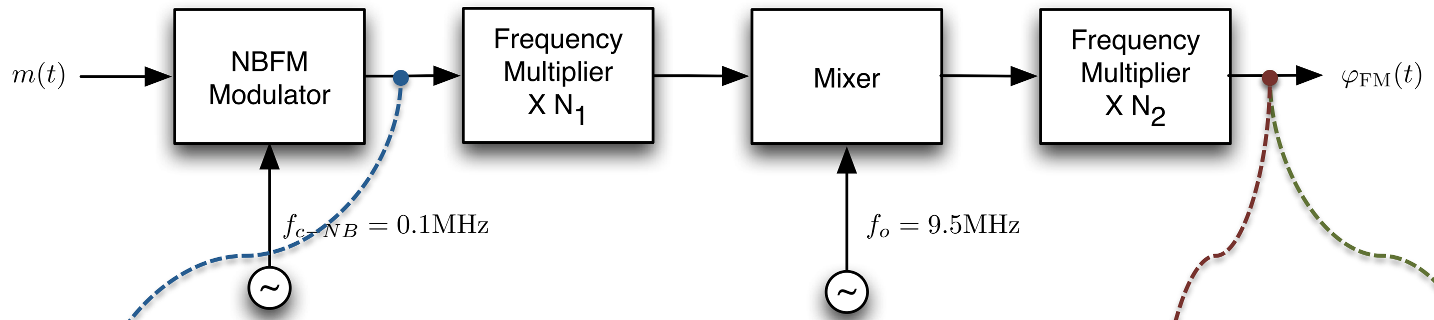


	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\text{NB}}$	$N_1 \Delta f_{\text{NB}}$	$N_2 N_1 \Delta f_{\text{NB}}$	Δf_{WB}
f_c	$f_{c-\text{NB}}$	$N_1 f_{c-\text{NB}}$	$f_o - N_1 f_{c-\text{NB}}$	$N_2 f_0 - N_2 N_1 f_{c-\text{NB}}$	$f_{c-\text{WB}}$

$$N_2 N_1 \Delta f_{\text{NB}} = \Delta f_{\text{WB}}$$
$$N_2 f_o - N_2 N_1 f_{c-\text{NB}} = f_{c-\text{WB}}$$

$$\boxed{\begin{aligned} 20 N_2 N_1 &= 75000 \\ 9.5 N_2 - 0.1 N_2 N_1 &= 100 \end{aligned}}$$

Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	Δf_{NB}	$N_1 \Delta f_{\text{NB}}$	$N_1 \Delta f_{\text{NB}}$	$N_2 N_1 \Delta f_{\text{NB}}$	Δf_{WB}
f_c	$f_{c-\text{NB}}$	$N_1 f_{c-\text{NB}}$	$f_o - N_1 f_{c-\text{NB}}$	$N_2 f_0 - N_2 N_1 f_{c-\text{NB}}$	$f_{c-\text{WB}}$

$$N_2 N_1 \Delta f_{\text{NB}} = \Delta f_{\text{WB}}$$

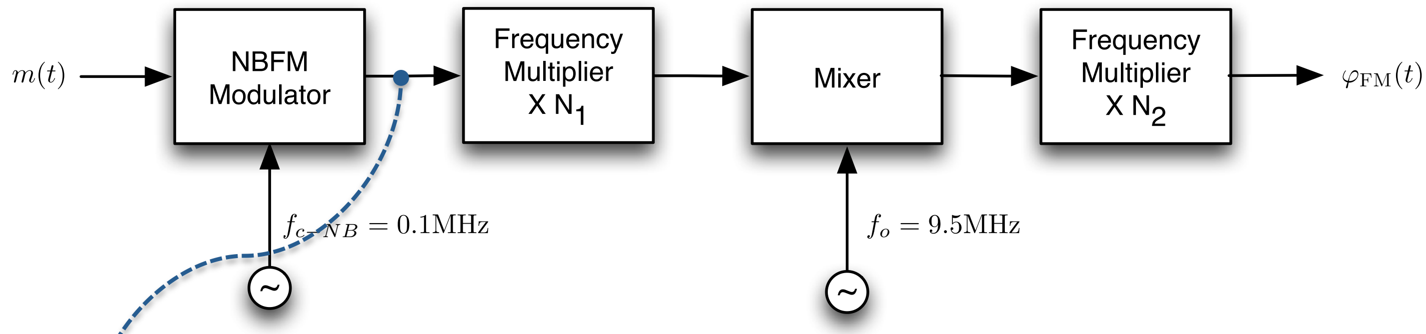
$$N_2 f_o - N_2 N_1 f_{c-\text{NB}} = f_{c-\text{WB}}$$

$$20 N_2 N_1 = 75000$$

$$9.5 N_2 - 0.1 N_2 N_1 = 100$$

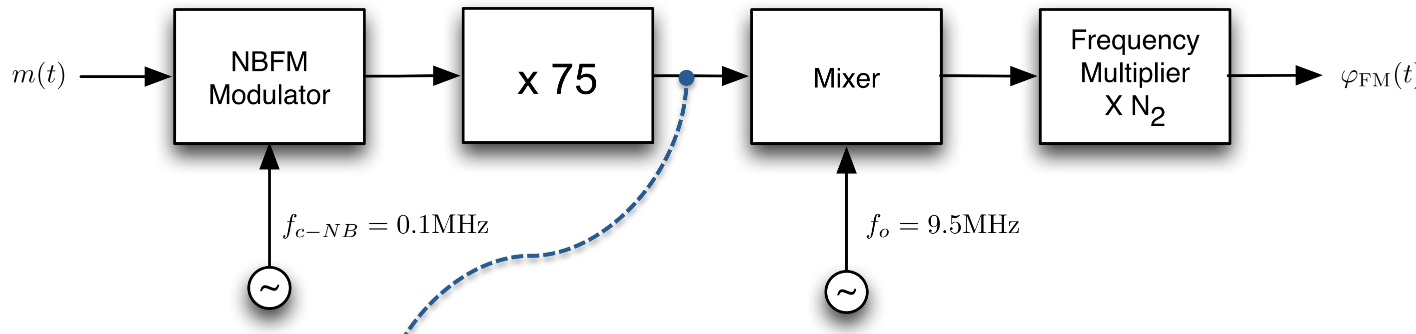
$$\Rightarrow \boxed{N_1 = 75 \quad N_2 = 50}$$

Indirect method – An Example



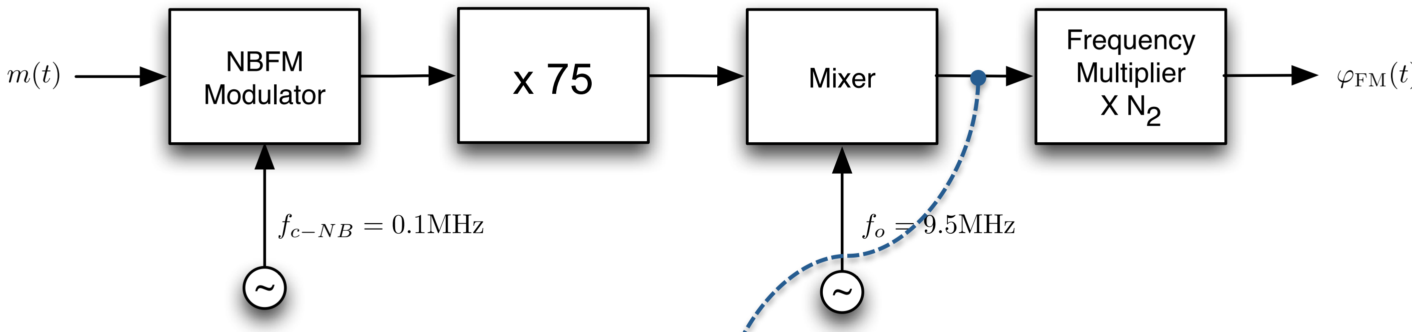
	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	20	$N_1 \Delta f_{\mathrm{NB}}$	$N_1 \Delta f_{\mathrm{NB}}$	$N_2 N_1 \Delta f_{\mathrm{NB}}$	Δf_{WB}
f_c	0.1M	$N_1 f_{c-\mathrm{NB}}$	$f_o - N_1 f_{c-\mathrm{NB}}$	$N_2 f_0 - N_2 N_1 f_{c-\mathrm{NB}}$	$f_{c-\mathrm{WB}}$

Indirect method – An Example



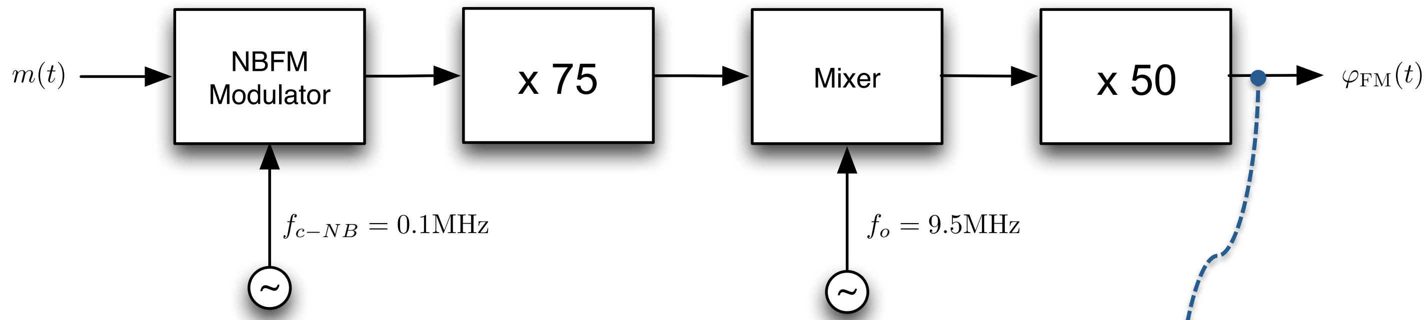
	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	20	1500	$N_1 \Delta f_{\mathrm{NB}}$	$N_2 N_1 \Delta f_{\mathrm{NB}}$	Δf_{WB}
f_c	0.1M	7.5M	$f_o - N_1 f_{c-\mathrm{NB}}$	$N_2 f_0 - N_2 N_1 f_{c-\mathrm{NB}}$	$f_{c-\mathrm{WB}}$

Indirect method – An Example



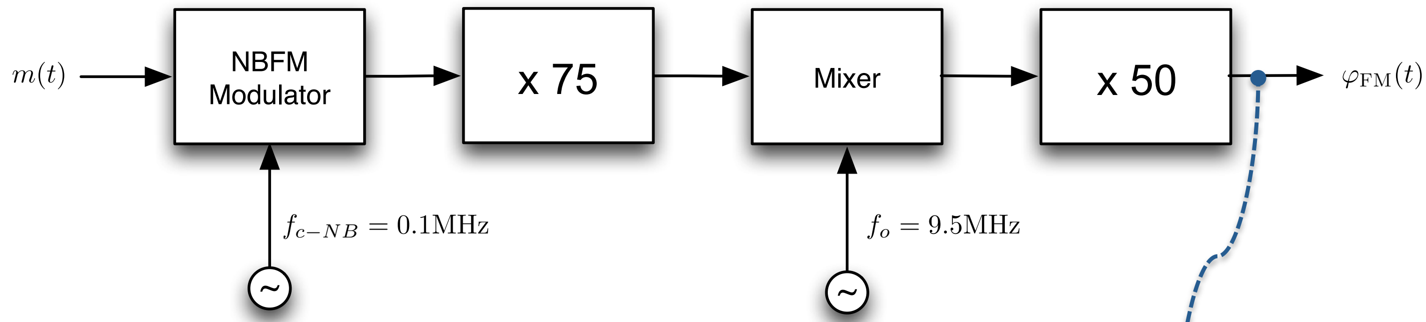
	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	20	1500	1500	$N_2 N_1 \Delta f_{\mathrm{NB}}$	Δf_{WB}
f_c	0.1M	7.5M	2.0M	$N_2 f_0 - N_2 N_1 f_{c-\mathrm{NB}}$	$f_{c-\mathrm{WB}}$

Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	20	1500	1500	75,000	Δf_{WB}
f_c	0.1M	7.5M	2.0M	100M	$f_{c-\mathrm{WB}}$

Indirect method – An Example



	NBFB	After Multiplier-1	After Mixer	After Multiplier-2	WBFM
Δf	20	1500	1500	75,000	Δf_{WB}
f_c	0.1M	7.5M	2.0M	100M	$f_{c-\mathrm{WB}}$

Design targets met

- **Time- and frequency-domain description of angle modulated signals**
  - Phase Modulated (PM) signals
  - Frequency Modulated (FM) signals
  - Bandwidth of FM signals
- **Effects of nonlinearities on modulated signals**
  - Amplitude Modulated (AM) signals
  - Frequency Modulated (FM) signals
- **Demodulation of FM signals**
  - PLL based FM demodulation
- **FM Stereo Broadcasting**
  - Stereo signal multiplexing
  - Stereo signal demodulation
  - Tips, tricks, standards …

Demodulation of FM signals

The Phase-Locked Loop (PLL) is a versatile component frequently used in communication systems. Applications of PLL include:

- Locking onto and tracking changes in the instantaneous frequency of the incoming signal.
- Narrowband filtering.
- Frequency synthesis.

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- Locking onto and tracking changes in the instantaneous frequency of the incoming signal.
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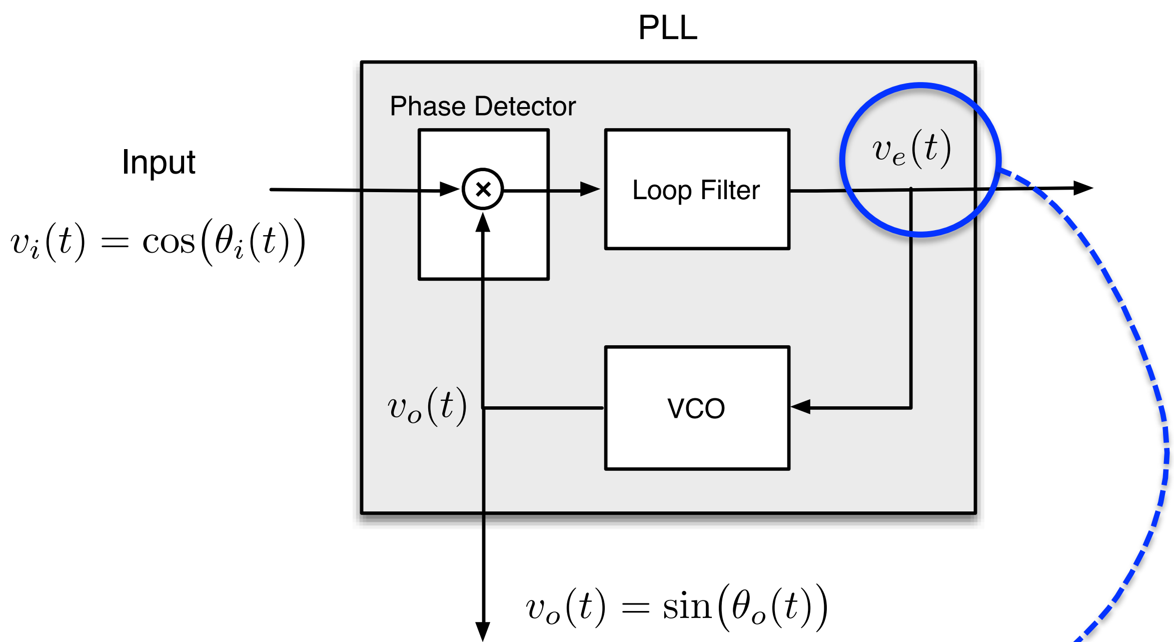
This property of the PLL makes it perfectly suitable for FM signal demodulation.

PLL is a feedback control system such where the output of the loop filter controls the instantaneous frequency of the VCO:



$$f_o(t) = \frac{1}{2\pi}\frac{d\theta_o(t)}{dt} = f_o + K v_e(t)$$

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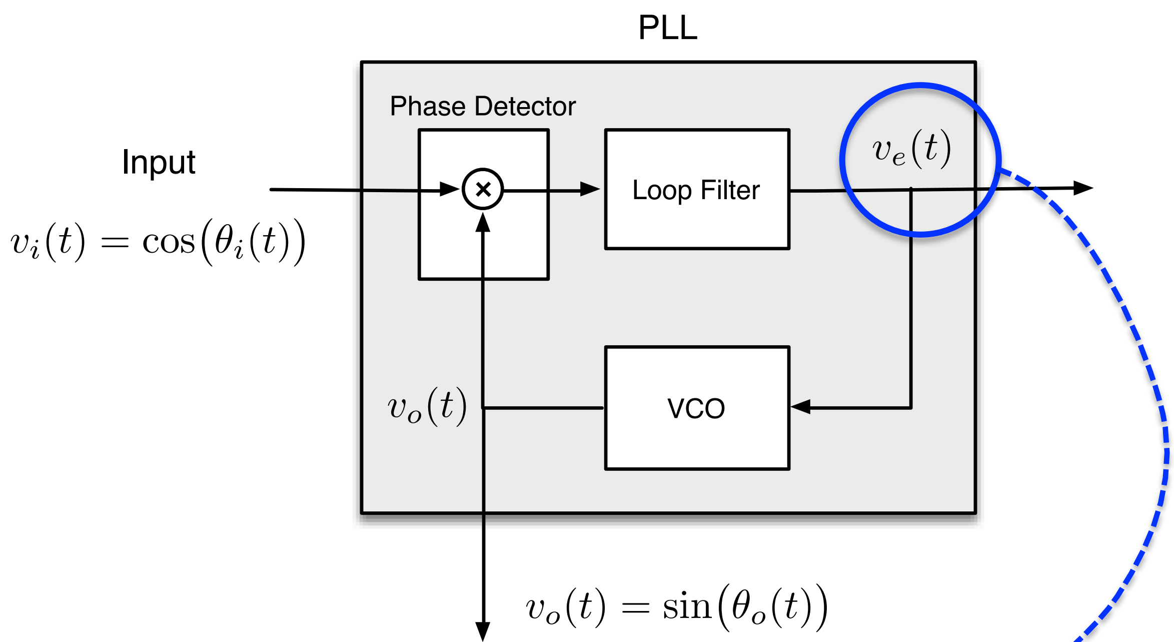


$$f_o(t) = \frac{1}{2\pi}\frac{d\theta_o(t)}{dt} = f_o + K v_e(t)$$

or equivalently:

$$\boxed{v_o(t) = \sin\left(2\pi f_o t + K\int v_e(\lambda)d\lambda\right)}$$

Assume:

- Input is the FM signal: $v_i(t) = \cos\left(2\pi f_c t + K_f \int m(\lambda) d\lambda\right)$
- VCO free-running frequency is set to the carrier frequency f_c.

Assume:

- Input is the FM signal: $v_i(t) = \cos\left(2\pi f_c t + K_f \int m(\lambda) d\lambda\right)$
- VCO free-running frequency is set to the carrier frequency f_c.

How will the PLL operate?

- The feedback action will drive the time-varying phase of the VCO output to match the time-varying phase of the input $\theta_o(t) \rightarrow \theta_i(t)$ or equivalently:

Output

Input

$$v_o(t) = \sin\Big(\underbrace{2\pi f_c t + K\int v_e(\lambda)d\lambda}_{\theta_o(t)}\Big) \qquad v_i(t) = \cos\Big(\underbrace{2\pi f_c t + K_f\int m(\lambda)d\lambda}_{\theta_i(t)}\Big)$$

$$\theta_o(t) \longrightarrow \theta_i(t)$$

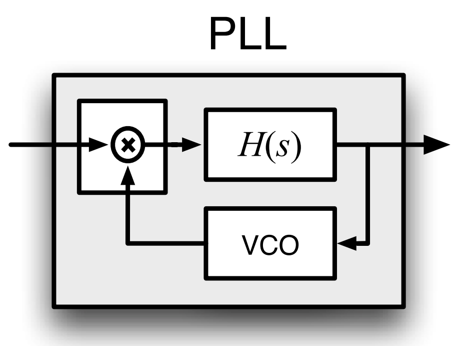
$$v_e(t) \longrightarrow m(t)$$

so that:

$$\boxed{\begin{aligned} v_o(t) &= \sin\Big(2\pi f_c t + K\int v_e(\lambda)d\lambda\Big) \\ &= \sin\Big(2\pi f_c t + K\int m(\lambda)d\lambda\Big) \end{aligned}}$$

Therefore, we can use a PLL tuned to f_c to demodulate FM signals:

$$\varphi_{\mathrm{FM}}(t) = \cos(\,\omega_c t + K_f \int m(\lambda)d\lambda\,)$$



$$\sim m(t)$$

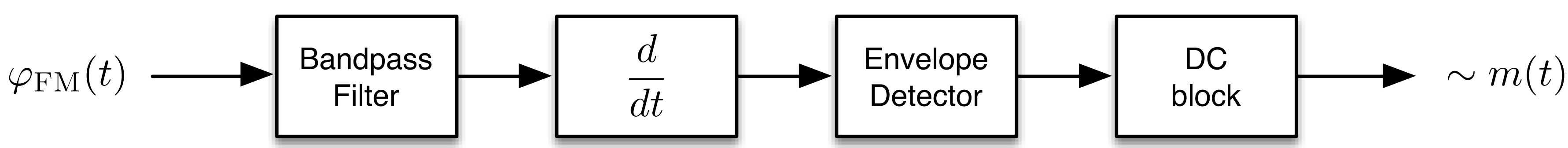
There are other FM demodulation techniques, including:

Envelope Detection

$$\frac{d\varphi_{\mathrm{FM}}(t)}{dt} = \frac{d}{dt}\left[A_c \cos\left(\omega_c t + K_f \int m(\lambda)d\lambda\right)\right]$$

$$= -A_c\left[\omega_c + K_f m(t)\right] \sin\left(\omega_c t + K_f \int m(\lambda)d\lambda\right)$$

$$= A_c \underbrace{\left[\omega_c + K_f m(t)\right]}_{\text{time-varying \textbf{envelope} with } \omega_c \gg K_f m(t)} \sin\left(\omega_c t + \left[K_f \int m(\lambda)d\lambda\right] - \pi\right)$$



There are other FM demodulation techniques, including:

- Envelope detection;
- Slope detection;
- Balanced discriminator based FM demodulation,
- Balanced zero-crossing FM detection.

- **Time- and frequency-domain description of angle modulated signals**
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- **Generation of FM signals**
  - Indirect Method

- **FM Stereo Broadcasting**
  - Stereo signal multiplexing
  - Stereo signal demodulation
  - Tips, tricks, standards …

FM Stereo Broadcasting

Objective

Design a stereophonic FM broadcasting system that:

- **conforms with FM broadcasting regulations** already in place;
- is **backward compatible** with existing monophonic FM receivers.

Key FM Broadcasting Parameters

- Maximum / Peak frequency deviation …………………… $[\Delta f]_{max} = 75$ kHz
- Message signal bandwidth ……………………………………. $B_m = 15$ kHz
- Transmission bandwidth (by Carson's Rule) …… $B_T \approx 2(\Delta f + B_m) = 180$ kHz
- FM Broadcasting regulations ………………………………… $B_T = 200$ kHz

Key FM Broadcasting Parameters

- Maximum allowed frequency deviation …………………… $[\Delta f]_{\mathrm{max}} = 75$ kHz
- Audio signal bandwidth ………………………………………. $B_{\mathrm{m}} = 15$ kHz
- Transmission bandwidth (by Carson's Rule) …… $B_{\mathrm{T}} \approx 2\,(\Delta f + B_{\mathrm{m}}) = 180$ kHz
- Transmission/Channel bandwidth (regulations) …………………… 200 kHz

---

Design Considerations

- Initial FM broadcasting was mono only.
- Transmission of an stereo FM signal has to be within the allocated FM broadcasting channel bandwidth.
- Stereo receivers have to be backward compatible with monophonic receivers.

Step 1: Create the baseband signal $m(t)$

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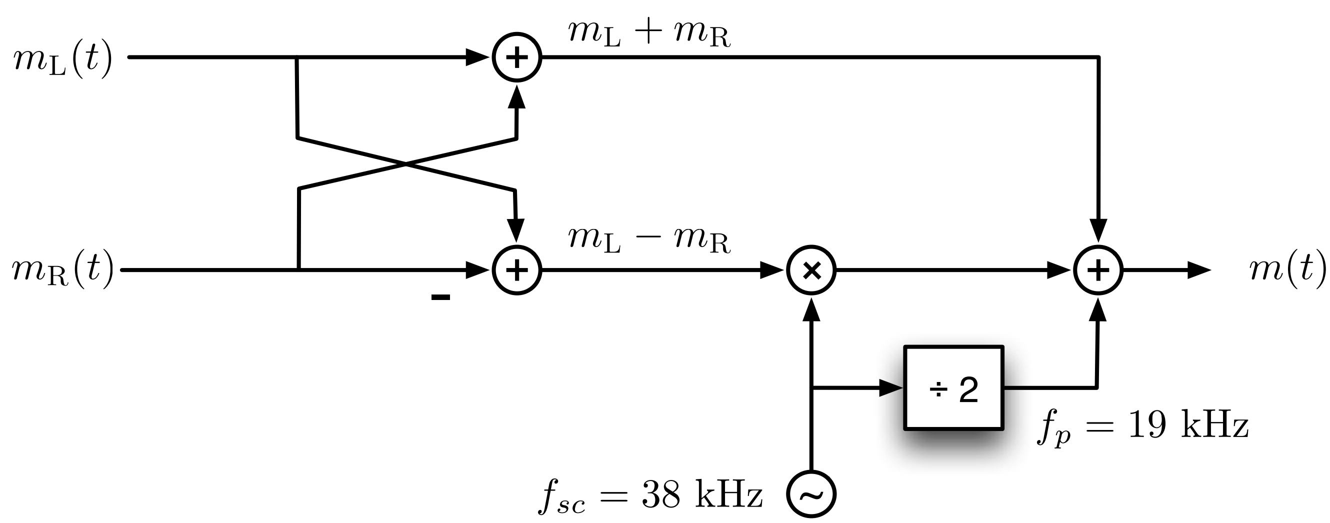

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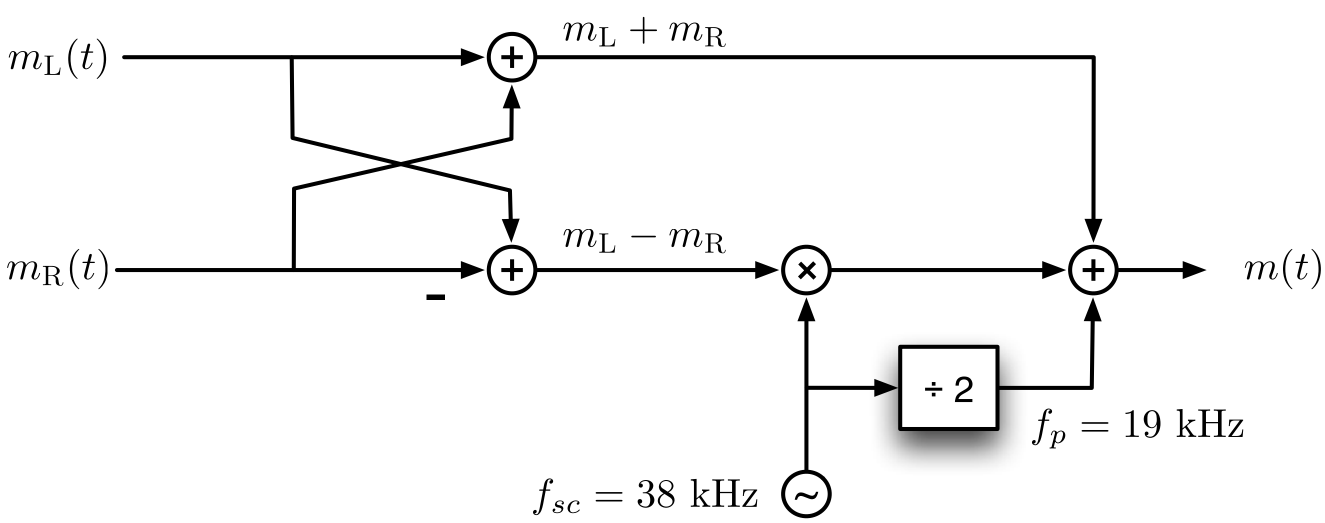
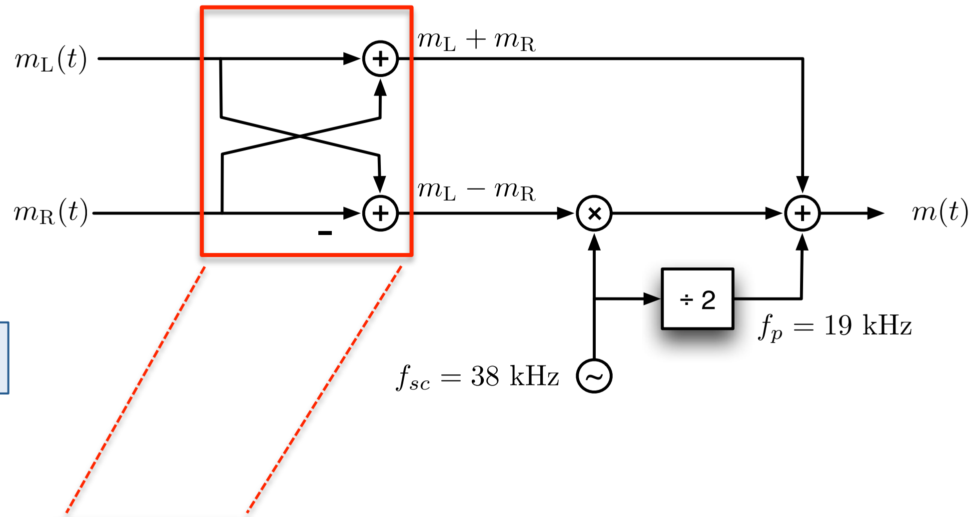
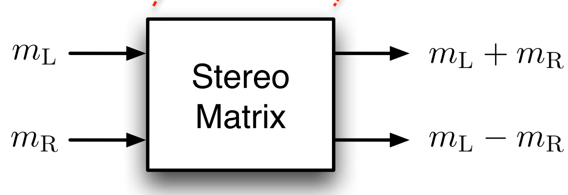


Analyze the components of $m(t)$

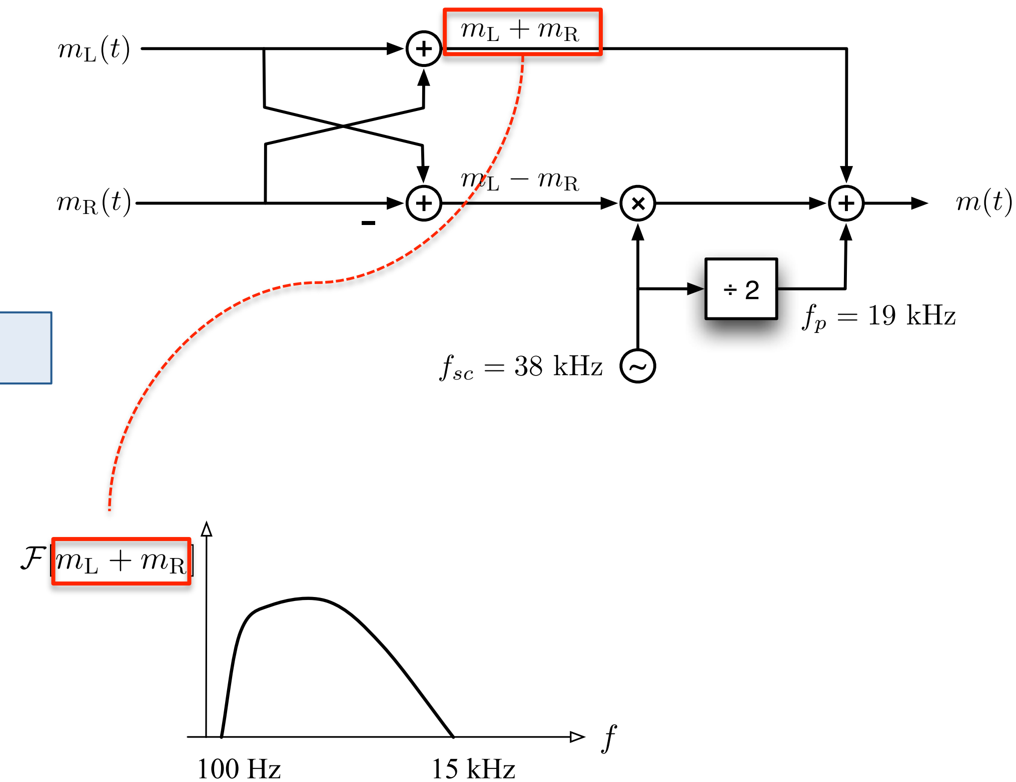


Stereo Matrix



$$\begin{bmatrix} m_{\rm L} + m_{\rm R} \\ m_{\rm L} - m_{\rm R} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} m_{\rm L} \\ m_{\rm R} \end{bmatrix}$$

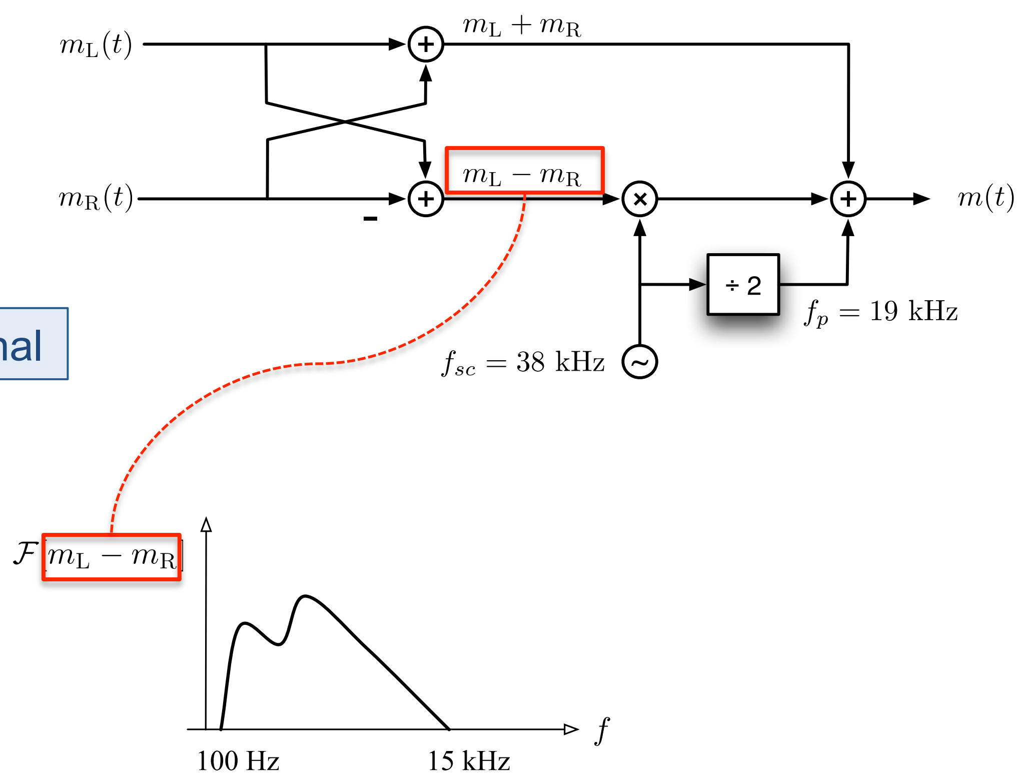
- **Backward compatibility:** cannot transmit $m_{\rm L}$ and $m_{\rm R}$ separately.
- **Monophonic receiver:** will receive only the mono signal $[m_{\rm L} + m_{\rm R}]$.
- **Stereo Matrix:** has to be non-singular for proper de-matrixing.



Mono Signal

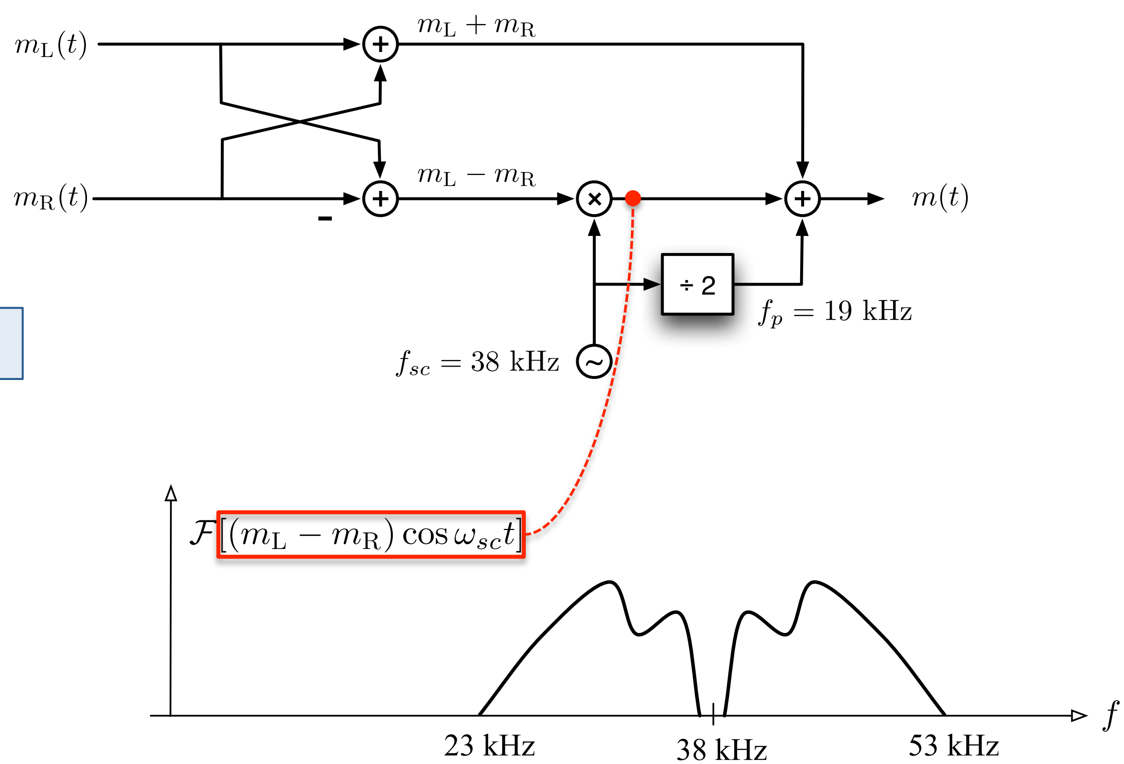
Bandwidth: m_L and m_R and therefore $[m_L + m_R]$ are bandlimited to 15 kHz.

Difference Signal

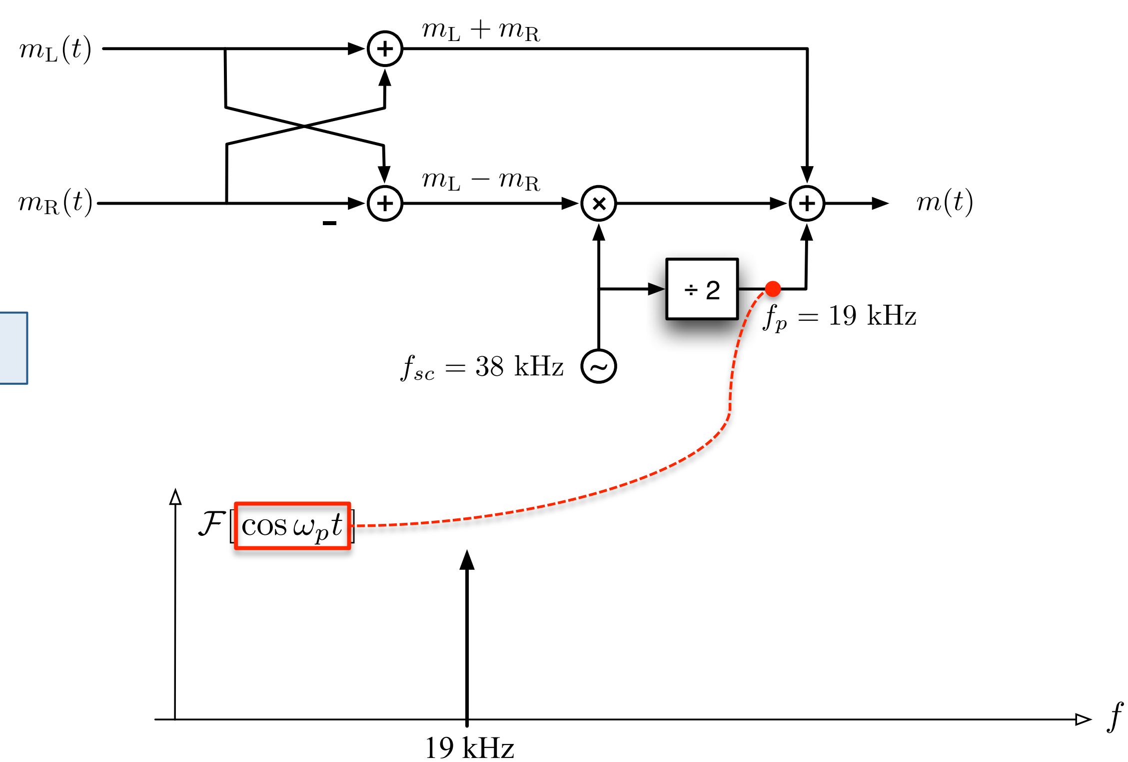


Bandwidth: m_L and m_R and therefore $[m_L - m_R]$ are bandlimited to 15 kHz.

DSB-SC Signal



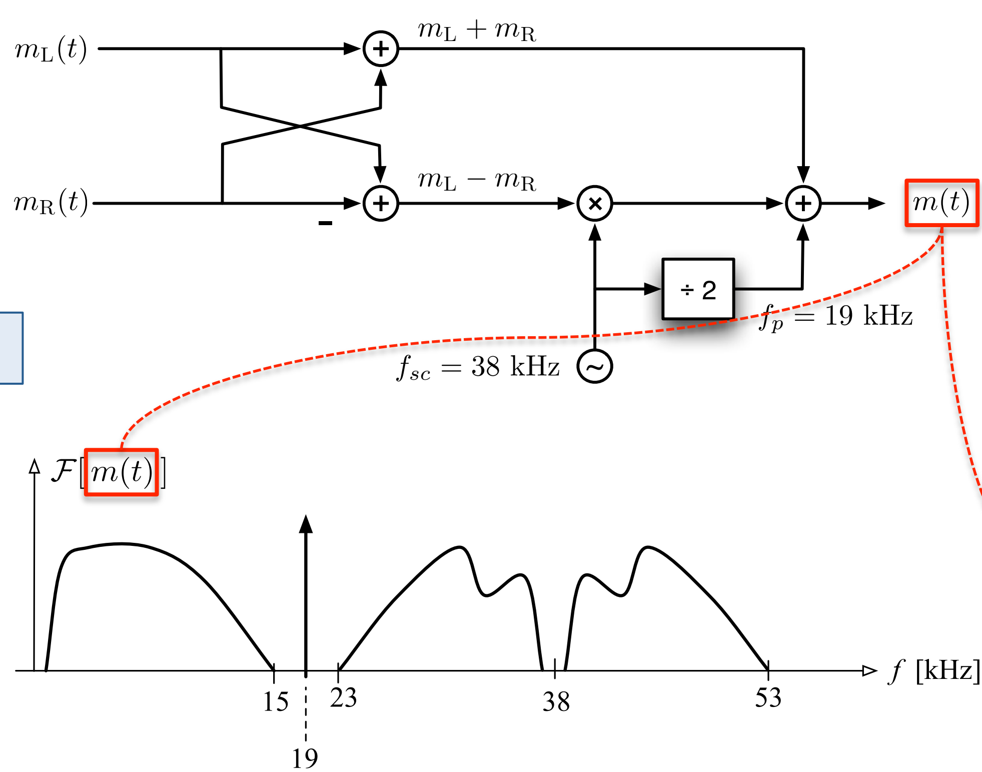
DSB-SC signal: $(m_L - m_R)\cos\omega_{sc}t$ with carrier frequency $f_{sc} = 38$ kHz.



Pilot Tone

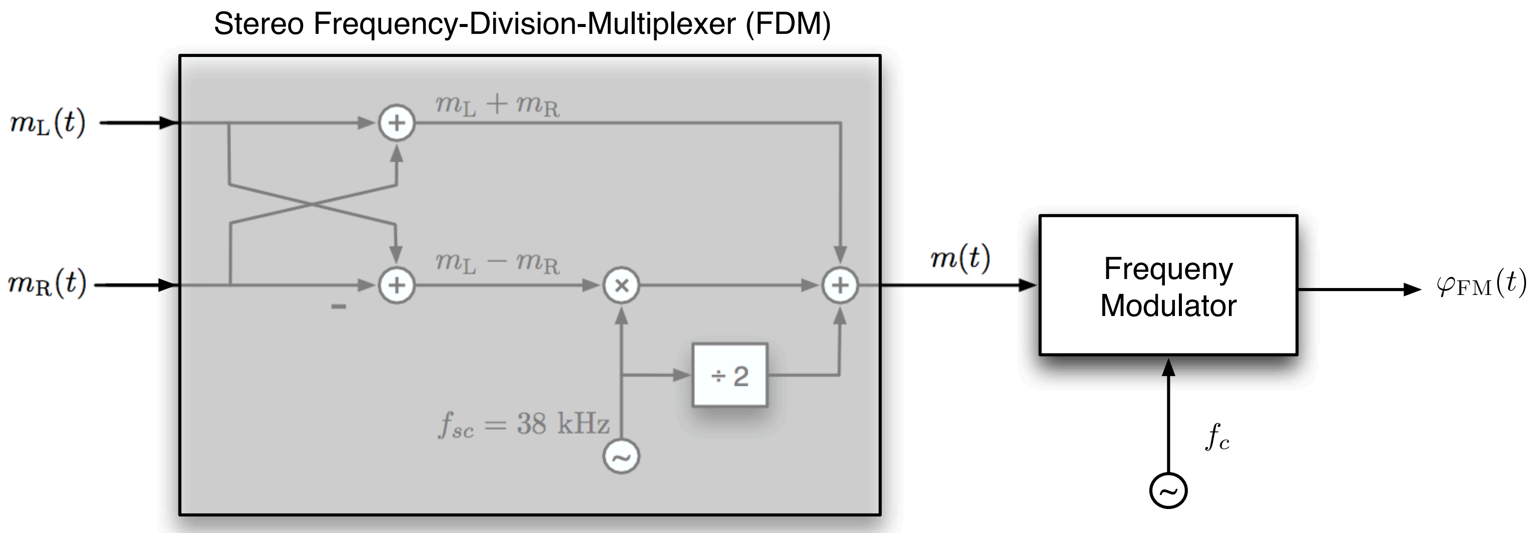
- A **Pilot Tone** at frequency $f_p = 19$ kHz is added for **coherent demodulation of the DSC-SC signal** $(m_L - m_R)\cos\omega_{sc}t$.
- The **Pilot Tone** also indicates stereo transmission.

Baseband Signal



$$m(t) = \left[m_{\mathrm{L}} + m_{\mathrm{R}}\right] + \left[m_{\mathrm{L}} - m_{\mathrm{R}}\right] \cos \omega_{sc} t + K_p \cos \omega_p t$$

Step 2: Generate a WBFM signal from the composite baseband signal $m(t)$



$$\varphi_{FM}(t) = A_c \cos\left(\omega_c t + K_f \int_0^t m(\lambda) d\lambda\right)$$

Demodulation of FM Broadcast Signals

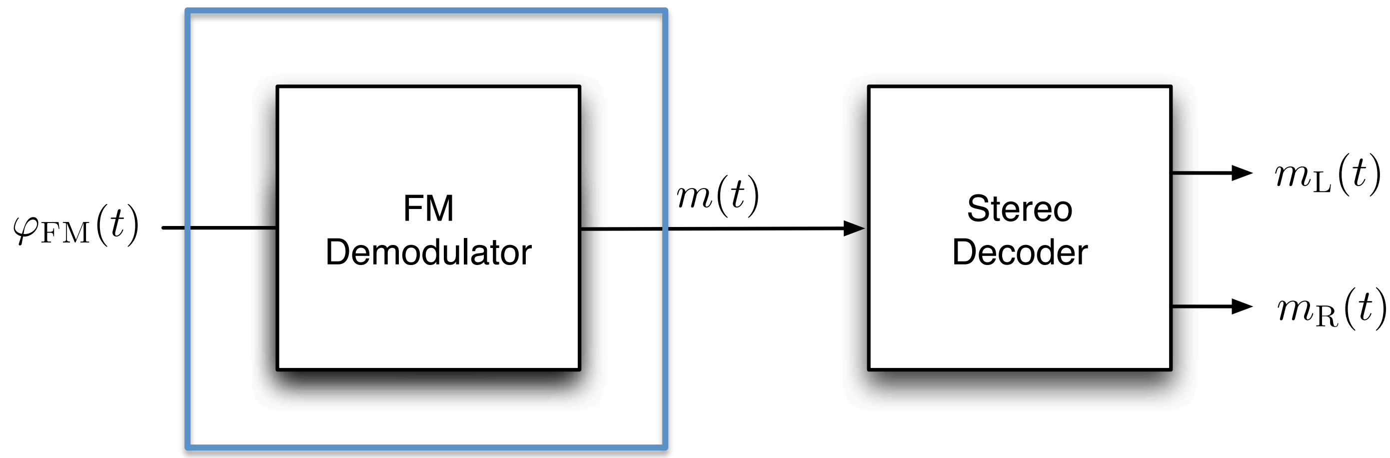
Objective

Design a stereo FM receiver:

- **demodulate** the wideband FM signal $\varphi_{\mathrm{FM}}(t)$, and
- **generate** the left- and right channel audio signals $m_{\mathrm{L}}(t)$ and $m_{\mathrm{R}}(t)$.

Demodulation of FM Broadcast Signals

Step 1: Demodulate $\varphi_{\mathrm{FM}}(t)$ to generate the composite baseband signal $m(t)$



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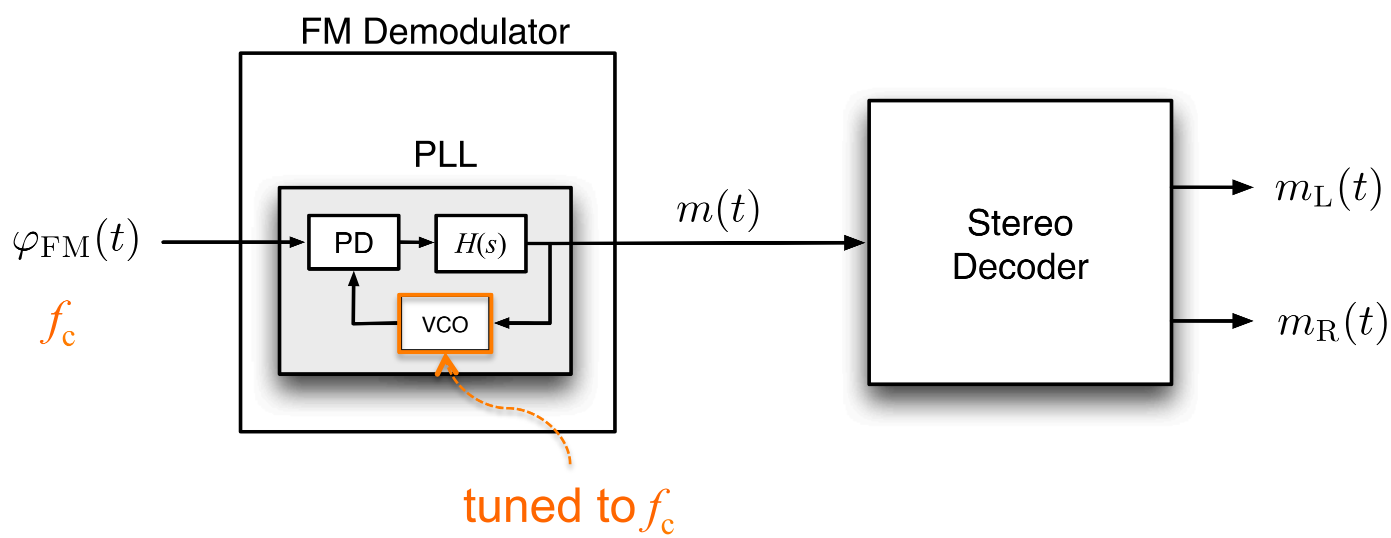

- Use a PLL with its VCO tuned to the **carrier frequency** f_c of the incoming FM signal to demodulate and generate the baseband signal $m(t)$.

Demodulation of FM Broadcast Signals

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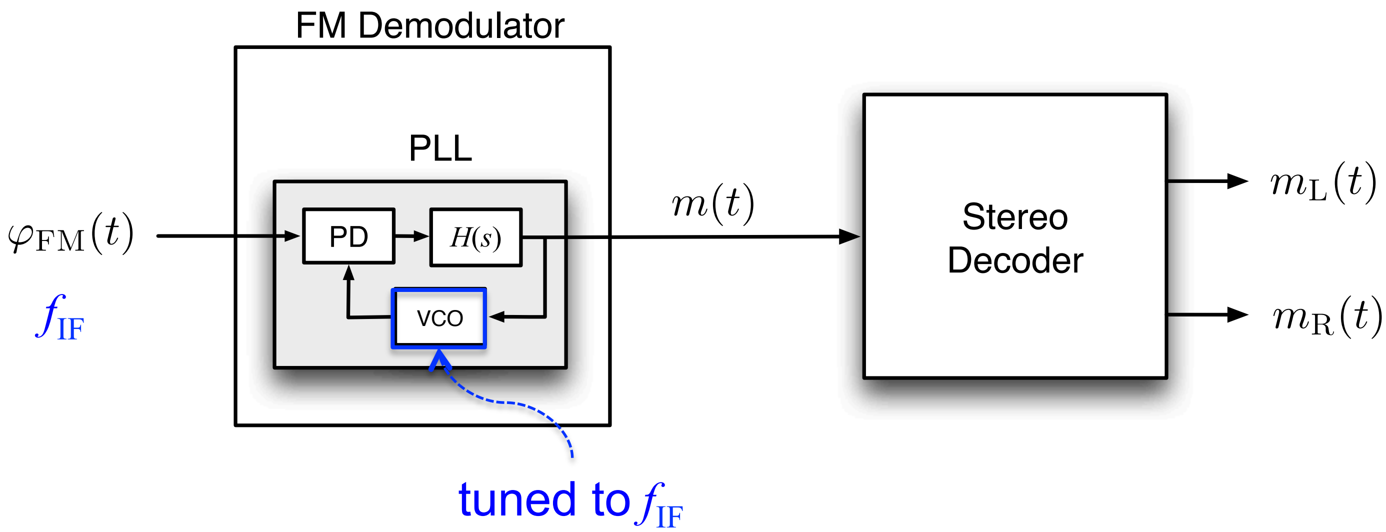
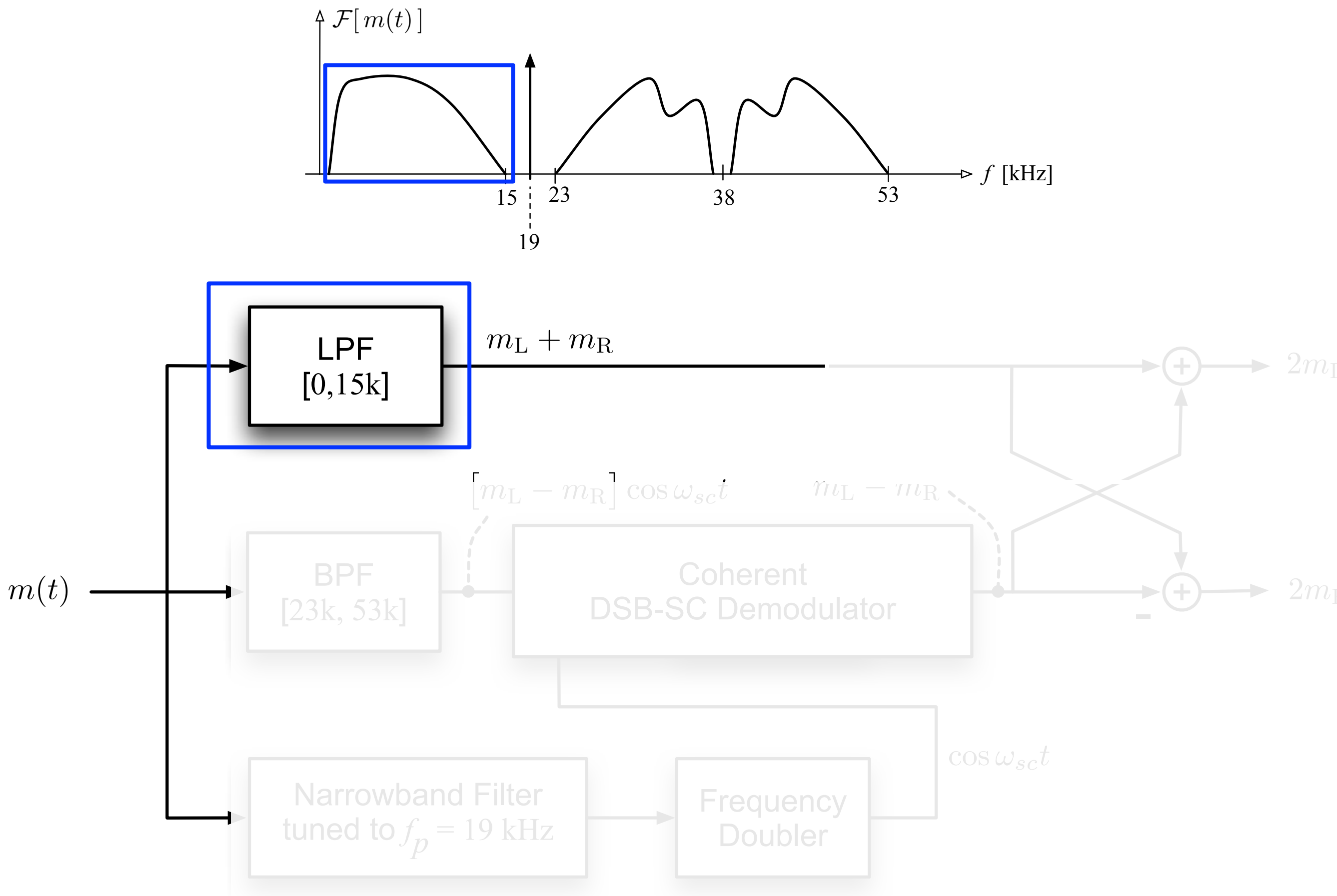


- Use a PLL with its VCO tuned to the carrier frequency f_c of the incoming FM signal to demodulate and generate the baseband signal $m(t)$.
- If the incoming FM signal is from the **output of the IF stage** of a superheterodyne receiver, then the VCO must be tuned f_{IF}.

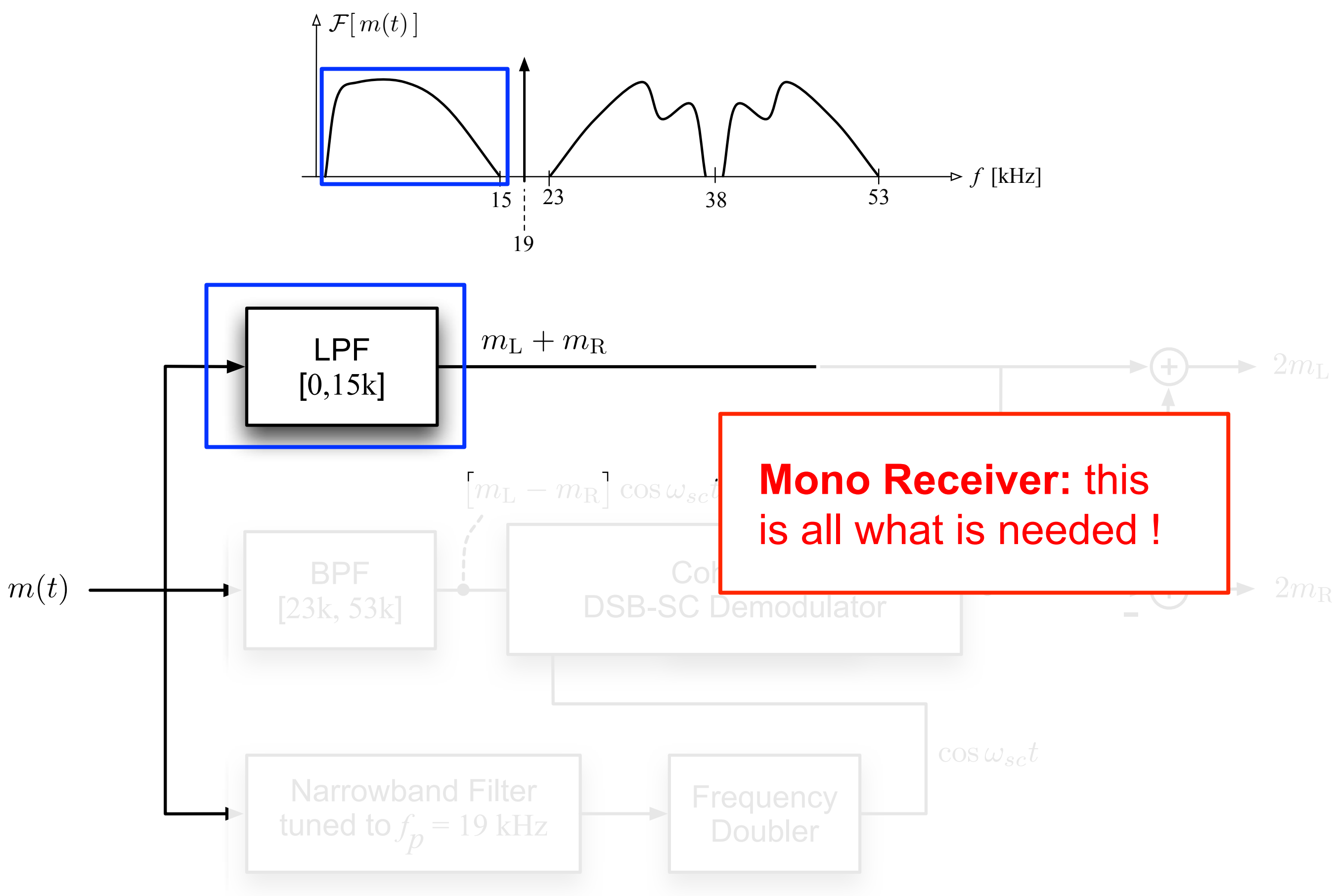
Demodulation of FM Broadcast Signals

Step 2: Demultiplex the composite baseband signal $m(t)$



Demodulation of FM Broadcast Signals

Step 2: Demultiplex the composite baseband signal $m(t)$



Demodulation of FM Broadcast Signals

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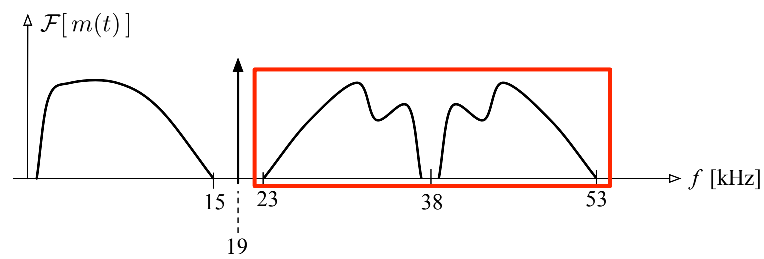

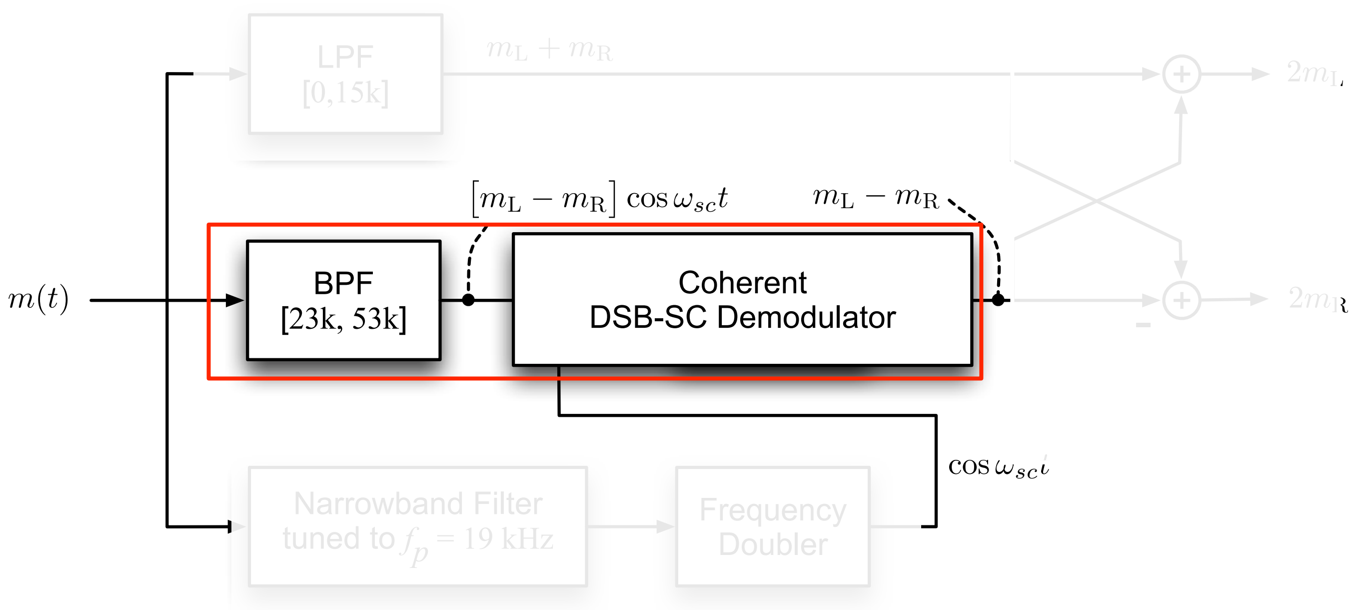

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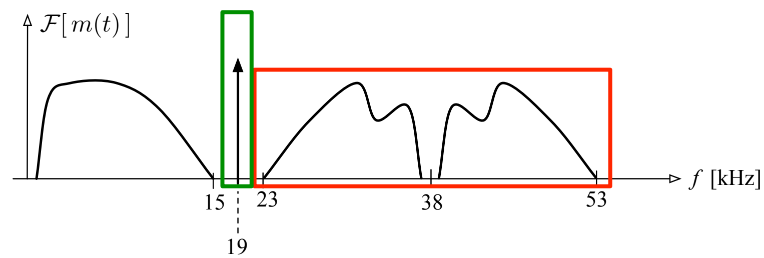

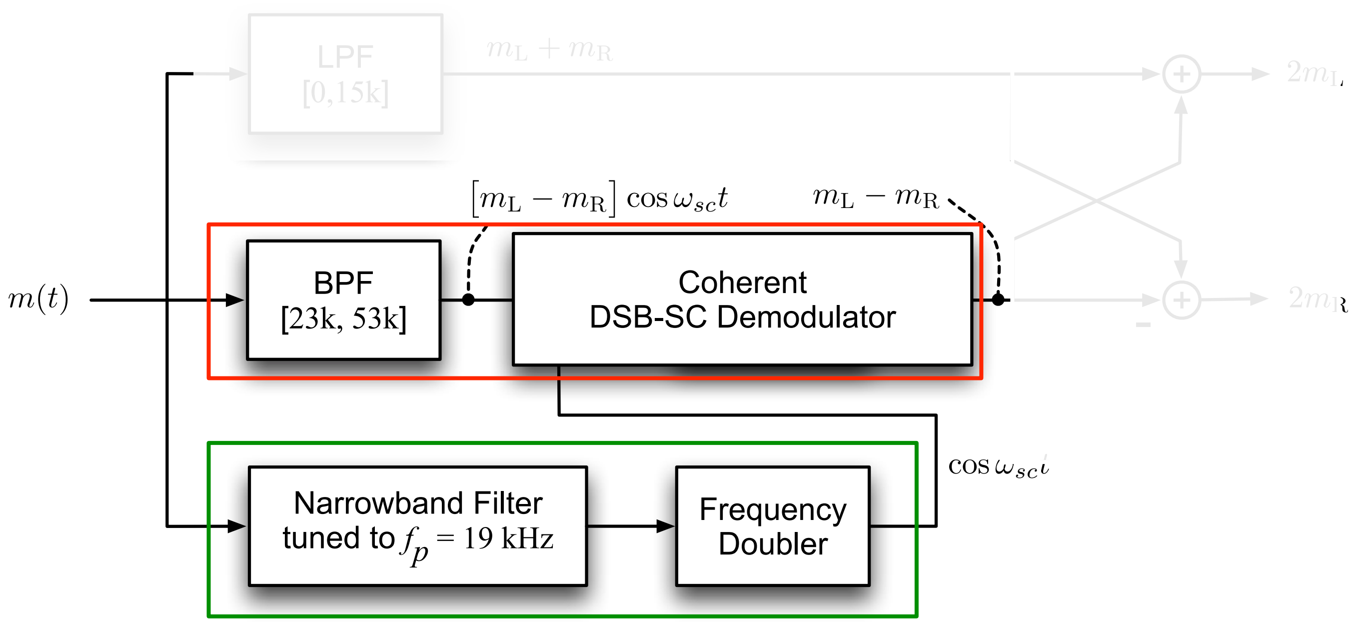

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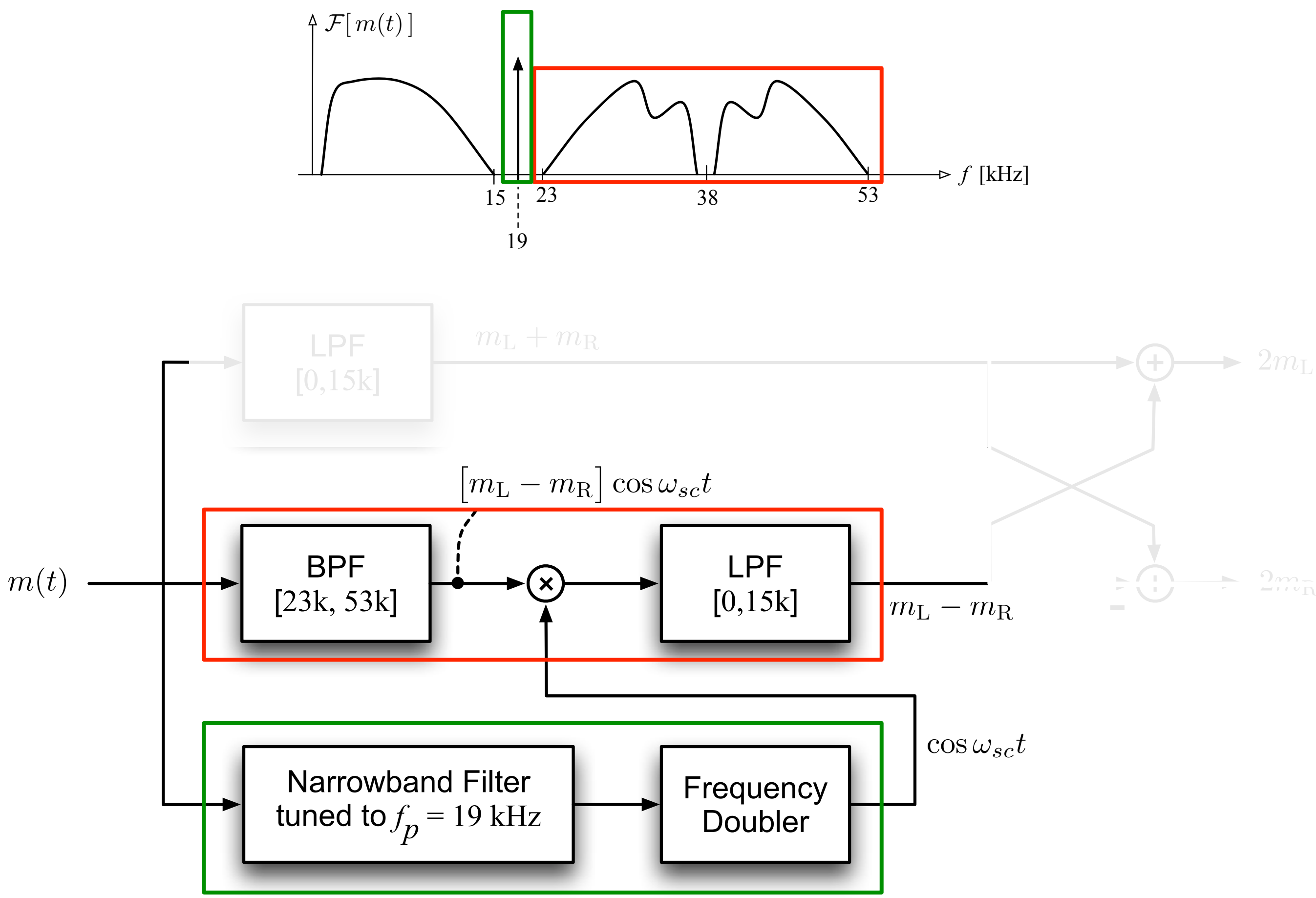

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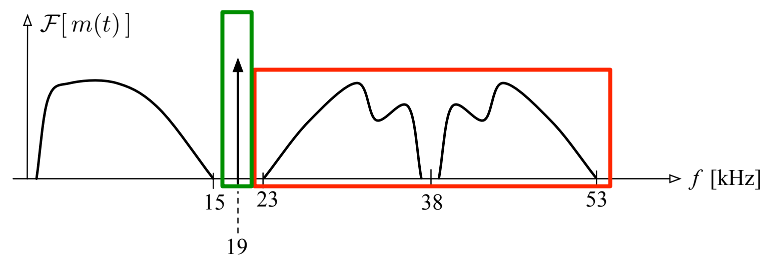

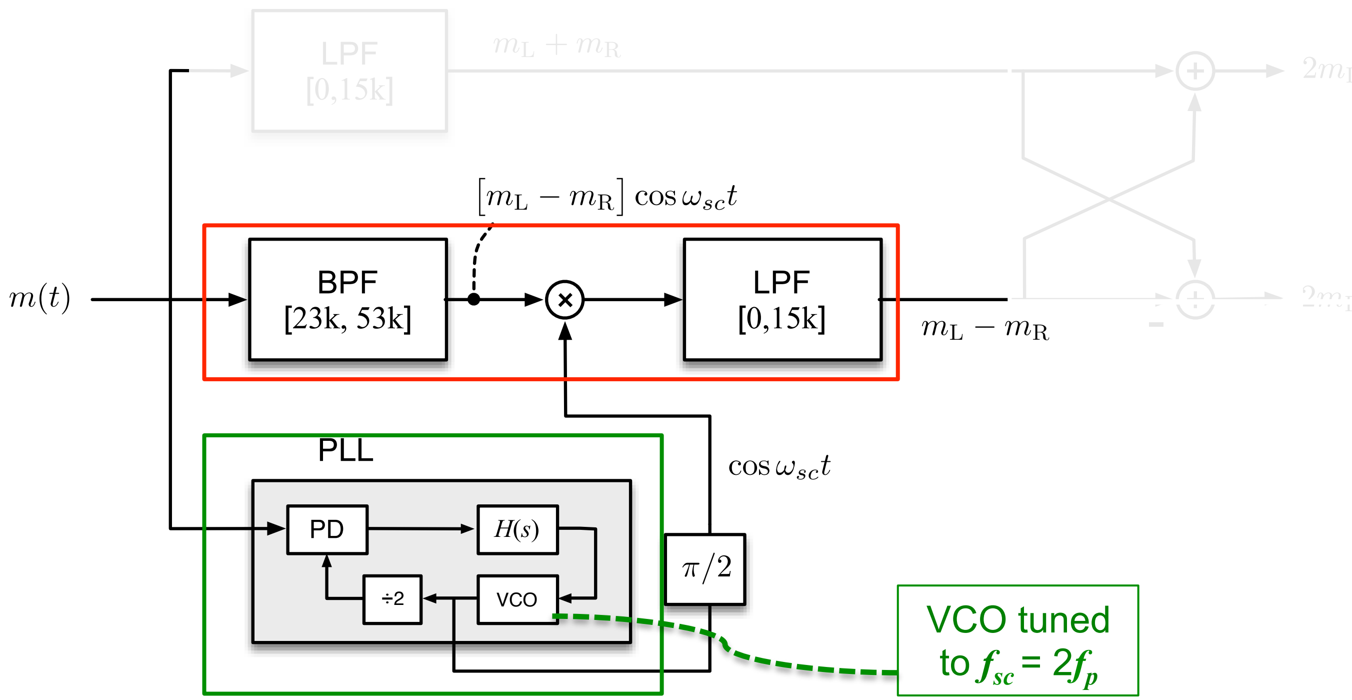

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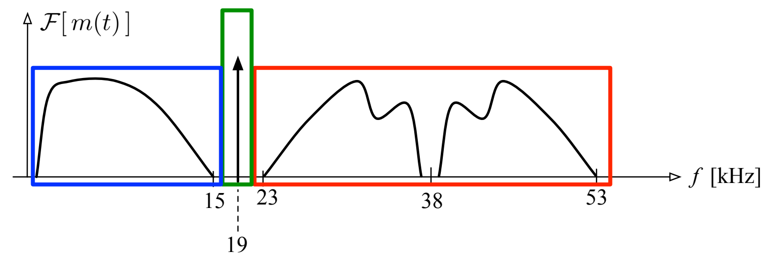
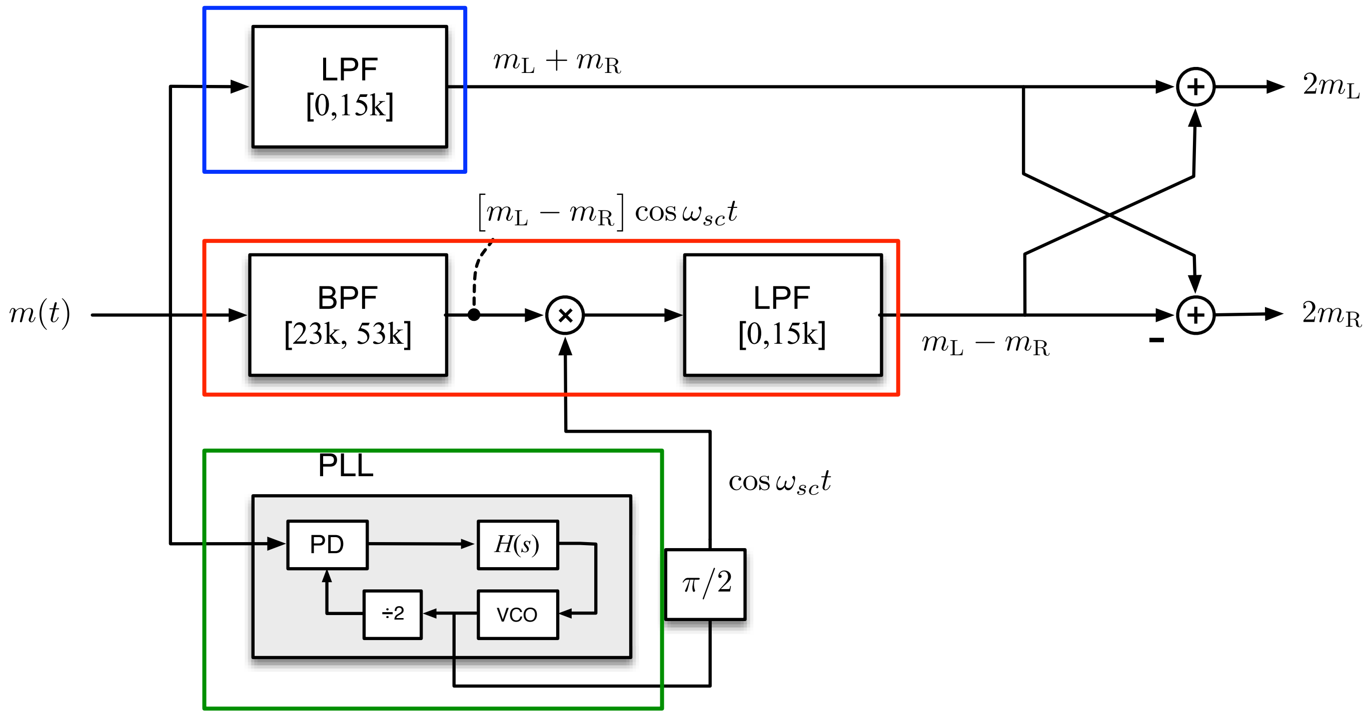
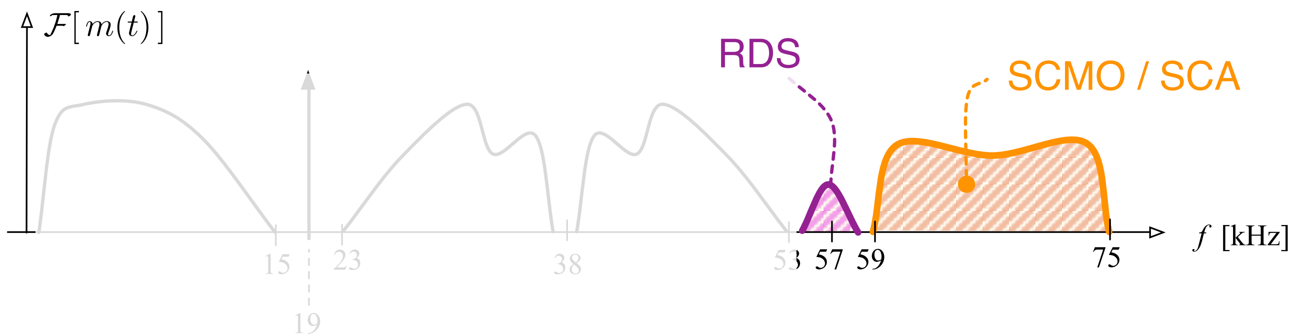


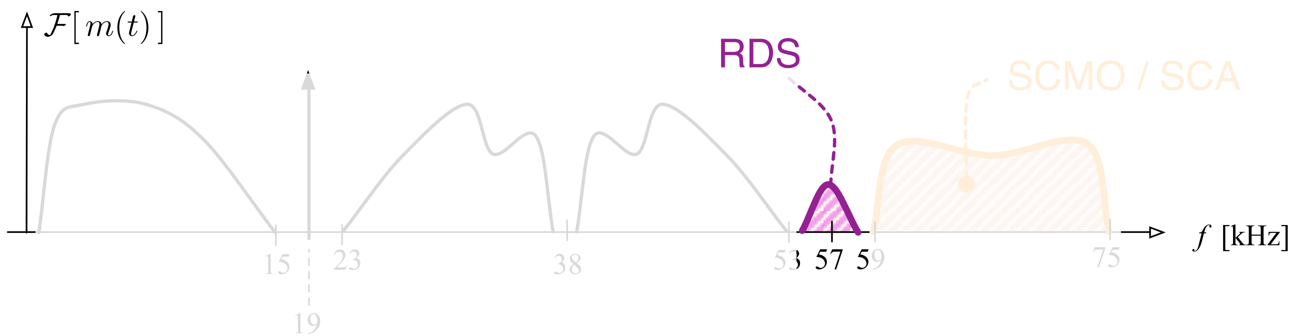


FM Broadcasting: SCMO/SCA

Actual baseband signal $m(t)$ may have other components



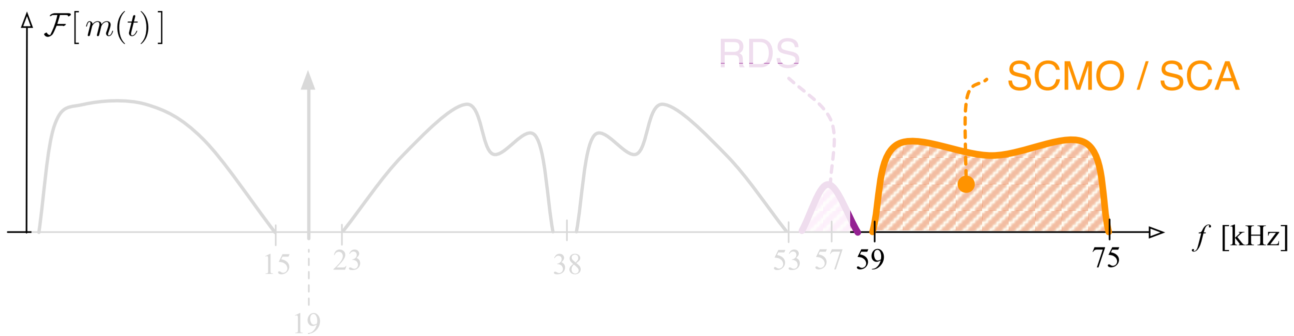
FM Broadcasting: SCMO/SCA



- **Radio (Broadcast) Data Service (RDS)**: communications protocol standard for embedding small amounts of digital information such as:
  - alternative frequencies,
  - program identification/service type (station call letters ...),
  - radio text (title and artist of the current song ...),
  - traffic message,
  - ...

Data at 1187.5 bits per second on a 57-kHz subcarrier (3 x 19 kHz).

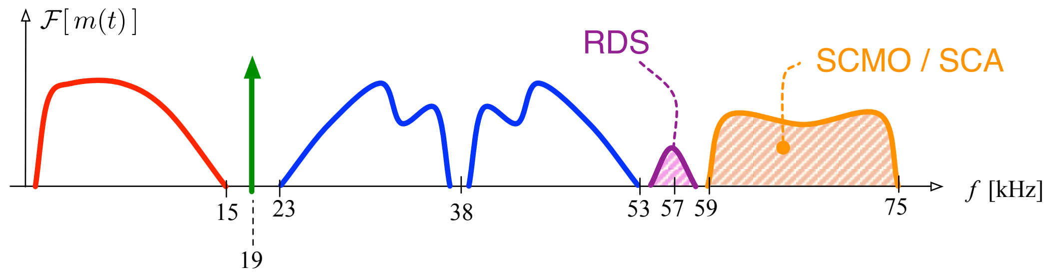
FM Broadcasting: SCMO/SCA



- **Subsidiary Communications Multiplex Operation (SCMO)** in Canada **Subsidiary Communications Authorization (SCA)** in the United States: broadcast additional services such as:
  - background music (MUZAK),
  - paging systems,
  - alternate audio/language tracks …

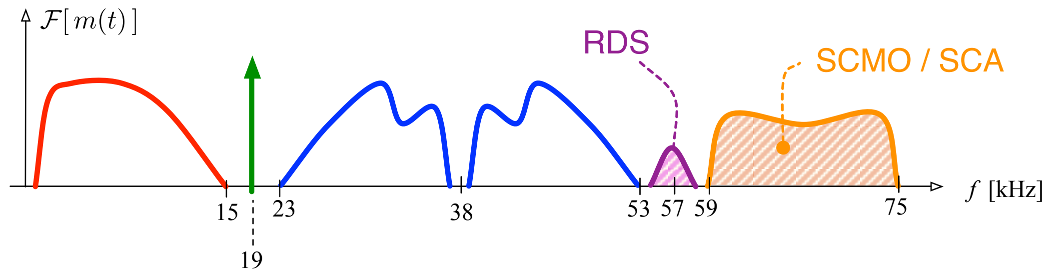
The SCMO/SCA signal is typically an FM signal with bandwidth 5 kHz.

FM Broadcasting: Allocation of Peak Frequency Deviation



$$m(t) = \boxed{[m_L + m_R]} + \boxed{[m_L - m_R]\cos\omega_{sc}t} + \boxed{K_p \cos\omega_p t} + \boxed{K_r \varphi_r(t)} + \boxed{K_s \varphi_s(t)}$$

FM Broadcasting: Allocation of Peak Frequency Deviation



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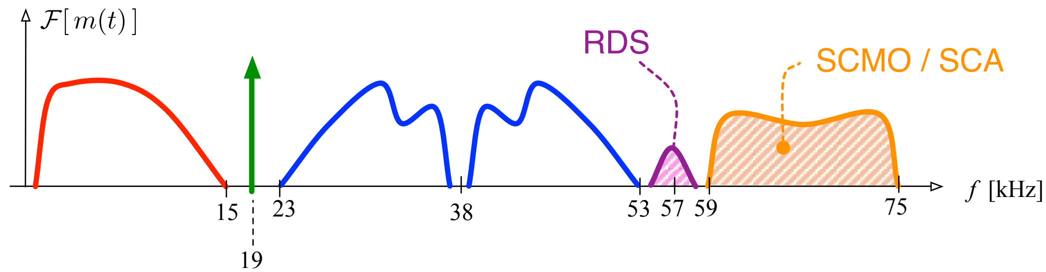
FM Modulator

$$\varphi_{\text{FM}}(t) = A_c\cos\left(\omega_c t + K_f\int m(\lambda)d\lambda\right)$$

$$= A_c\cos\left(\omega_c t + \boxed{\text{Mono}} + \boxed{\text{Stereo}} + \boxed{\text{Pilot}} + \boxed{\text{RDS}} + \boxed{\text{SCMO}}\right)$$

FM Broadcasting: Allocation of Peak Frequency Deviation



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↓

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- Modulating signal $m(t)$ bandwidth .... $B_{\mathrm{m}} = 75$ kHz
- Maximum/Peak frequency deviation .... $[\,\Delta f\,]_{\max} = 75$ kHz

not much flexibility for modulation

FM Broadcasting: Allocation of Peak Frequency Deviation

For proper signal modulation, components of $m(t)$ are allocated portions of the total allowable $[\Delta f]_{\max}$

Pilot Tone

Adjust its sensitivity parameter such that

$$[\Delta f]_{pilot} \leq 0.10\,[\Delta f]_{max} = 7.5 \text{ kHz}$$

$$\beta_{pilot} = \frac{[\Delta f]_{pilot}}{f_{pilot}} = \frac{7.5}{19} = 0.395$$

almost NBFM

FM Broadcasting: Allocation of Peak Frequency Deviation

For proper signal modulation, components of $m(t)$ are allocated portions of the total allowable $[\,\Delta f\,]_{\max}$

$$\varphi_{\mathrm{FM}}(t) = A_c \cos\left(\omega_c t + \boxed{\text{Mono}} + \boxed{\text{Stereo}} + \boxed{\text{Pilot}} + \boxed{\text{RDS}} + \boxed{\text{SCMO}}\right)$$

	Monaural Station	Stereo Station	Stereo no SCMO
Mono	70%	80%	90%
Stereo			
Pilot		10%	10%
SCMO / SCA	30%	10%	

If **RDS** is also used, then it uses 5% of the total allowable $[\,\Delta f\,]_{\max}$

Higher percentage of $[\,\Delta f\,]_{\max}$ → more signal power

FM Broadcasting: Further comments

Observations: FM Signal Power

- $P_{\mathrm{FM}} =$ **total power** in an FM signal.
- P_{FM} is **independent** of β (modulation index) or $\mathcal{D}$ (deviation index).
- $P_{\mathrm{FM}} = P_{\mathrm{useful}}(\beta) + P_{\mathrm{wasted}}(\beta)$

FM Broadcasting: Further comments

Observations: FM Signal Power

- $P_{\mathrm{FM}} =$ **total power** in an FM signal.
- P_{FM} is **independent** of β (modulation index) or $\mathcal{D}$ (deviation index).
- $P_{\mathrm{FM}} = P_{\mathrm{useful}}(\beta) + P_{\mathrm{wasted}}(\beta)$
- Increase β $\longrightarrow$ increase $P_{\mathrm{useful}}(\beta)$
- To increase $P_{\mathrm{useful}}(\beta)$ **maximize $\boldsymbol{\beta}$**.

Can we increase β arbitrarily?

FM Broadcasting: Further comments

Observations: Increasing β

- β values are **limited by $[\Delta f]_{max}$**
- β is maximum when the modulating **signal** hits **its maximum amplitude**.
- Program material (e.g. music, audio …) is **dynamic**:
  - hits peak amplitude only few times during the program;
  - most of time program material is at a much lower amplitude level
  - much lower modulation level (6-8%) resulting in lower P_{useful}
- Typical FM station: average $B_T \approx 46$ kHz whereas the maximum allowable $B_T = 200$ kHz.

What to do?

FM Broadcasting: Further comments

Observations: How to achieve higher P_{useful}

- Many FM stations use **compressors** to even out the peaks and valleys of the program material.
- Compressed signals allow **higher average** Δf , β and therefore $P_{\text{useful}}(\beta)$

FM Broadcasting: Further comments

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- Stations with heavy compression $B_T \approx 140\text{-}150$ kHz
- Price to pay → program with drastically **reduced dynamic range!...**

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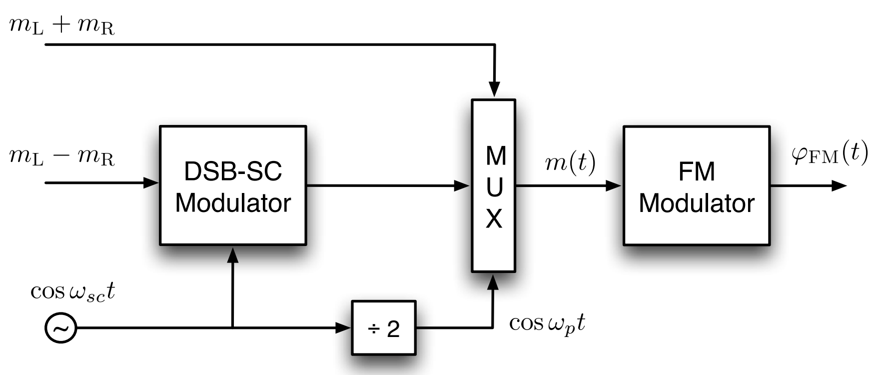
Reduced Dynamic Range: Good or Bad ?

FM Broadcasting Standards

- **Assigned carrier frequency ….. :** 88.1-107.9 MHz in 0.2 MHz increments
- **Channel bandwidth ……………. :** 200 kHz
- **Carrier frequency stability …… :** ± 2 kHz
- **Peak Frequency Deviation……. :** 75 kHz
- **Modulation ……………………… :** $\beta \approx 5$ for $\Delta f = 75$ kHz and $B_m = 15$ kHz
- **Audio frequency response …... :** 50 Hz − 15 kHz
  following a 75-μs or 125-μs pre-emphasis
- **Maximum licensed power ……. :** 100 kW (carrier power)

A systematic approach is all what is needed …



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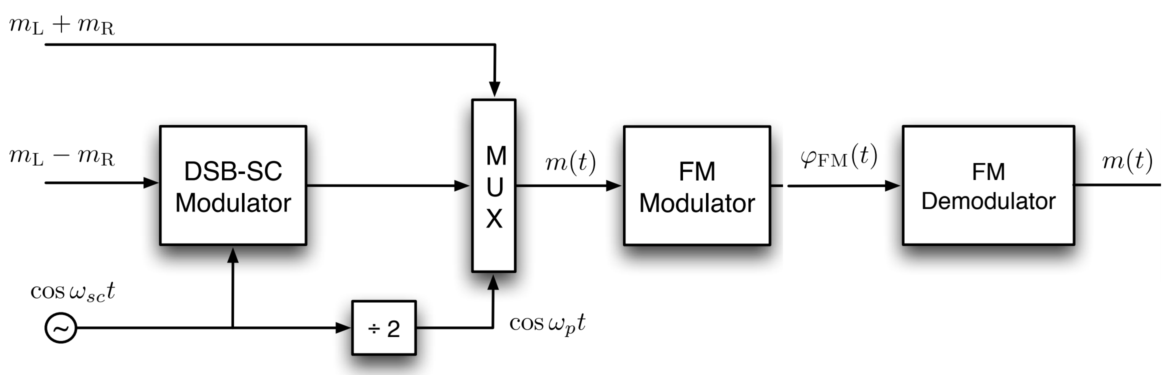

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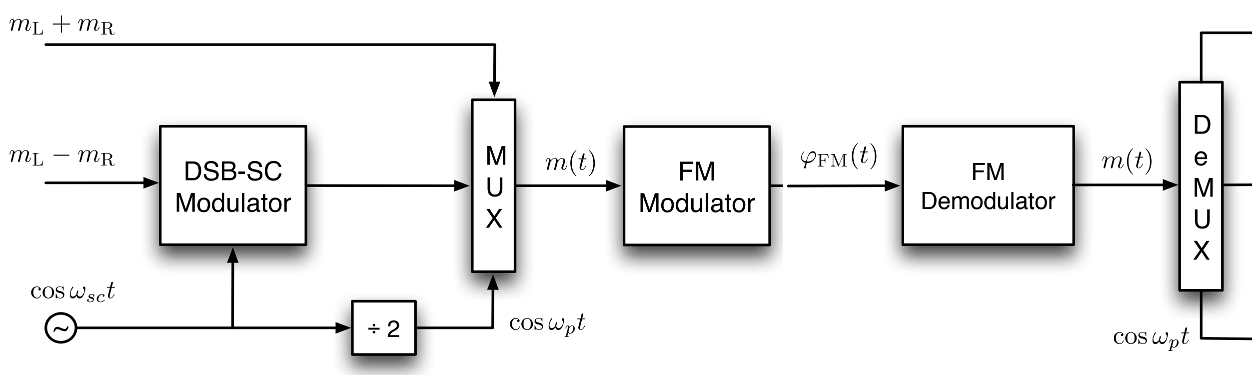

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