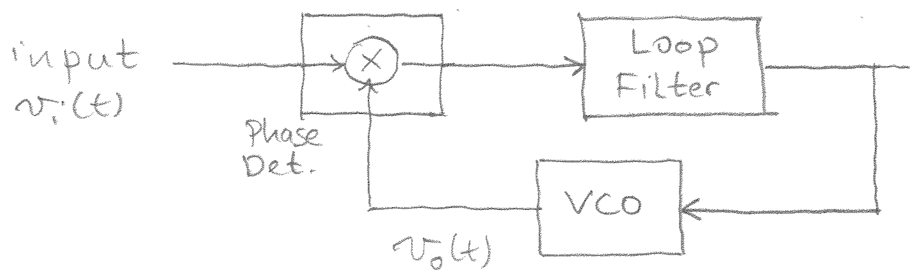
PHASED LOCKED LOOP (PLL)

PLL is a closed loop feedback system. Its action is to lock the frequency of a voltage controlled oscillator (VCO) to that of the incoming signal and to track the changes in the instantenous frequency of the incoming signal.

There are three basic functional units:

1. Phase detector (acts as a multiplier)
2. Loop filter (is a LPF.)
3. Voltage controlled oscillator, the VCO.



$v_i(t) = \cos(\omega_c t + \varphi_i(t))$

$v_o(t) = \sin(\omega_o t + \varphi_o(t))$

- If $v_i(t) = \emptyset \Rightarrow$ PD output $= \emptyset$
  control/error voltage to VCO $= \emptyset$
  VCO continues to operate at its free running frequency f_o (also referred to as the centre frequency of the PLL).

- If $v_i(t) \neq \emptyset \Rightarrow$ PD generates a non-zero output
  Since PD acts as a multiplier, it generates a signal with frequency components $f_c \pm f_o$

$$v_i(t)\, v_o(t) \sim \cos(\omega_c t + \varphi_i) \sin(\omega_o t + \varphi_o)$$
$$\sim \left[\sin((\omega_c - \omega_o)t + (\varphi_c - \varphi_o)) + \sin((\omega_c + \omega_o)t + (\varphi_c + \varphi_o))\right]$$

- If $|f_o - f_c| > f_L$ (a threshold), then both components in the product $v_i(t) v_o(t)$, i.e., the sum & difference terms, are eliminated by the loop filter, the input to the VCO is still zero and the PLL does not acquire a lock.

- If $|f_o - f_c| < \mu$, the sum component will be eliminated by the Loop filter. But the difference term will be within the passband of the loop filter and it is applied as a control signal to the VCO
- This control signal will cause the VCO frequency to vary in a direction which reduces the error signal.
- If f_c is sufficiently close to f_o, the feedback action of the Loop causes the VCO to lock to the instantenous frequency of the input signal.
- Once in lock, the VCO frequency is identical to that of the input signal. The action of the Loop is to cause $(\varphi_c - \varphi_o)$ to take on that value which is required to generate the VCO control voltage necessary to change the frequency of the VCO from its free running frequency to that of the input signal frequency.
- This action allows the PLL to "track" any instantenous frequency changes of the input once Lock has been acquired.
- Initially, we adjust the VCO so that when the control voltage is zero, the following two conditions are met

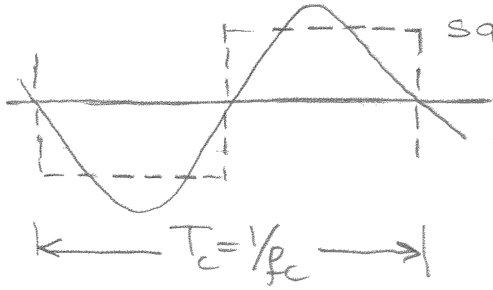
  ① $f_o = f_c$

  ② VCO output has $90°$ phase shift with respect to the unmodulated carrier wave:
  - unmodulated carrier $\cos \omega_c t$
  - free-running VCO output $\sin \omega_c t$

  Then the PD output will be

  $$\sin(\varphi_o(t) - \varphi_i(t)) = \phi.$$

- The VCO waveform used in most PLLs is a square-wave, i.e.



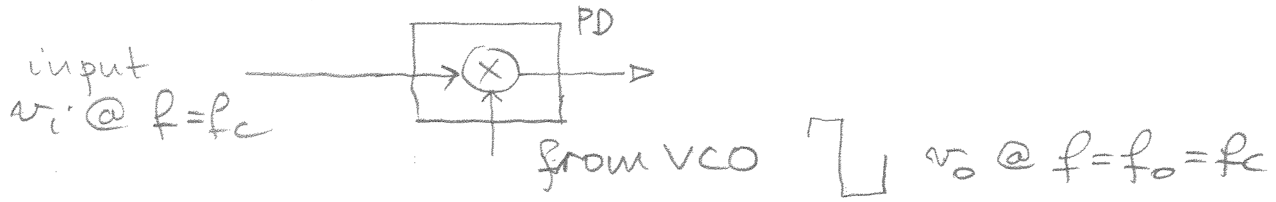
square-wave } use instead
sine-wave } of

Why:
Square wave is simpler to generate. Also since PD = [multiply] + [filter] + [amplitude scale] and

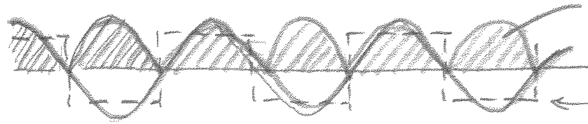
$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2\pi(2n-1) f_o t)$$

after multiplication and filtering in the PD all high-order terms will be eliminated. With only the fundamental in the above series remaining the net effect is the same.

- Let us demonstrate why we need the 90° phase difference in the two inputs to the PD.



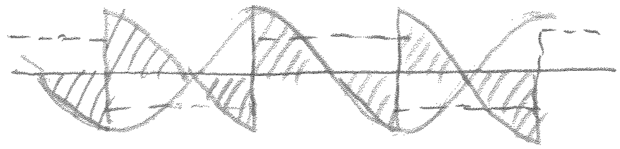
CASE 1 $v_i = \cos \omega_c t$, $v_o = \cos \omega_o t = \cos \omega_c t$ } $\varphi_e = \varphi_o(t) - \varphi_i(t) = \phi$.



area after multiplication

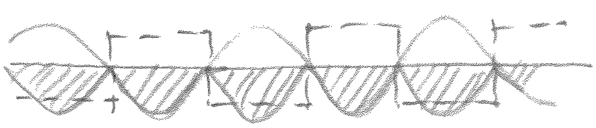
PD output = $\boxed{\text{maximum positive}}$

CASE 2 $v_i = \cos \omega_c t$, $v_o = \sin \omega_o t = \sin \omega_c t = \cos(\omega_c t - \pi/2)$ } $\varphi_e = -\pi/2$



output = area after mutiplication = $\boxed{\phi}$.

CASE 3 $v_i = \cos \omega_c t$, $v_o = -\cos \omega_o t$ } $\varphi_e(t) = -\pi$



output = area after mult.
= $\boxed{\text{mininum, negative}}$

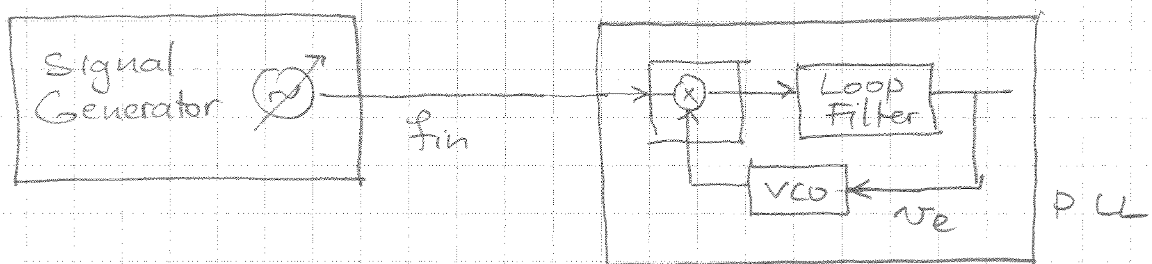
Some Definitions

- **Free-running frequency / centre frequency**
  This is the frequency at which the VCO operates when not locked to an input signal. Pre-set by external components (R, L, ...) to PLL.

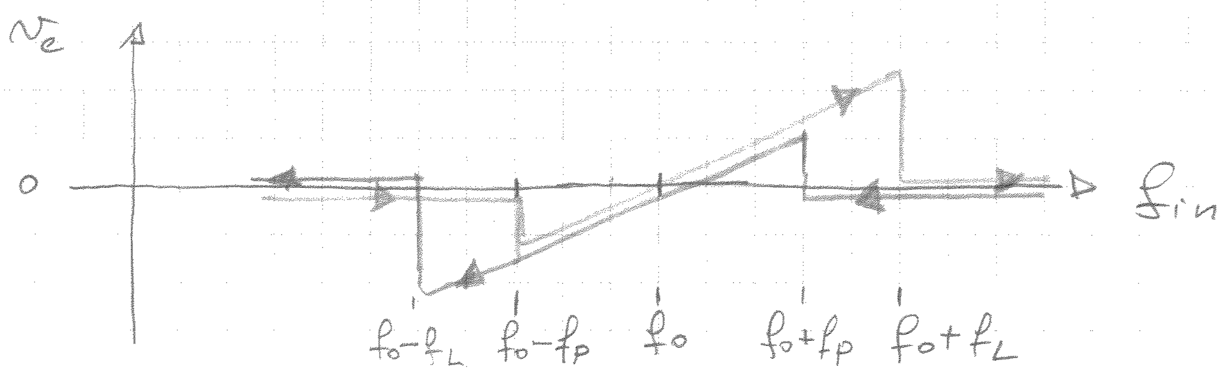
- **Lock range / tracking / hold-in range**
  The frequency range in the vicinity of f_0 over which the PLL, once locked to the input, will remain in lock, $\boxed{f_L}$

- **Capture range / pull-in / acquisition range**
  The maximum initial frequency difference between the input signal and the VCO output for which the PLL can acquire lock, $\boxed{f_P}$

How to measure these parameters?



- Fix VCO free-running frequency at f_0.
- Change (both increase and decrease) f_{in}
- Measure the input to VCO, i.e., v_e, the control voltage based on the error signal generated by the PD
- No error $\Rightarrow v_e = \phi$ or No-Lock $\Rightarrow v_e = \phi$.

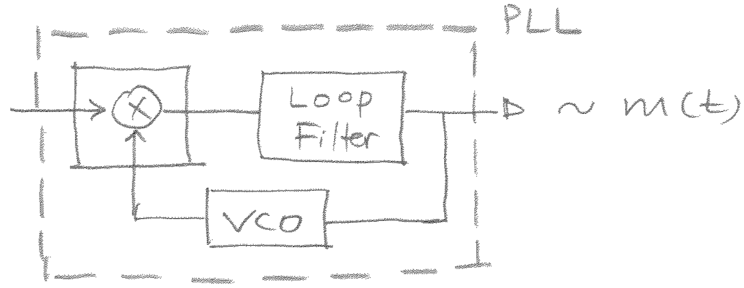


APPLICATION EXAMPLES

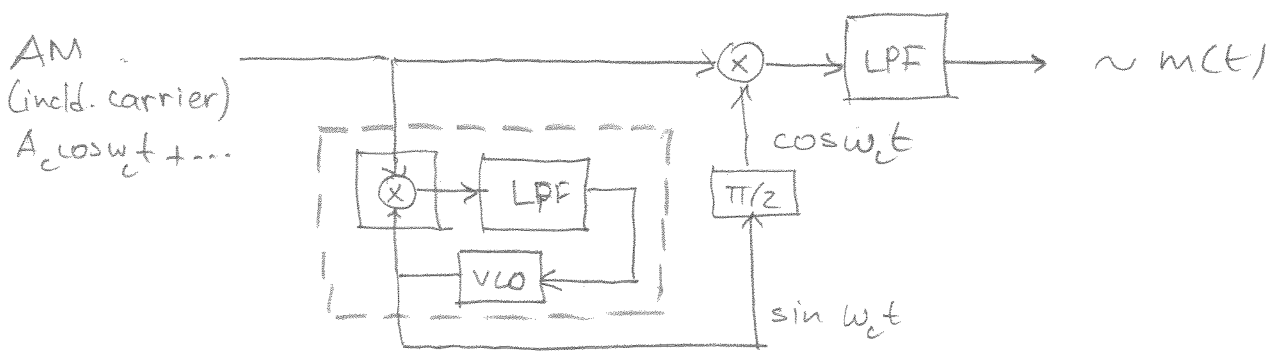
① Narrow BW PLL: narrow-band tracking filter.

② Wider BW: FM Demodulation.

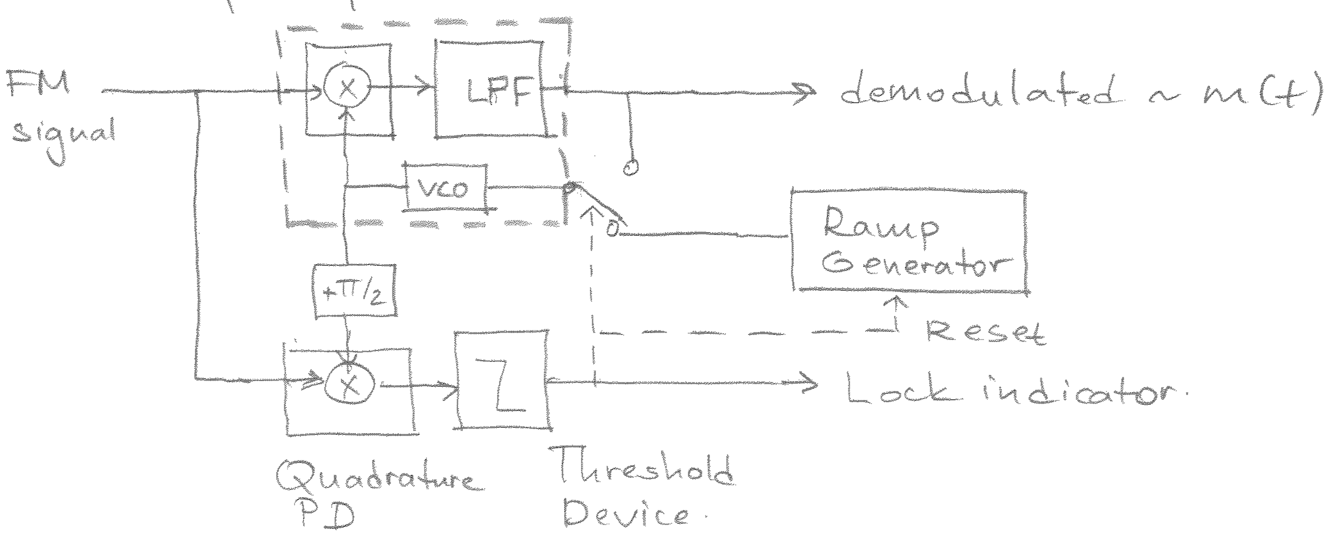
$$v_i(t) = \cos\left(\omega_c t + k_f \int_0^t m(\lambda)\,d\lambda\right)$$



③ Coherent/Sychronous) demodulation (consider AM with separate carrier term)

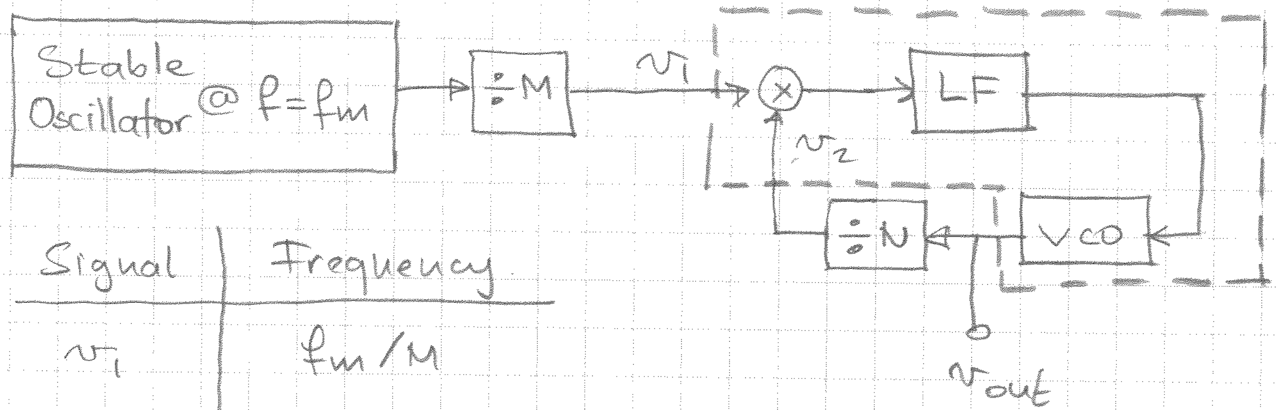


④ PLL Sweep acquisition



VCO frequency can be swept to search for a signal to Lock on.
For example, the above set-up can be used as a "seek" function we encounter in car/home radios.

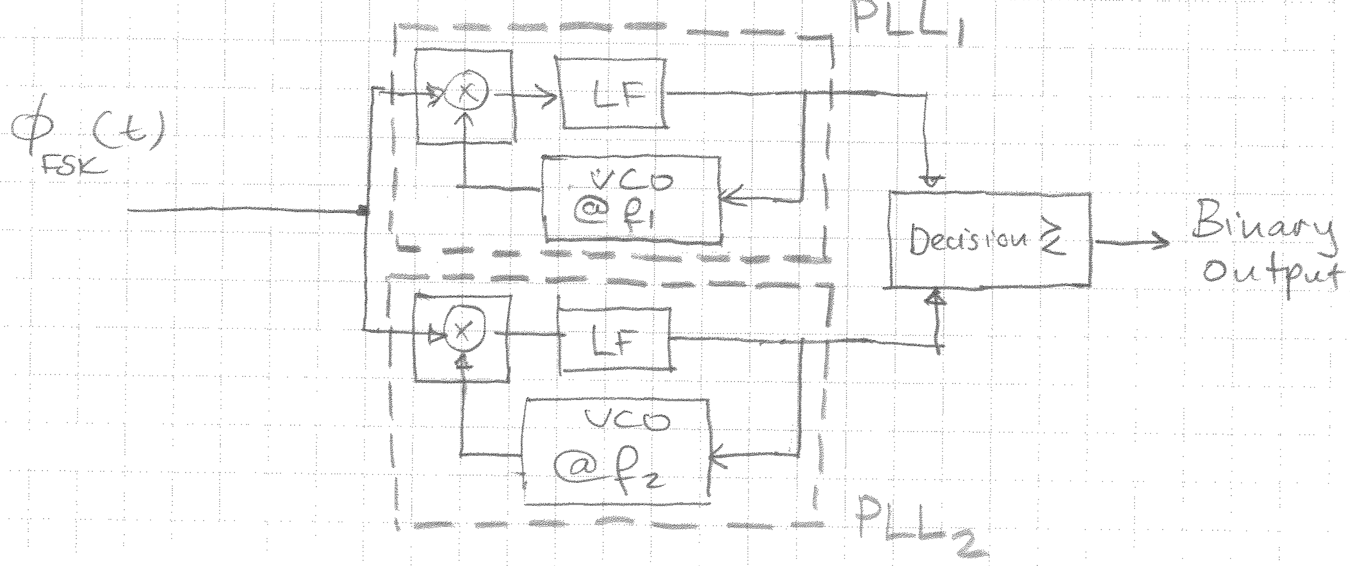
(5) Frequency Synthesis



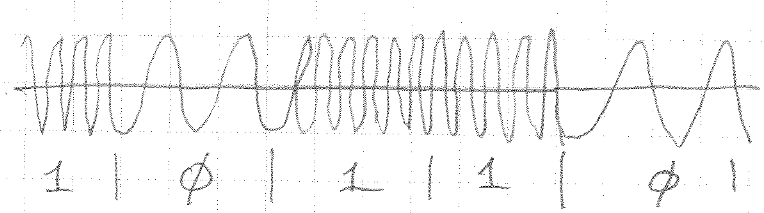
Signal	Frequency
v_1	f_m / M
v_2	Locked at $f_2 = f_m / M$
v_0	$f_{out} / N = f_2 = f_m / M$

$$\Longrightarrow \boxed{f_{out} = \frac{N}{M} f_m}$$

(6) Coherent FSK demodulation.



Binary "1" $\longrightarrow f_1$
Binary "∅" $\longrightarrow f_2$



For example caller ID signal detection.