ELE 635 Communication Systems

Assignment Problems

and

Solutions

Winter 2015

CONTENTS

Note

Assignment problems shown in this document are from the 3*rd* Edition of the course reference text. Mapping of assignment problems from this earlier edition to problems in the 4*th* Edition is shown on pages (iii)–(viii).

Problems from the 3rd edition

(2.1-1) Find the energies of the signals:

a) sin t, $0 \leq t \leq \pi$
b) –sin t, $0 \leq t \leq \pi$
c) 2 sin t, $0 \leq t \leq \pi$
d) sin (t-2π), $2\pi \leq t \leq 4\pi$

Comment on the effect on energy of sign change, time shifting or doubling of the signal. What is the effect on the energy if the signal is multiplied by k?

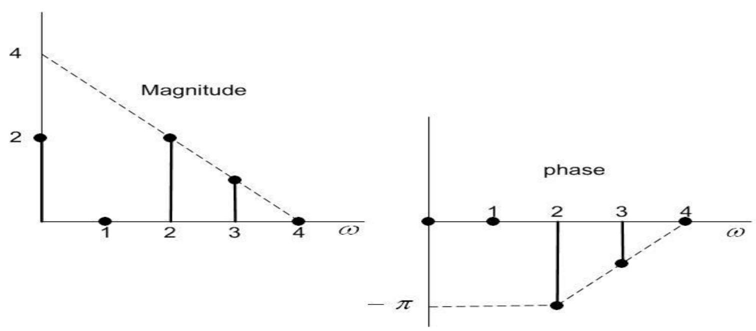
(2.9-2) of the 3rd edition is the same as (2.9-2) of 4th edition with the following change:

$$g(t) = 3\cos t + \cos(5t - 2\pi/3) + 2\cos(8t + 2\pi/3)$$

(2.9-3) Figure below shows the trigonometric Fourier spectra of a periodic signal *g(t)*.

a) By inspection of the figure, find the trigonometric Fourier series representing *g(t)*.
b) By inspection of the figure, sketch the exponential Fourier spectra of *g(t)*.
c) By inspection of the exponential Fourier spectra obtained in part (b), find the exponential Fourier series of *g(t)*.

Show that the series found in parts (a) and (c) are equivalent.



(3.1-5) of the 3rd edition is the same as (3.1-4) of the 4th edition

(3.1-7) of the 3rd edition is the same as (3.1-6) of the 4th edition with the following changes:

Figure (a): bandwidth = π/2; Figure (b): bandwidth= ω_o.

(3.2-2) Show that the Fourier transform of rect (t-5) is $sinc(\frac{\omega}{2})e^{-j5\omega}$.

(3.3-6) of the 3rd edition is the same as (3.3-6) of the 4th edition with the following changes:

Figures (b) and (c): Frequency range is from π to 3π

(3.3-7) Using the frequency-shifting property, find the inverse Fourier transform of the following spectra:

a) $G(\omega) = rect(\frac{\omega-4}{2}) + rect(\frac{\omega+4}{2})$
b) $G(\omega) = \Delta(\frac{\omega-4}{4}) + \Delta(\frac{\omega+4}{4})$

(3.3-10) of the 3rd edition is the same as (3.3-9) of the 4th edition with the following change:

Bandwidth of the filter is W rad/sec

(3.4-1) = (3.4-2) 4th edition

(3.5-3) Determine the maximum bandwidth of a signal that can be transmitted through the low-pass RC filter in fig. 3.28a with R=1000 and C=10^{-9} if, over this bandwidth, the amplitude response (gain) variation is to be within 5% and the time delay variation is to be within 2%.

(6.1-1) = (6.1-1) 4th edition with the following change:

Figure (b) bandwidth = 15,000 Hz

(6.1-2) Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals:

(a) $sinc\ (100\pi t)$; (b) $sinc^2(100\pi t)$; (c) $sinc\ (100\pi t) + sinc\ (50\pi t)$; (d) $sinc\ (100\pi t) + 3sinc^2(60\pi t)$; (e) $sinc\ (50\pi t) sinc\ (100\pi t)$.

(6.1-4) A signal $g(t) = sinc^2(5\pi t)$ is sampled (using uniformly space impulses) at a rate of (i) 5 Hz; (ii) 10 Hz; (iii) 20 Hz. For each of the three cases:

a) Sketch the sampled signal.
b) Sketch the spectrum of the sampled signal.
c) Explain whether you can recover the signal *g(t)* from the sampled signal.
d) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the spectrum of the output signal.

(6.2-2) = (6.2-1) 4th edition

(6.2-9) = (6.2-10) 4th edition with the following change:

Sample rate = 50% higher than the Nyquist rate

(6.2-10) = (6.2-11) 4th edition with the following change:

10-bit quantizer;

(4.2-1) For each of the following baseband signals: (i) $m(t) = \cos 1000t$; (ii) $m(t) = 2\cos 1000t + \cos 2000t$; (iii) $m(t) = \cos 1000t \cos 3000t$:

a) Sketch the spectrum of *m(t)*.
b) Sketch the spectrum of the DSB-SC signal $m(t) = \cos 10{,}000t$.
c) Identify the upper sideband (USB) and the lower sideband (LSB) spectra.
d) Identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC, USB, and LSB spectra. Explain the nature of frequency shifting in each case.

(4.2-2) Repeat Prob. 4.2-1 [parts (a), (b), and (c) only] if: (i) $m(t) = sinc(100t)$; (ii) $m(t) = e^{-|t|}$; (iii) $m(t) = e^{-|t-1|}$. Observe that $e^{-|t-1|}\ is\ e^{-|t|}$ delayed by 1 second. For the last case you need to consider both the amplitude and the phase spectra.

(4.2-3) Repeat Prob. 4.2-1 [parts (a), (b), and (c) only] for $m(t) = e^{-|t|}$ if the carrier is $\cos\left(10{,}000t - \frac{\pi}{4}\right)$.

(4.2-4) = (4.2-3) 4th edition

(4.2-6) = (4.2-5) 4th edition

(4.3-1) Show that coherent (synchronous) demodulation can demodulate the AM signal

$$[A + m(t)] \cos \omega_c t$$

regardless of the value of A.

(4.3-2) = (4.3-3) 4th edition

(4.3-3) For the AM signal in Prob. 4.3-2 with $\mu = 0.8$:

a) Find the amplitude and power of the carrier.
b) Find the sideband power and the power efficiency η.

(4.3-4)

a) Sketch the DSB-SC signal corresponding to $m(t) = \cos 2\pi t$.

b) This DSB-SC signal is applied at the input of an envelope detector. Show that the output of the envelope detector is not $m(t)$, but $|m(t)|$. Show that, in general, if an AM signal $[A + m(t)] \cos \omega_c t$ is envelope-detected, the output is $[A + m(t)]$. Hence, show that the condition for recovering *m(t)* from the envelope detector is $A + m(t) > 0$ for all *t*.

(4.5-1) A modulating signal $m(t)$ is given by:

a) $m(t) = \cos 100t$
b) $m(t) = \cos 100t + 2\cos 300t$
c) $m(t) = \cos 100t \cos 500t$

In each case:

i. Sketch the spectrum of $m(t)$.
ii. Find and sketch the spectrum of the DSB-SC signal $2m(t) \cos 1000t$.
iii. From the spectrum obtained in (ii), suppress the LSB spectrum to obtain the USB spectrum.
iv. Knowing the USB spectrum in (ii), write the expression $\varphi_{USB}(t)$ for the USB signal.
v. Repeat (iii) and (iv) to obtain the LSB signal $\varphi_{LSB}(t)$.

(4.5-2) For the signals in Prob. 4.5-1, determine $\varphi_{LSB}(t)$ and $\varphi_{USB}(t)$ if the carrier frequency $\omega_c = 1000$.

(4.5-3) Find $\varphi_{LSB}(t)$ and $\varphi_{USB}(t)$ for the modulating signal $m(t) = B\ sinc(2\pi Bt)$ with *B*= 1000 and carrier frequency $\omega_c = 10000\,\pi$. Following this do it yourself steps:

a) Sketch spectra of $m(t)$ and the corresponding DSB-SC signal $2m(t) \cos \omega_c t$.
b) To find the LSB spectrum, suppress the USB in the DSB-SC spectrum found in (a).
c) Find the LSB signal, which is the inverse Fourier transform of the LSB spectrum found in part(b). Follow the similar procedure to find $\varphi_{USB}(t)$.

(4.5-5) An LSB signal is demodulated synchronously. Unfortunately, the local carrier is not $2\cos \omega_c$t as required, but is $2\cos[(\omega_c + \Delta\omega)$t + $\delta)]$. Show that:

a) When $\delta = 0$, the output *y(t)* is the signal *m(t)* with all its spectral components shifted (offset) by $\Delta\omega$.
b) When $\Delta\omega = 0$, the output is the signal *m(t)* with phases of all its spectral components shifted by δ.

In each of these cases, explain the nature of distortion.

(4.5-6) = (4.4-7) 4th edition

(4.8-1) A transmitter transmits an AM signal with a carrier frequency of 1500 kHz. When an inexpensive radio receiver (which has a poor selectivity in its RF-stage bandpass filter) is tuned to 1500 kHz, the signal is heard loud and clear. This same signal is also heard (not as strong) at another dial setting. State, with reasons, at what frequency you will hear this station. The IF frequency is 455 kHz.

(4.8-2) Consider a superheterodyne receiver designed to receive the frequency band of 1 to 30 MHz with IF frequency 8 MHz. What is the range of frequencies generated by the local oscillator for this receiver? An incoming signal with carrier frequency 10 MHz is received at the 10 MHz setting. At this setting of the receiver we also get interference from a signal with some other carrier frequency if the receiver RF stage bandpass filter has poor selectivity. What is the carrier frequency of the interfering signal?

(5.1-3) = (5.1-4) 4^{th} edition with the following change:

$$\omega_c = 10{,}000$$

(5.2-1) = (5.2-3) 4^{th} edition with the following change in $m(t)$:

$$m(t) = 2\cos 100t + 18\cos 2000\pi t$$

(5.2-2) = (5.2-4) 4^{th} edition

(5.2-3)=(5.2-5) 4^{th} edition

(5.2-4)=(5.2-6) 4^{th} edition

(5.2-5) Estimate the bandwidth of $\varphi_{PM}(t)$ and $\varphi_{FM}(t)$ in Prob. 5.1-2. Assume the bandwidth of $m(t)$ to be the fifth harmonic frequency of $m(t)$.

(5.2-7)=(5.2-8) 4^{th} edition with the following change: $m(t) = e^{-t^2}$.

(5.3-1)=(5.3-2) 4^{th} edition

(5.3-2)=(5.3-1) 4^{th} edition

(11.1-4) Determine $\overline{x(t)}$ and $R_x(t_1, t_2)$ for the random process: $x(t) = a\cos(\omega t + \theta)$, where ω and θ are constants and a is an RV uniformly distributed in the range (*-A, A*). Also determine whether this is a wide-sense stationary process.

(11.1-8)=(9.1-9) 4^{th} edition

(11.2-3)=(9.2-4) 4^{th} edition with the following change:

$$S_x(\omega) = k\, rect(\frac{\omega}{4\pi B})$$

(12.2-1) = (10.2-1) 4th edition with the following changes:

$S_n(\omega) = 10^{-10}$; signal bandwidth=4 kHz; SNR>30dB; $H_c(\omega) = 10^{-4}$.

(11.5-1)=(9.8-1) 4th edition

(11.5-2) =(9.8-2) 4th edition with (c) $f_c = 90\ kHz$.

(11.5-3)=(9.8-3) 4th edition

(12.1-1) A certain telephone channel has $H_c(\omega) = 10^{-3}$ over the signal band. The message signal PSD is $S_m(\omega) = \beta rect\left(\frac{\omega}{2\alpha}\right), with\ \alpha = 8000\pi$. The channel noise PSD is $S_n(\omega) = 10^{-8}$. If the output SNR at the receiver is required to be at least 30dB, what is the minimum transmitted power required? Calculate the value of β corresponding to this power.

(12.2-2)=(10.2-2) 4th edition

(12.2-3)=(10.2-6) 4th edition

(12.2-4)=(10.2-3) 4th edition

(12.3-1)=(10.3-1) 4th edition with the following changes:

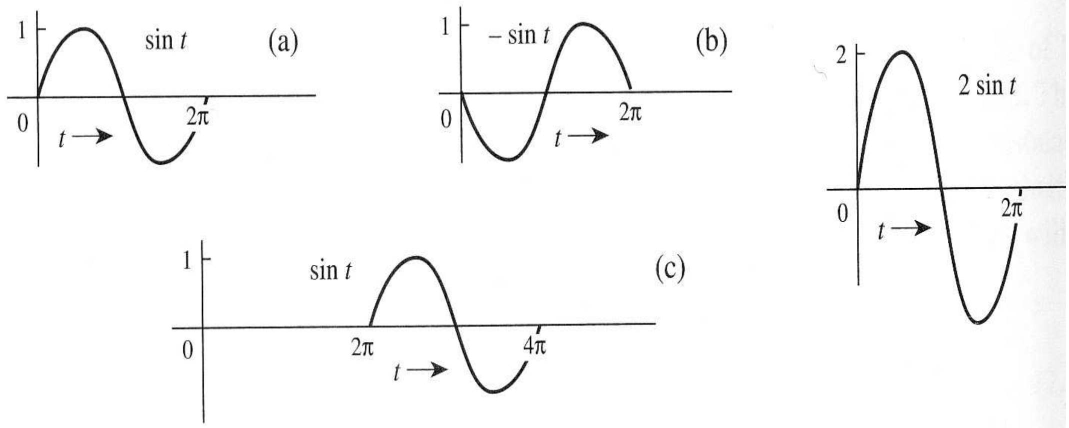
$\beta = 2;\ \alpha = 10^{-4}$.

1. ASSIGNMENT 1

1.1 Assignment 1 Problems

- **2.1-1**

2.1-1 Find the energies of the signals shown in Fig. P2.1-1. Comment on the effect on energy of change, time shifting or doubling of the signal. What is the effect on the energy if the sign multiplied by k?



- **2.1-7**: Show that the power of a signal $g(t)$ given by

$$g(t) = \sum_{k=m}^{n} D_k e^{j\omega_k t} \qquad \omega_i \neq \omega_k \quad \text{for all} \quad i \neq k$$

is (Parseval's theorem)

$$P_g = \sum_{k=m}^{n} |D_k|^2$$

- **2.1-8**: Determine the power and the rms value for each of the following signals:

(a) $10\cos(100t + \pi/3)$ (b) $10\cos(100t + \pi/3) + 16\sin(150t + \pi/5)$

(c) $(10 + 2\sin 3t)\cos 10t$ (d) $10\cos 5t \, \cos 10t$

(e) $10\sin 5t \, \cos 10t$ (f) $e^{j\alpha t} \, \cos\omega_0 t$

2.4-1 Simplify the following expressions:

(a) $\left(\frac{\sin t}{t^2+2}\right)\delta(t)$ **(b)** $\left(\frac{j\omega+2}{\omega^2+9}\right)\delta(\omega)$

(c) $\left[e^{-t}\cos(3t-60°)\right]\delta(t)$ **(d)** $\left[\frac{\sin \frac{\pi}{2}(t-2)}{t^2+4}\right]\delta(t-1)$

(e) $\left(\frac{1}{j\omega+2}\right)\delta(\omega+3)$ **(f)** $\left(\frac{\sin k\omega}{\omega}\right)\delta(\omega)$

Hint: Use Eq. (2.18). For part **(f)** use L'Hôpital's rule.

2.4-2 Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} g(\tau)\delta(t-\tau)\,d\tau$ **(b)** $\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)\,d\tau$

(c) $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\,dt$ **(d)** $\int_{-\infty}^{\infty} \delta(t-2)\sin \pi t\,dt$

(e) $\int_{-\infty}^{\infty} \delta(t+3)e^{-t}\,dt$ **(f)** $\int_{-\infty}^{\infty} (t^3+4)\delta(1-t)\,dt$

(g) $\int_{-\infty}^{\infty} g(2-t)\delta(3-t)\,dt$ **(h)** $\int_{-\infty}^{\infty} e^{(x-1)}\cos\frac{\pi}{2}(x-5)\delta(x-3)\,dx$

Hint: $\delta(x)$ is located at $x=0$. For example, $\delta(1-t)$ is located at $1-t=0$; that is, at $t=1$, and so on.

2.9-1 For each of the periodic signals in Fig. P2.8-4, find exponential Fourier series and sketcl corresponding spectra.

2.9-2 A periodic signal $g(t)$ is expressed by the following Fourier series:

$$g(t) = 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t + \frac{2\pi}{3}\right)$$

(a) Sketch the amplitude and phase spectra for the trigonometric series.

(b) By inspection of spectra in part **(a)**, sketch the exponential Fourier series spectra.

(c) By inspection of spectra in part **(b)**, write the exponential Fourier series for $g(t)$.

2.9-3 Figure P2.9-3 shows the trigonometric Fourier spectra of a periodic signal $g(t)$.

(a) By inspection of Fig. P2.9-3, find the trigonometric Fourier series representing $g(t)$.

(b) By inspection of Fig. P2.9-3, sketch the exponential Fourier spectra of $g(t)$.

(c) By inspection of the exponential Fourier spectra obtained in part **(b)**, find the expone Fourier series for $g(t)$.

(d) Show that the series found in parts **(a)** and **(c)** are equivalent.

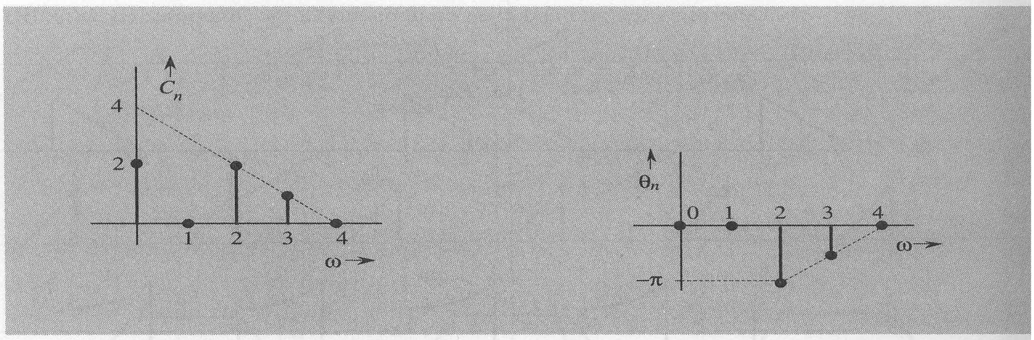


Figure P2.9-3

1.2 Assignment 1 Solutions

- **2.1-1** Let us denote the signal in question by $g(t)$ and its energy by E_g. For parts (a) and (b)

$$E_g = \int_0^{2\pi} \sin^2 t dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos(2t)\, dt = \pi + 0 = \pi$$

$$(c) \qquad E_g = \int_{2\pi}^{4\pi} \sin^2 t dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos(2t)\, dt = \pi + 0 = \pi$$

$$(d) \qquad E_g = \int_0^{2\pi} (2\sin t)^2 dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos(2t)\, dt\right] = 4[\pi + 0] = 4\pi$$

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way, we can show that the energy of $kg(t)$ is $k^2 E_g$.

- **2.1-7**:

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g(t)\cdot g^*(t) dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n}\sum_{r=m}^{n} D_k D_r^* e^{j(\omega_k-\omega_r)t}$$

The integrals of the cross-product terms (when $k \neq r$) are finite because the integrands are periodic signals (made up of sinusoids). These terms, when divided by $T \to \infty$, yields zero. The remaining terms ($k = r$) yields

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n} |D_k|^2\, dt = \sum_{k=m}^{n} |D_k|^2$$

- **2.1-8** (a) Power of a sinusoid of amplitude C is $C^2/2$ (Eq. (2.6a)) regardless of its frequency ($\omega \neq 0$) and phase. Therefore, in this case, $P = (10)^2/2 = 50$.

  (b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids (Eq. (2.6b)). Therefore, in this case, $P = (10)^2/2 + (16)^2/2 = 178$.

  (c) $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. Hence, from (Eq. (2.6b)), $P = (10)^2/2 + 1/2 + 1/2 = 51$

  (d) $10\cos 5t \cos 10t = 5(\cos 5t + \cos 15t)$. Hence, P=25;

  (e) $10\sin 5t \cos 10t = 5(\sin 15t - \sin 5t)$. Hence, P=25;

  (f) $e^{j\alpha t}\cos\omega_0 t = 1/2[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)}]$. Using the results obtained in Prob. 2.1-7, we obtain $P = 1/4 + 1/4 = 1/2$

- **2.4-1** Using the fact that $g(x)\delta(x) = g(0)\delta(x)$, we have (a) 0, (b) $\frac{2}{9}\delta(\omega)$ (c) $\frac{1}{2}\delta(t)$ (d) $-\frac{1}{5}\delta(t-1)$ (e) $\frac{1}{2-j3}\delta(\omega+3)$ (f) $k\delta(\omega)$ (use L'Hopital's rule).

- **2.4-2** In these problems remember that impulse $\delta(x)$ is located at $x = 0$. Thus, an impulse $\delta(t - \tau)$ is located at $\tau = t$, and so on.

(a) The impulse is located at $\tau = t$ and $g(\tau)$ at $\tau = t$ is $g(t)$. Therefore,

$$\int_{-\infty}^{\infty} g(\tau)\delta(t-\tau)d\tau = g(t)$$

(b) The impulse $\delta(\tau)$ is at $\tau = 0$ and $g(t-\tau)$ at $\tau = 0$ is $g(t)$. Therefore,

$$\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)d\tau = g(t)$$

Using similar arguments, we obtain (c) 1, (d) 0, (e) e^3, (f) 5, (g) $g(-1)$ (h) $-e^2$

- **2.9-1** (a) $T_0 = 4, \omega_0 = \pi/2$. Also, $D_0 = 0$ (by inspection),

$$D_n = \frac{1}{2\pi}\int_{-1}^{1} e^{-j(n\pi/2)t}\,dt - \int_{1}^{3} e^{-j(n\pi/2)t}\,dt = \frac{2}{\pi n}\sin\frac{n\pi}{2} \quad |n| \geq 1$$

(b) $T_0 = 10\pi, \omega_0 = 2\pi/10\pi = 1/5$.

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n}{5}t} \quad \text{where}$$

$$D_n = \frac{1}{10\pi}\int_{-\pi}^{\pi} e^{-j\frac{n}{5}t}\,dt = \frac{j}{2\pi n}\left(-2j\sin\frac{n\pi}{5}\right) = \frac{1}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

(c)

$$g(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jnt} \quad \text{where by inspection } D_0 = 0.5$$

$$D_n = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{t}{2\pi}e^{-jnt}\,dt = \frac{j}{2\pi n}$$

so that $|D_n| = \frac{1}{2\pi n}$ and $\angle D_n = \pi/2$ when $n > 0$ and $-\pi/2$ when $n < 0$.

(d) $T_0 = \pi, \omega_0 = 2$ and $D_0 = 0$.

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt} \quad \text{where}$$

$$D_n = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4t}{\pi}e^{-j(2n)t}\,dt = \frac{-j}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

(e) $T_0 = 3, \omega_0 = \frac{2\pi}{3}$.

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{2\pi n}{3}t} \quad \text{where}$$

$$D_n = \frac{1}{3}\int_{0}^{1} t\, e^{-j\frac{2\pi n}{3}t}\,dt = \frac{3}{4\pi^2 n^2}\left[e^{-j\frac{2\pi n}{3}}\left(\frac{j2\pi n}{3}+1\right) - 1\right]$$

(f) $T_0 = 6, \omega_0 = \frac{\pi}{3}$, and $D_0 = 0.5$

$$g(t) = 0.5 + \sum_{n=-\infty}^{\infty} D_n e^{j\frac{\pi n t}{3}} \quad \text{where}$$

$$D_n = \frac{1}{6}\left[\int_{-2}^{-1}(t+2)e^{-j\frac{\pi n t}{3}}\,dt + \int_{-1}^{1} e^{-j\frac{\pi n t}{3}}\,dt + \int_{1}^{2}(-t+2)\,e^{-j\frac{\pi n t}{3}}\,dt\right] = \frac{3}{\pi^2 n^2}\left(\cos\frac{n\pi}{3} - \cos\frac{2n\pi}{3}\right)$$

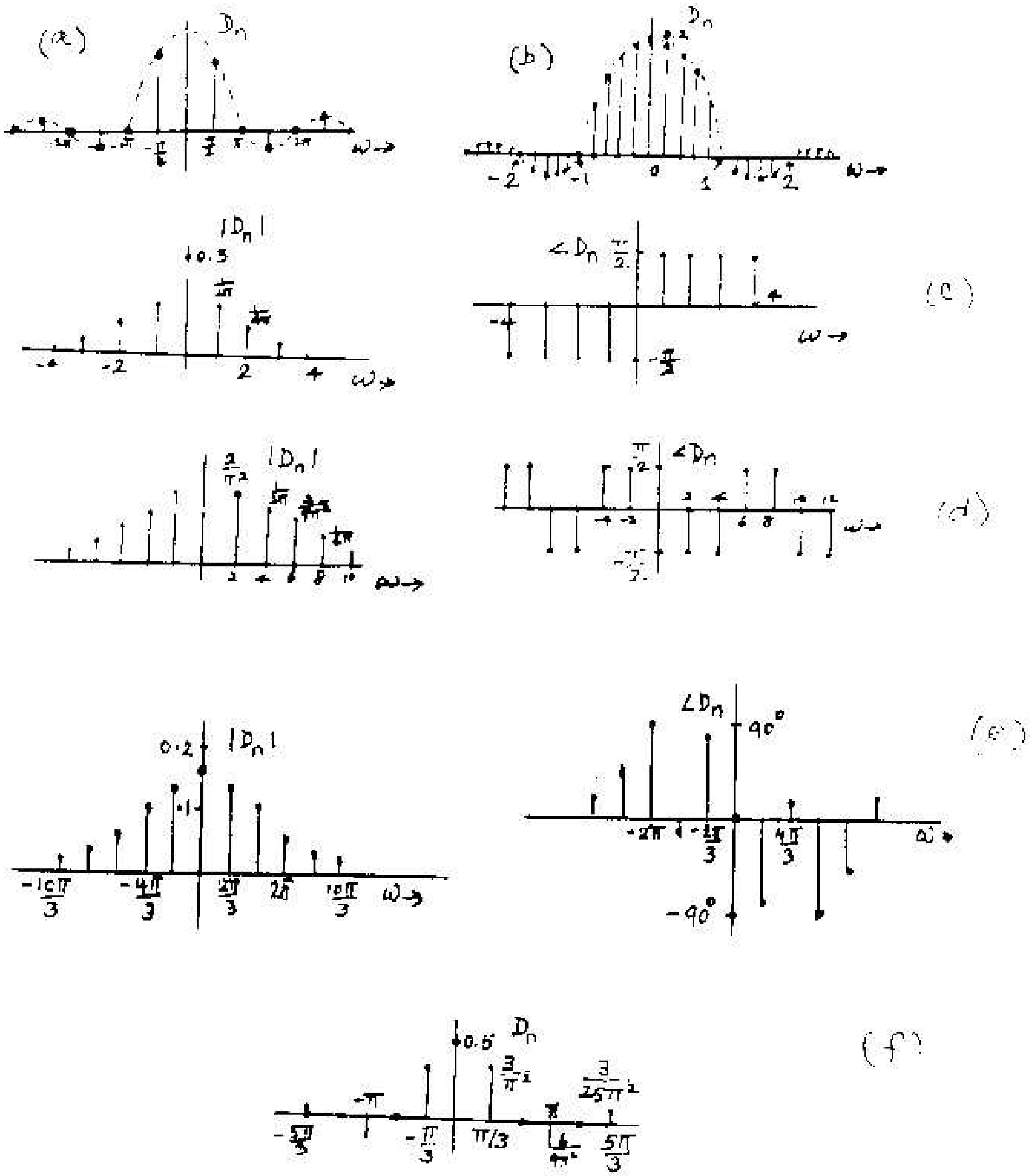


Fig. 1.1: Solution 2-9-1.

- **2-9.2** (a)

$$g(t) = 3\cos t + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right)$$

For a compact trigonometric form, all terms must have cosine form and amplitudes must be positive. For this reason, we rewrite $g(t)$ as

$$\begin{aligned} g(t) &= 3\cos t + \cos\left(5t - \frac{\pi}{6} - \frac{\pi}{2}\right) + 2\cos\left(8t - \frac{\pi}{3} - \pi\right) \\ &= 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t - \frac{4\pi}{3}\right) \end{aligned}$$

(b) By inspection of the trigonometric spectra in Fig.2a, we plot the exponential spectra as shown in Fig.2b. By inspection of exponential spectra in Fig.2a, we obtain

$$\begin{aligned} g(t) &= \frac{3}{2}\left(e^{jt} + e^{-jt}\right) + \frac{1}{2}\left[e^{j(5t-2\pi/3)} + e^{-j(5t-2\pi/3)}\right] + \left[e^{j(8t-4\pi/3)} + e^{-j(8t-4\pi/3)}\right] \\ &= \frac{3}{2}e^{jt} + \left(\frac{1}{2}e^{-j2\pi/3}\right)e^{j5t} + \left(e^{-j4\pi/3}\right)e^{j8t} + \frac{3}{2}e^{-jt} + \left(\frac{1}{2}e^{j2\pi/3}\right)e^{-j5t} + \left(e^{j4\pi/3}\right)e^{-j8t} \end{aligned}$$

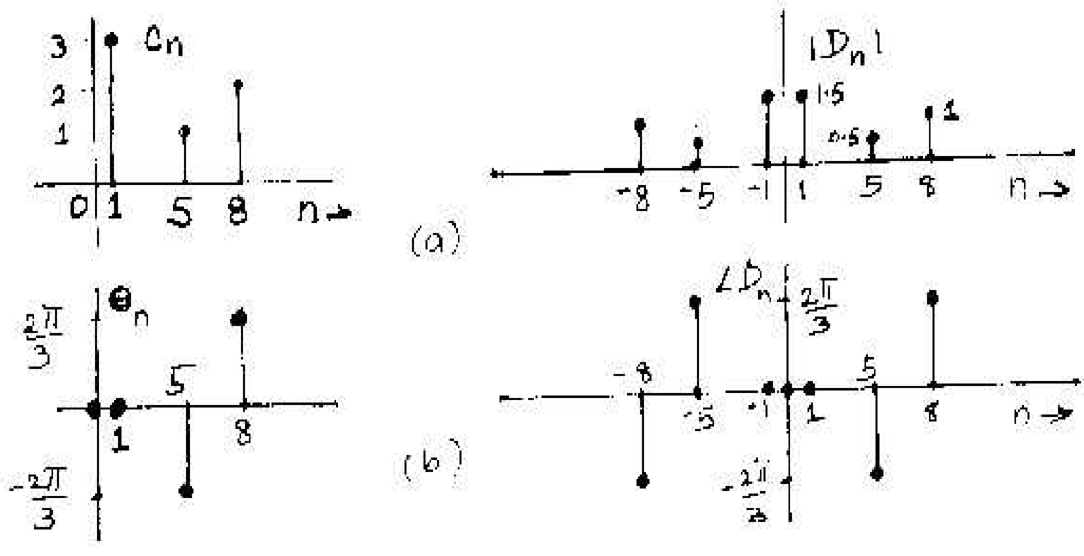


Fig. 1.2: Solution 2-9-2.

- **2-9.3** (a)

$$g(t) = 2 + 2\cos(2t - \pi) + \cos(3t - \pi/2) = 2 - 2\cos(2t) + \sin(3t)$$

(b) The exponential spectra are shown below.

(c) By inspection of exponential spectral,

$$\begin{aligned} g(t) &= 2 + \left[e^{(2t-\pi)} + e^{-j(2t-\pi)}\right] + \frac{1}{2}\left[e^{j(3t-\pi/2)} + e^{-j(3t-\pi/2)}\right] \\ &= 2 + 2\cos(2t - \pi) + \cos(3t - \pi/2) \end{aligned}$$

(d) Observe that the two expressions (trigonometric and exponential Fourier series) are equivalent.

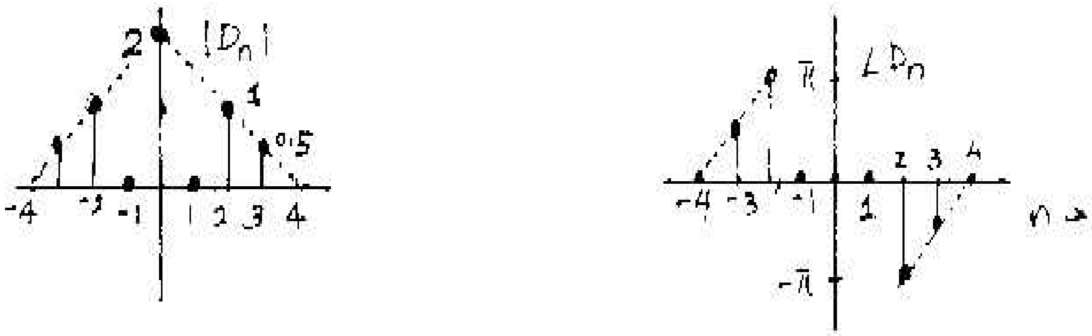


Fig. 1.3: Solution 2-9-3.

2. ASSIGNMENT 2

2.1 Assignment 2 Problems

3.1-5 From definition (3.8a), find the Fourier transforms of the signals shown in Fig. P3.1-5.

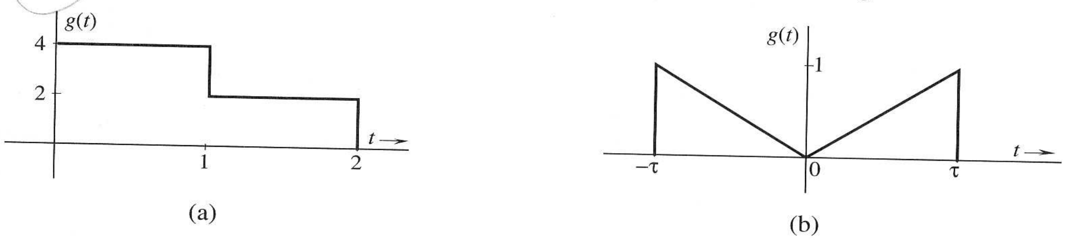


Figure P3.1-5

3.1-6 From definition (3.8b), find the inverse Fourier transforms of the spectra shown in Fig. P3.1-6.

Figure P3.1-6

3.1-7 From definition (3.8b), find the inverse Fourier transforms of the spectra shown in Fig. P3.1-7.

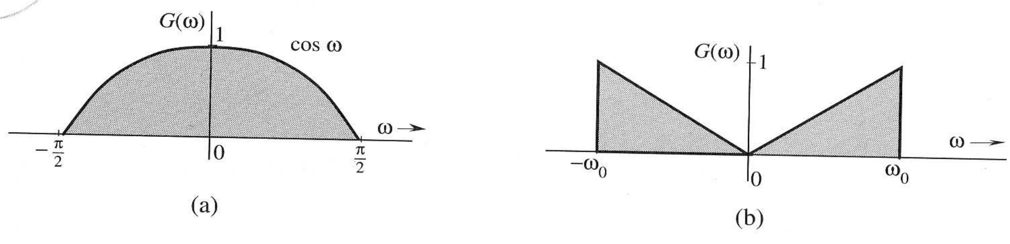


Figure P3.1-7

- **3.2-2**: From the definition of Fourier Transform, show that the Fourier transform of $\text{rect}(t-5)$ is $\text{sinc}(\omega/2)e^{j5\omega}$

3.3-2 The Fourier transform of the triangular pulse $g(t)$ in Fig. P3.3-2a is given as

$$G(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1)$$

Using this information, and the time-shifting and time-scaling properties, find the Fourier transforms of the signals shown in Fig. P3.3-2b, c, d, e, and f. *Hint*: Time inversion in $g(t)$ results in the pulse $g_1(t)$ in Fig. P3.3-2b; consequently $g_1(t) = g(-t)$. The pulse in Fig. P3.3-2c can be expressed as $g(t-T) + g_1(t-T)$ [the sum of $g(t)$ and $g_1(t)$ both delayed by T]. The pulses in Fig. P3.3-2d and e both can be expressed as $g(t-T) + g_1(t+T)$ [the sum of $g(t)$ delayed by T and $g_1(t)$ advanced by T] for some suitable choice of T. The pulse in Fig. P3.3-2f can be obtained by time-expanding $g(t)$ by a factor of 2 and then delaying the resulting pulse by 2 seconds [or by first delaying $g(t)$ by 1 second and then time-expanding by a factor of 2].

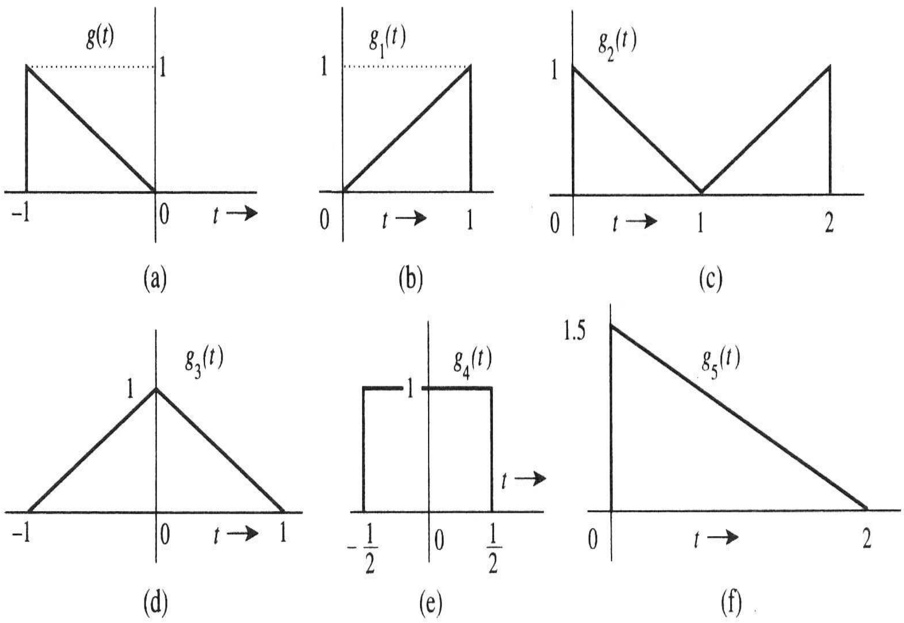


Figure P3.3-2

3.3-6 The signals in Fig. P3.3-6 are modulated signals with carrier $\cos 10t$. Find the Fourier transforms of these signals using the appropriate properties of the Fourier transform and Table 3.1. Sketch the amplitude and phase spectra for parts **(a)** and **(b)**. *Hint*: These functions can be expressed in the form $g(t)\cos \omega_0 t$.

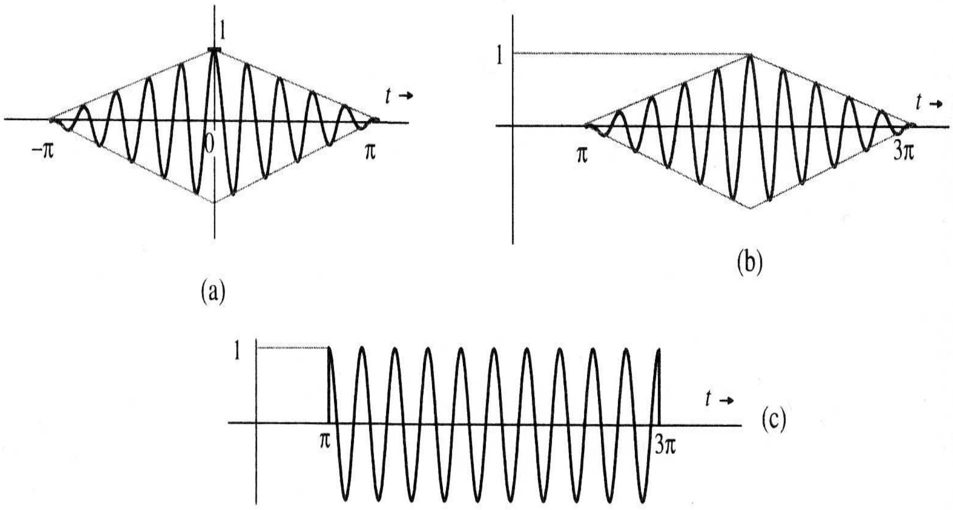


Figure P3.3-6

3.3-7 Using the frequency-shifting property and Table 3.1, find the inverse Fourier transform of the spectra shown in Fig. P3.3-7.

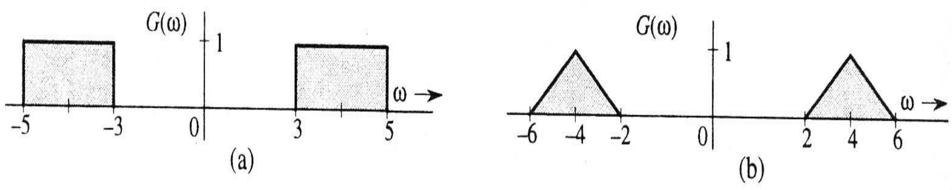


Figure P3.3-7

3.3-10 The process of recovering a signal $g(t)$ from the modulated signal $g(t) \cos \omega_0 t$ is called **demodulation**. Show that the signal $g(t) \cos \omega_0 t$ can be demodulated by multiplying it with $2 \cos \omega_0 t$ and passing the product through a low-pass filter of bandwidth W rad/s [the bandwidth of $g(t)$]. Assume $W < \omega_0$. *Hint*: $2 \cos^2 \omega_0 t = 1 + \cos 2\omega_0 t$. Recognize that the spectrum of $g(t) \cos 2\omega_0 t$ is centered at $2\omega_0$ and will be suppressed by a low-pass filter of bandwidth W rad/s.

3.4-1 Signals $g_1(t) = 10^4 \text{rect}(10^4 t)$ and $g_2(t) = \delta(t)$ are applied at the inputs of the ideal low-pass filters $H_1(\omega) = \text{rect}(\omega/40,000\pi)$ and $H_2(\omega) = \text{rect}(\omega/20,000\pi)$ (Fig. P3.4-1). The outputs $y_1(t)$ and $y_2(t)$ of these filters are multiplied to obtain the signal $y(t) = y_1(t)y_2(t)$.

(a) Sketch $G_1(\omega)$ and $G_2(\omega)$.

(b) Sketch $H_1(\omega)$ and $H_2(\omega)$.

(c) Sketch $Y_1(\omega)$ and $Y_2(\omega)$.

(d) Find the bandwidths of $y_1(t)$, $y_2(t)$, and $y(t)$.

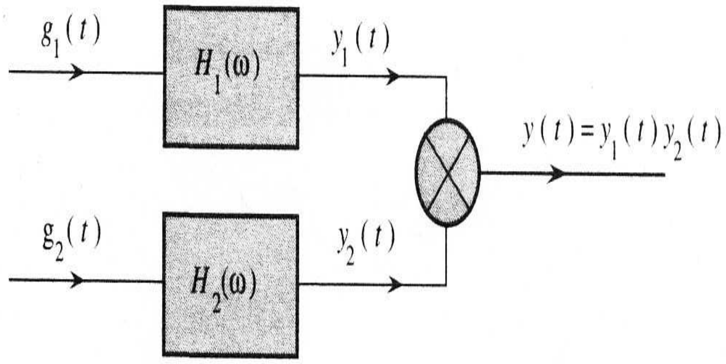


Figure P3.4-1

2.2 Assignment 2 Solutions

- **3.1-5** (a)

$$G(\omega) = \int_0^1 4e^{-j\omega t}\,dt + \int_1^2 2e^{-j\omega t}\,dt = \frac{4 - 2e^{-j\omega} - 2e^{-j2\omega}}{j\omega}$$

(b)

$$G(\omega) = \int_{-\tau}^{0} -\frac{t}{\tau}\,e^{-j\omega t}\,dt + \int_0^{\tau} \frac{t}{\tau}\,e^{-j\omega t}\,dt = \frac{2}{\tau\omega^2}[\cos\omega\tau + \omega\tau\sin\omega\tau - 1]$$

This results could also be derived by observing that $g(t)$ is an even function. Therefore, from the result in Prob. 3.1-1,

$$G(\omega) = \frac{2}{\tau}\int_0^{\tau} t\,\cos\omega t\,dt = \frac{2}{\tau\omega^2}[\cos\omega\tau + \omega\tau\sin\omega\tau - 1]$$

- **3.1-7** (a)

$$g(t) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos\omega\,e^{j\omega t}\,d\omega = \frac{e^{j\omega t}}{2\pi(1-t^2)}(jt\cos\omega + \sin\omega)_{-\pi/2}^{\pi/2} = \frac{1}{\pi(1-t^2)}\cos\frac{\pi t}{2}$$

(b)

$$g(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} G(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\left[\int_{-\omega_0}^{\omega_0} G(\omega)\cos\omega t d\omega + j\int_{-\omega_0}^{\omega_0} G(\omega)\sin\omega t d\omega\right]$$

Because $G(\omega)$ is even function, the second integral on the right-hand side vanishes. Also the integrand of the first term is an even function. Therefore,

$$g(t) = \frac{1}{\pi}\int_0^{\omega_0} \frac{\omega}{\omega_0}\cos t\omega\,d\omega = \frac{1}{\pi\omega_0 t^2}[\cos\omega_0 t + \omega_0 t\sin\omega_0 t - 1]$$

- **3.2-2** The function $\mathrm{rect}(t-5)$ is centered at $t = 5$, has a width of unity, and its value over this interval is unity. Hence

$$\begin{aligned} G(\omega) &= \int_{4.5}^{5.5} e^{-j\omega t}\,dt = -\frac{1}{j\omega}e^{-j\omega t}|_{4.5}^{5.5} = \frac{1}{j\omega}[e^{-j4.5\omega} - e^{-j5.5\omega}] \\ &= \frac{e^{-j4.5\omega}}{j\omega}[e^{j\omega/2} - e^{-j\omega/2}] = \frac{e^{-j5\omega}}{j\omega}[2j\sin\frac{\omega}{2}] = \mathrm{sinc}\left(\frac{\omega}{2}\right)e^{-j5\omega} \end{aligned}$$

- **3.3-2 Fig.(b)** $g_1(t) = g(-t)$ and

$$G_1(\omega) = G(-\omega) = \frac{1}{\omega^2}[e^{-j\omega} + j\omega\,e^{-j\omega} - 1]$$

Fig.(c) $g_2(t) = g(t-1) + g_1(t-1)$. Therefore

$$G_3(\omega) = [G(\omega) + G_1(\omega)]e^{-j\omega} = [G(\omega) + G(-\omega)]e^{-j\omega} = \frac{2e^{-j\omega}}{\omega^2}[\cos\omega + \omega\sin\omega - 1]$$

Fig.(d) $g_3(t) = g(t-1) + g_1(t+1)$ and

$$G_4(\omega) = G(\omega)e^{-j\omega} + G(-\omega)e^{j\omega} = \frac{1}{\omega^2}[2 - 2\cos\omega] = \frac{4}{\omega^2}\sin^2\frac{\omega}{2} = \text{sinc}^2\left(\frac{\omega}{2}\right)$$

Fig.(e) $g_4(t) = g(t-1/2) + g_1(t+1/2)$. Therefore

$$\begin{aligned} G_4(\omega) &= G(\omega)e^{-j\omega/2} + G_1(\omega)e^{j\omega/2} = \frac{e^{-j\omega/2}}{\omega^2}[e^{j\omega} - j\omega e^{j\omega} - 1] + \frac{e^{j\omega/2}}{\omega^2}[e^{-j\omega} + j\omega e^{-j\omega} - 1] \\ &= \frac{1}{\omega^2}\left[2\omega\sin\frac{\omega}{2}\right] = \text{sinc}\left(\frac{\omega}{2}\right) \end{aligned}$$

Fig.(f) $g_5(t)$ can be obtained in three steps: (i) time-expanding $g(t)$ by a factor 2; (ii) then delaying it by 2 seconds; (iii) and multiplying it by 1.5 (we may interchange the sequence for steps (i) and (ii)]. The first step (time0expansion by a factor 2) yields

$$f\left(\frac{t}{2}\right) \Leftrightarrow 2G(2\omega) = \frac{1}{2\omega^2}(e^{j2\omega} - 2j\omega e^{j2\omega} - 1)$$

Second step of time delay of 2 secs, yields,

$$f\left(\frac{t-2}{2}\right) \Leftrightarrow \frac{1}{2\omega^2}(e^{j2\omega} - 2j\omega e^{j2\omega} - 1)e^{-j2\omega} = \frac{1}{2\omega^2}(1 - j2\omega - e^{-j2\omega})$$

The third step of multiplying the resulting signal by 1.5 yields

$$g_5(t) = 1.5f\left(\frac{t-2}{2}\right) \Leftrightarrow \frac{3}{4\omega^2}(1 - j2\omega - e^{-j2\omega})$$

3.3-4 From time-shifting property

$$g(t \pm T) \Longleftrightarrow G(\omega)e^{\pm j\omega T}$$

Therefore

$$g(t+T) + g(t-T) \Longleftrightarrow G(\omega)e^{j\omega T} + G(\omega)e^{-j\omega T} = 2G(\omega)\cos \omega T$$

We can use this result to derive transforms of signals in Fig. P3.3-4.
(a) Here $g(t)$ is a gate pulse as shown in Fig. S3.3-4a.

$$g(t) = \text{rect}\left(\frac{t}{2}\right) \Longleftrightarrow 2\,\text{sinc}(\omega)$$

Also $T = 3$. The signal in Fig. P3.3-4a is $g(t+3) + g(t-3)$, and

$$g(t+3) + g(t-3) \Longleftrightarrow 4\,\text{sinc}(\omega)\cos 3\omega$$

(b) Here $g(t)$ is a triangular pulse shown in Fig. S3.3-4b. From the Table 3.1 (pair 19)

$$g(t) = \Delta\left(\frac{t}{2}\right) \Longleftrightarrow \text{sinc}^2\left(\frac{\omega}{2}\right)$$

Also $T = 3$. The signal in Fig. P3.3-4b is $g(t+3) + g(t-3)$, and

$$g(t+3) + g(t-3) \Longleftrightarrow 2\,\text{sinc}^2\left(\frac{\omega}{2}\right)\cos 3\omega$$

Fig. S3.3-4

3.3-6 Fig. (a) The signal $g(t)$ in this case is a triangle pulse $\Delta(\frac{t}{2\pi})$ (Fig. S3.3-6) multiplied by $\cos 10t$.

$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t$$

Also from Table 3.1 (pair 19) $\Delta(\frac{t}{2\pi}) \Longleftrightarrow \pi\,\text{sinc}^2(\frac{\pi\omega}{2})$ From the modulation property (3.35), it follows that

$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t \Longleftrightarrow \frac{\pi}{2}\left\{\text{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$$

The Fourier transform in this case is a real function and we need only the amplitude spectrum in this case as shown in Fig. S3.3-6a.

Fig. (b) The signal $g(t)$ here is the same as the signal in Fig. (a) delayed by 2π. From time shifting property, its Fourier transform is the same as in part (a) multiplied by $e^{-j\omega(2\pi)}$. Therefore

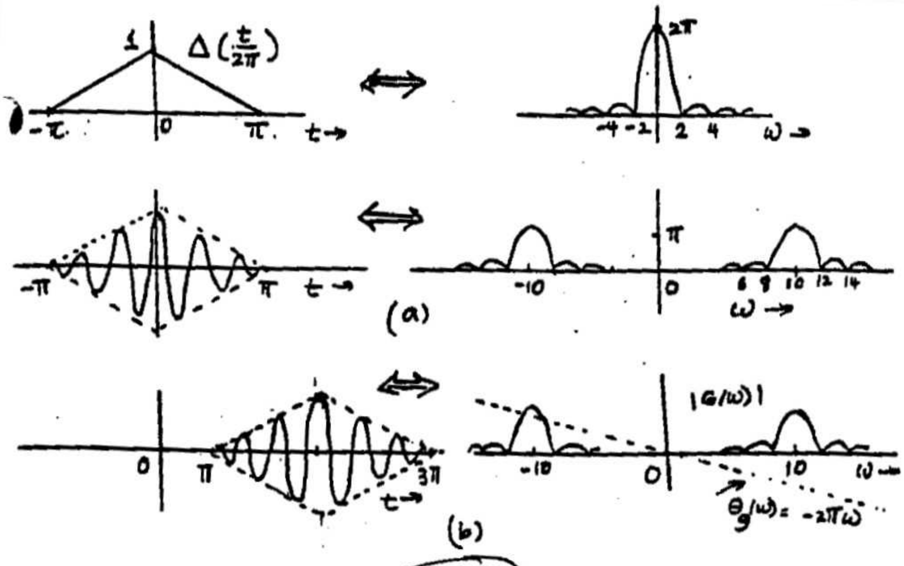


Fig. S3.3-6

$$G(\omega) = \frac{\pi}{2}\left\{\text{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}e^{-j2\pi\omega}$$

The Fourier transform in this case is the same as that in part (a) multiplied by $e^{-j2\pi\omega}$. This multiplying factor represents a linear phase spectrum $-2\pi\omega$. Thus we have an amplitude spectrum [same as in part (a)] as well as a linear phase spectrum $\angle G(\omega) = -2\pi\omega$ as shown in Fig. S3.3-6b. the amplitude spectrum in this case as shown in Fig. S3.3-6b.

Note: In the above solution, we first multiplied the triangle pulse $\Delta(\frac{t}{2\pi})$ by cos $10t$ and then delayed the result by 2π. This means the signal in Fig. (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi})\cos 10(t-2\pi)$.

We could have interchanged the operation in this particular case, that is, the triangle pulse $\Delta(\frac{t}{2\pi})$ is first delayed by 2π and then the result is multiplied by cos $10t$. In this alternate procedure, the signal in Fig. (b) is expressed as $\Delta(\frac{t-2\pi}{2\pi})\cos 10t$.

This interchange of operation is permissible here only because the sinusoid cos $10t$ executes integral number of cycles in the interval 2π. Because of this both the expressions are equivalent since $\cos 10(t-2\pi) = \cos 10t$.

Fig. (c) In this case the signal is identical to that in Fig. b, except that the basic pulse is rect$(\frac{t}{2\pi})$ instead of a triangle pulse $\Delta(\frac{t}{2\pi})$. Now

$$\text{rect}\left(\frac{t}{2\pi}\right) \Longleftrightarrow 2\pi\,\text{sinc}(\pi\omega)$$

Using the same argument as for part (b), we obtain

$$G(\omega) = \pi\{\text{sinc}[\pi(\omega+10)] + \text{sinc}[\pi(\omega-10)]\}e^{-j2\pi\omega}$$

3.3-7 (a)

$$G(\omega) = \text{rect}\left(\frac{\omega-4}{2}\right) + \text{rect}\left(\frac{\omega+4}{2}\right)$$

Also

$$\frac{1}{\pi}\text{sinc}(t) \Longleftrightarrow \text{rect}\left(\frac{\omega}{2}\right).$$

Therefore

$$g(t) = \frac{2}{\pi}\text{sinc}(t)\cos 4t$$

(b)

$$G(\omega) = \Delta\left(\frac{\omega+4}{4}\right) + \Delta\left(\frac{\omega-4}{4}\right)$$

Also

$$\frac{1}{\pi}\text{sinc}^2(t) \Longleftrightarrow \Delta\left(\frac{\omega}{4}\right)$$

Therefore

$$g(t) = \frac{2}{\pi}\text{sinc}^2(t)\cos 4t$$

3.3-10

A basic demodulator is shown in Fig. S3.3-10a. The product of the modulated signal $g(t)\cos\omega_0 t$ with $2\cos\omega_0 t$ yields

$$g(t)\cos\omega_0 t \times 2\cos\omega_0 t = 2g(t)\cos^2\omega_0 t = g(t)[1+\cos 2\omega_0 t] = g(t) + g(t)\cos 2\omega_0 t$$

The product contains the desired $g(t)$ (whose spectrum is centered at $\omega = 0$) and the unwanted signal $g(t)\cos 2\omega_0 t$ with spectrum $\frac{1}{2}[G(\omega+2\omega_0)+G(\omega-2\omega_0)]$, which is centered at $\pm 2\omega_0$. The two spectra are nonoverlapping because

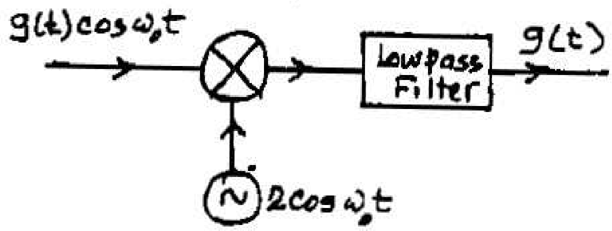


Fig. S3.3-10

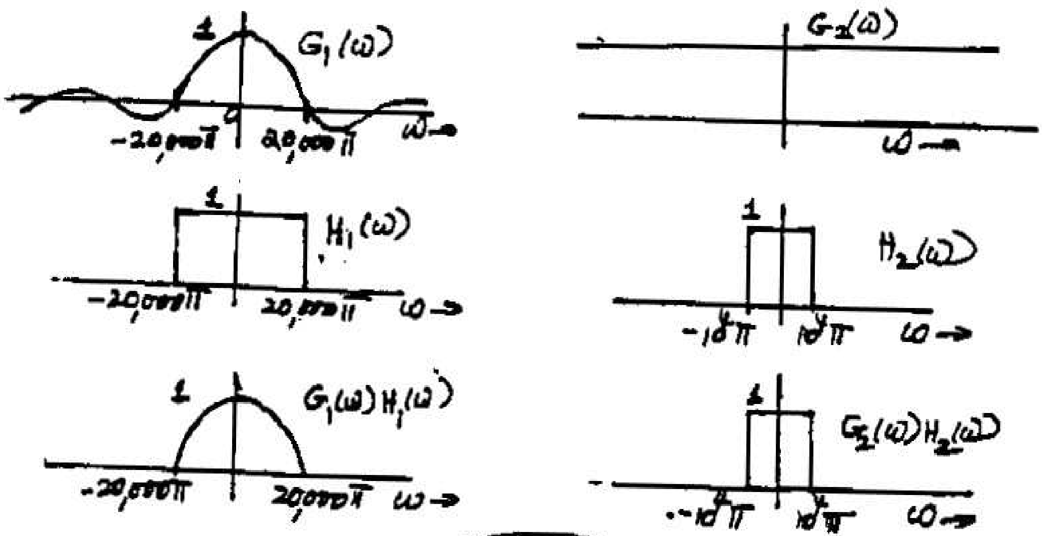


Fig. S3.4-1

$W < \omega_0$ (See Fig. S3.3-10b). We can suppress the unwanted signal by passing the product through a lowpass filter as shown in Fig. S3.3-10a.

3.4−1

$$G_1(\omega) = \text{sinc}\left(\frac{\omega}{20000}\right) \quad \text{and} \quad G_2(\omega) = 1$$

Figure S3.4-1 shows $G_1(\omega)$, $G_2(\omega)$, $H_1(\omega)$ and $H_2(\omega)$. Now

$$Y_1(\omega) = G_1(\omega)H_1(\omega)$$
$$Y_2(\omega) = G_2(\omega)H_2(\omega)$$

The spectra $Y_1(\omega)$ and $Y_2(\omega)$ are also shown in Fig. S3.4-1. Because $y(t) = y_1(t)y_2(t)$, the frequency convolution property yields $Y(\omega) = Y_1(\omega) * Y_2(\omega)$. From the width property of convolution, it follows that the bandwidth of $Y(\omega)$ is the sum of bandwidths of $Y_1(\omega)$ and $Y_2(\omega)$. Because the bandwidths of $Y_1(\omega)$ and $Y_2(\omega)$ are 10 kHz, 5 kHz, respectively, the bandwidth of $Y(\omega)$ is 15 kHz.

3.5-3 From the results in Example 3.16

$$|H(\omega)| = \frac{a}{\sqrt{\omega^2 + a^2}} \qquad a = \frac{1}{RC} = 10^6$$

Also $H(0) = 1$. Hence if ω_1 is the frequency where the amplitude response drops to 0.95, then

$$|H(\omega_1)| = \frac{10^6}{\sqrt{\omega_1^2 + 10^{12}}} = 0.95 \Longrightarrow \omega_1 = 328,684$$

Moreover, the time delay is given by (see Example 3.16)

$$t_d(\omega) = \frac{a}{\omega^2 + a^2} \Longrightarrow t_d(0) = \frac{1}{a} = 10^{-6}$$

If ω_2 is the frequency where the time delay drops to 0.98% of its value at $\omega = 0$, then

$$t_d(\omega_2) = \frac{10^6}{\omega_2^2 + 10^{12}} = 0.98 \times 10^{-6} \Longrightarrow \omega_2 = 142,857$$

We select the smaller of ω_1 and ω_2, that is $\omega = 142,857$, where both the specifications are satisfied. This yields a frequency of 22,736.4 Hz.

3.5-4 There is a typo in this example. The time delay tolerance should be 4% instead of 1%.
The band of $\Delta\omega = 2000$ centered at $\omega = 10^5$ represents the frequency range from 0.99×10^5 to 1.01×10^5. Let us consider the gains and the time delays at the band edges. From Example 3.16

$$|H(\omega)| = \frac{a}{\sqrt{\omega^2 + a^2}} \qquad t_d(\omega) = \frac{a}{\omega^2 + a^2} \qquad a = 10^3$$

At the edges of the band

$$|H(0.99 \times 10^5)| = \frac{10^3}{\sqrt{(0.99 \times 10^5)^2 + 10^6}} = 10.1 \times 10^{-3}, \quad \text{and} \quad |H(1.01 \times 10^5)| = \frac{10^3}{\sqrt{(1.01 \times 10^5)^2 + 10^6}} = 9.901 \times 10^{-3}$$

The gain variation over the band is only 1.99%. Similarly, we find the time delays at the band edges as

$$t_d(0.99 \times 10^5) = \frac{10^3}{(0.99 \times 10^5)^2 + 10^6} = \frac{1}{(99)^2 10^3}, \quad \text{and} \quad t_d(1.01 \times 10^5) = \frac{10^3}{(1.01 \times 10^5)^2 + 10^6} = \frac{1}{(101)^2 \times 10^3}$$

The time delay variation over the band is 4%. Hence, the transmission may be considered distortionless. The signal is transmitted with a gain and time delay at the center of the band, that is at $\omega = 10^5$. We also find $|H(10^5) \approx 0.01$ and $t_d(10^5) \approx 1 -^{-7}$. Hence, if $g(t)$ is the input, the corresponding output is

$$y(t) = 0.01\, g(t - 10^{-7})$$

3. ASSIGNMENT 3

3.1 Assignment 3 Problems

6.1-1 Figure P6.1-1 shows Fourier spectra of signals $g_1(t)$ and $g_2(t)$. Determine the Nyquist interval and the sampling rate for signals $g_1(t)$, $g_2(t)$, $g_1^2(t)$, $g_2^3(t)$, and $g_1(t)g_2(t)$.

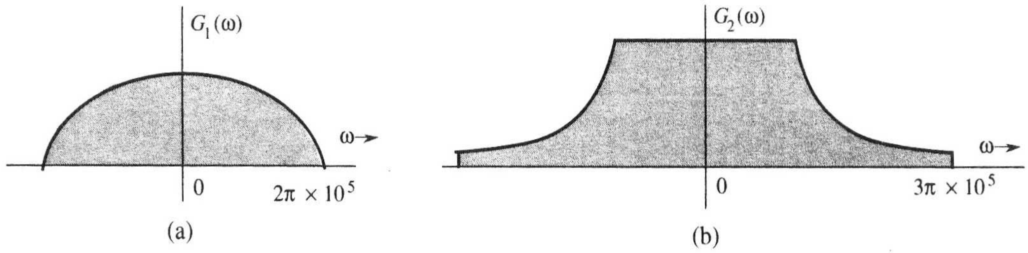


Figure P6.1-1

6.1-2 Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals: **(a)** sinc $(100\pi t)$; **(b)** sinc^2 $(100\pi t)$; **(c)** sinc $(100\pi t)$ + sinc $(50\pi t)$; **(d)** sinc $(100\pi t)$ + 3 sinc^2 $(60\pi t)$; **(e)** sinc $(50\pi t)$sinc $(100\pi t)$.

6.1-3 A signal $g(t)$ band-limited to B Hz is sampled by a periodic pulse train $p_{T_s}(t)$ made up of a rectangular pulse of width $1/8B$ seconds (centered at the origin) repeating at the Nyquist rate ($2B$ pulses per second). Show that the sampled signal $\overline{g}(t)$ is given by

$$\overline{g}(t) = \frac{1}{4}g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) g(t) \cos n\omega_s t \qquad \omega_s = 4\pi B$$

Show that the signal $g(t)$ can be recovered by passing $\overline{g}(t)$ through an ideal low-pass filter of bandwidth B Hz and a gain of 4.

6.1-4 A signal $g(t) = \text{sinc}^2(5\pi t)$ is sampled (using uniformly spaced impulses) at a rate of: **(i)** 5 Hz; **(ii)** 10 Hz; **(iii)** 20 Hz. For each of the three case:

(a) Sketch the sampled signal.

(b) Sketch the spectrum of the sampled signal.

(c) Explain whether you can recover the signal $g(t)$ from the sampled signal.

(d) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the

6.1-5 Signals $g_1(t) = 10^4 \text{rect}(10^4 t)$ and $g_2(t) = \delta(t)$ are applied at the inputs of ideal low-pass filters $H_1(\omega) = \text{rect}(\omega/40,000\pi)$ and $H_2(\omega) = \text{rect}(\omega/20,000\pi)$ (Fig. P6.1-5). The outputs $y_1(t)$ and $y_2(t)$ of these filters are multiplied to obtain the signal $y(t) = y_1(t)y_2(t)$. Find the Nyquist rate of $y_1(t)$, $y_2(t)$, and $y(t)$.

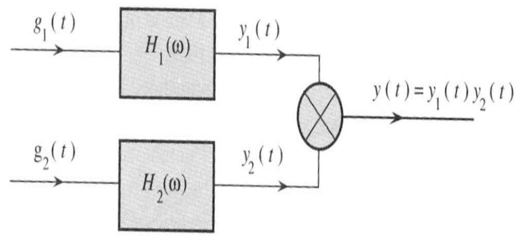


Figure P6.1-5

6.1-6 A zero-order hold circuit (Fig. P6.1-6) is often used to reconstruct a signal $g(t)$ from its samples.

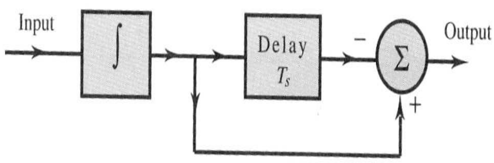


Figure P6.1-6

(a) Find the unit impulse response of this circuit.

(b) Find the transfer function $H(\omega)$ and sketch $|H(\omega)|$.

(c) Show that when a sampled signal $\overline{g}(t)$ is applied at the input of this circuit, the output is a staircase approximation of $g(t)$. The sampling interval is T_s.

6.2-2 A compact disc (CD) records audio signals digitally by using PCM. Assume the audio signal bandwidth to be 15 kHz.

(a) What is the Nyquist rate?

(b) If the Nyquist samples are quantized into $L = 65,536$ levels and then binary coded, determine the number of binary digits required to encode a sample.

(c) Determine the number of binary digits per second (bit/s) required to encode the audio signal.

(d) For practical reasons discussed in the text, signals are sampled at a rate well above the Nyquist rate. Practical CDs use 44,100 samples per second. If $L = 65,536$, determine the number of bits per second required to encode the signal, and the minimum bandwidth required to transmit the encoded signal.

6.2-3 A television signal (video and audio) has a bandwidth of 4.5 MHz. This signal is sampled, quantized, and binary coded to obtain a PCM signal.

(a) Determine the sampling rate if the signal is to be sampled at a rate 20% above the Nyquist rate.

(b) If the samples are quantized into 1024 levels, determine the number of binary pulses required to encode each sample.

(c) Determine the binary pulse rate (bits per second) of the binary-coded signal, and the minimum bandwidth required to transmit this signal.

6.2-6: A message signal $m(t)$ is transmitted by binary PCM without compression. If the SNR (signal-to-quantization-noise ratio) is required to be at least 47 dB, determine the minimum value of L required, assuming that $m(t)$ is sinusoidal. Determine the SNR obtained with this minimum L.

6.2-10: The output SNR of a 10-bit PCM ($N = 10$) was found to be 30 dB. The desired SNR is 42 dB. It was decided to increase the SNR to the desired value by increasing the number of quantization levels L. Find the required number of levels.

3.2 Assignment 3 Solutions

1. **6.1-1**: The bandwidth of $g_1(t)$ and $g_2(t)$ are 100 kHz and 150 kHz, respectively. Therefore,

   - the Nyquist sampling rates for $g_1(t)$ is 200 kHz, sampling interval $T_s = 1/200k = 5\mu s$
   - the Nyquist sampling rates for $g_2(t)$ is 300 kHz, sampling interval $T_s = 1/300k = 3.33\mu s$.
   - the bandwidth of $g_1^2(t)$ is 200 kHz, $f_{Nyq} = 400$ kHz, $f_{Nyq} = 1/400k = 0.25\mu s$.
   - the bandwidth of $g_2^3(t)$ is 450 kHz, $f_{Nyq} = 900$ kHz, $f_{Nyq} = 1/900k = 1.11\mu s$.
   - the bandwidth of $g_1(t) \cdot g_2(t)$ is 250 kHz, $f_{Nyq} = 500$ kHz, $f_{Nyq} = 1/500k = 2\mu s$.

2. **6.1-2**:

   - since
   $$\text{sinc}(100\pi t) \rightarrow 0.01\text{rect}\left(\frac{\omega}{200\pi}\right)$$
   the bandwidth of this signal is 100 π rad/s or 50 Hz. The Nyquist rate is 100 Hz (samples/sec).
   - $$\text{sinc}^2(100\pi t) \rightarrow 0.01\Delta\left(\frac{\omega}{400\pi}\right)$$
   the bandwidth of this signal is 200 π rad/s or 100 Hz. The Nyquist rate is 200 Hz (samples/sec).
   - $$\text{sinc}(100\pi t) + \text{sinc}(50\pi t) \rightarrow 0.01\text{rect}\left(\frac{\omega}{200\pi}\right) + 0.02\text{rect}\left(\frac{\omega}{100\pi}\right)$$
   the bandwidth of the first term on the right-hand side is 50 Hz and the second term is 25 Hz. Clearly the bandwidth of the composite signal is the higher of the two, that is, 100 Hz. The Nyquist rate is 200 Hz (samples/sec).
   - $$\text{sinc}(100\pi t) + 3\text{sinc}^2(60\pi t) \rightarrow 0.01\text{rect}\left(\frac{\omega}{200\pi}\right) + 0.05\Delta\left(\frac{\omega}{240\pi}\right)$$
   the bandwidth of the first term is 50 Hz and that of the second term is 60 Hz. The bandwidth of the sum is the higher of the two, that is, 60 Hz. The Nyquist sampling rate is 120 Hz.
   - $$\text{sinc}(50\pi t) \rightarrow 0.02\text{rect}\left(\frac{\omega}{100\pi}\right) \qquad \text{sinc}(100\pi t) \rightarrow 0.01\text{rect}\left(\frac{\omega}{200\pi}\right)$$
   The two signals have BW 25 Hz and 50 Hz respectively. The spectrum of the product of two signals is $1/(2\pi)$ times the convolution of their spectra. From width property of the convolution, the width of the convoluted signals is the sum of the widths of the signals convolved. Therefore, the BW of the product is 25+50=75 Hz. The Nyquist rate is 150 Hz.

3. **6.1-3**: The pulse train is a periodic signal with fundamental frequency $2B$ Hz. Hence, $\omega_s = 2\pi(2B) = 4\pi B$. The period is $T_0 = 1/(2B)$. It is an even function of t. Hence, the Fourier series for the pulse train can be expressed as

$$p_{T_s}(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos n\omega_s t$$

Using Eqs. (2.72), we obtain,

$$a_0 = C_0 = \frac{1}{T_0}\int_{-1/16B}^{1/16B} dt = \frac{1}{4}$$

and

$$a_n = C_n = \frac{2}{T_0}\int_{-1/16B}^{1/16B} \cos n\omega_s t \, dt = \frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right); \quad b_n = 0$$

Hence

$$\bar{g}(t) = g(t)p_{T_s}(t) = \frac{1}{4}g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right) g(t)\ \cos n\omega_s t$$

4. **6.1-4**: The BW of the signal $g(t)$ is 5 Hz (10π rad/s), since the FT as below:

$$g(t) = \text{sinc}^2(5\pi t) \quad \rightarrow \quad G(\omega) = 0.2\Delta\left(\frac{\omega}{20\pi}\right)$$

Therefore, the Nyquist rate is 10 Hz, and the Nyquist interval is $T = 1/10 = 0.1s$.

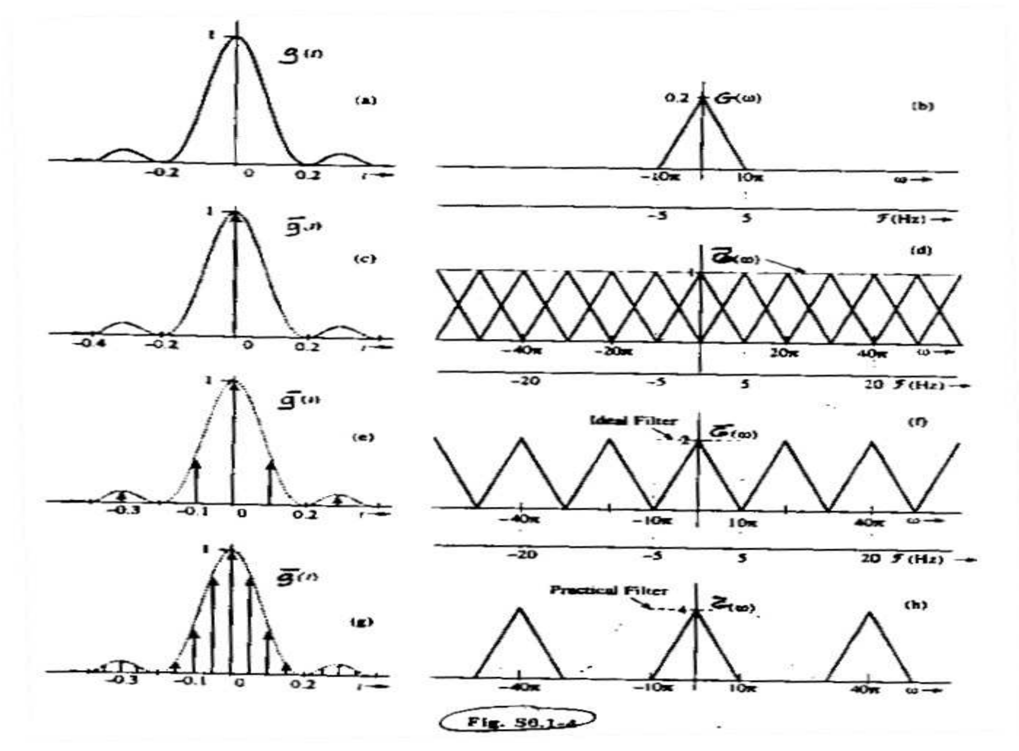


Fig. 3.1: Solution for 6.1-4.

- When $f_s = 5Hz$, the spectrum $\frac{1}{T}G(\omega)$ repeats every 5 Hz (10π rad/sec). The successive spectra overlap, and the spectrum $G(\omega)$ is not recoverable from $\bar{G}(\omega)$, that is, $g(t)$ cannot be recovered from its samples. If the sampled signal is passed through an ideal lowpass filter of BW 5 Hz, the output spectrum is $\text{rect}(\omega/20\pi)$, and the output signal is $10\text{sinc}(20\pi t)$, which is not the desired signal $\text{sinc}^2(5\pi t)$.
- When $f_s = 10Hz$, the spectrum $\bar{G}(\omega)$consists of back-to-back, nonoverlapping repetition of $\frac{1}{T}G(\omega)$ repeating every 10 Hz. Hence, $G(\omega)$ can be recovered from $\bar{G}(\omega)$ using an ideal lowpass filter of BW 5 Hz (Fig.1(f)), and the output is $10\text{sinc}^2(5\pi t)$.
- in the last case of oversampling ($f_s = 20$ Hz), with empty band between successive cycles. Hence, $G(\omega)$ can be recovered from $\bar{G}(\omega)$ using an ideal lowpass filter or even a practical lowpass filter. The output is $20\text{sinc}^2(5\pi t)$.

5. **6.1-5**: This scheme is analyzed fully in Problem 3.4-1, where we found the bandwidth of $y_1(t)$, $y_2(t)$, and $y(t)$ to be 10 kHz, 5 kHz, and 15 kHz, respectively. Hence, the Nyquist rates for the three signals are 20 kHz, 10 kHz, and 30 kHz, respectively.

6. **6.1-6**: (a) When the input to this filter is $b(t)$, the output of the summer is $\delta(t)-\delta(t-T)$. This acts as the input to the integrator. And, $h(t)$, the output of the integrator is

$$h(t) = \int_0^t [\delta(t) - \delta(t-T)]\, d\tau = u(t) - u(t-T) = \text{rect}\left(\frac{t - T/2}{T}\right)$$

The impulse response $h(t)$ is shown in the figure below.

(b) The transfer function of this circuit is

$$H(\omega) = T\text{sinc}\left(\frac{\omega T}{2}\right) e^{-j\omega T/2}$$

and

$$|H(\omega)| = T\left|\text{sinc}\left(\frac{\omega T}{2}\right)\right|$$

7. **6.2-2**:
(a): the bandwidth is 15 kHz. The Nyquist rate is 30 kHz.
(b): $65536 = 2^{16}$, so that 16 binary digits are needed to encode each sample.
(c): $30,000 \times 16 = 480,000$ bits/s.
(d): $44,100 \times 16 = 705,600$ bits/s.

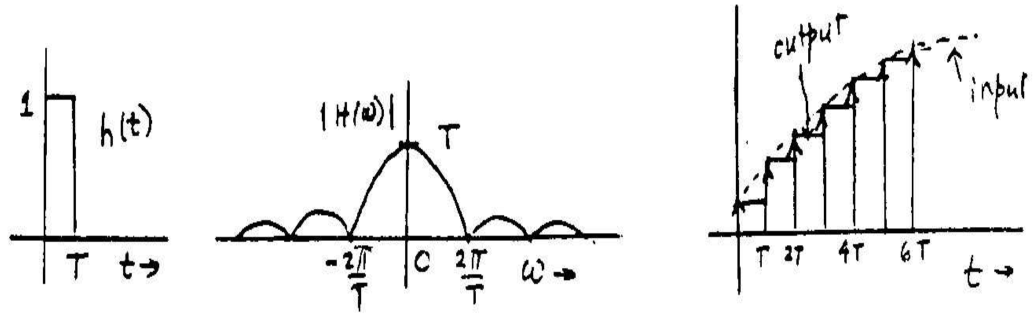


Figure S6.1-6

Fig. 3.2: Solution for 6.1-6.

6.2-9 (a) Nyquist rate = 2×10^6 Hz. The actual sampling rate is $1.5 \times (2 \times 10^6) = 3 \times 10^6$ Hz. Moreover, $L = 256$ and $\mu = 255$. From Eq. (6.18)

$$\frac{S_0}{N_0} = \frac{3L^2}{[\ln(\mu+1)]^2} = \frac{3(256)^2}{(\ln 256)^2} = 6394 = 38.06 \text{ dB}$$

(b) If we reduce the sampling rate and increase the value of L so that the same number of bits/second is maintained, we can improve the SNR (because of increased L) with the same bandwidth. In part (a), the sampling rate is 3×10^6 Hz and each sample is encoded by 8 bits ($L = 256$). Hence, the transmission rate is $8 \times 3 \times 10^6 = 24$ Mbits/second.
If we reduce the sampling rate to 2.4×10^6 (20% above the Nyquist rate), then for the same transmission rate (24 Mbits/s), we can have $(24 \times 10^6)/(2.4 \times 10^6) = 10$ bits/sample. This results in $L = 2^{10} = 1024$. Hence, the new SNR is

$$\frac{S_0}{N_0} = \frac{3L^2}{[\ln(\mu+1)]^2} = \frac{3(1024)^2}{(\ln 256)^2} = 102300 = 50.1 \text{ dB}$$

Clearly, the SNR is increased by more than 10 dB.

6.2-10 Equation (6.23) shows that increasing n by one bit increases the SNR by 6 dB. Hence, an increase in the SNR by 12 dB (from 30 to 42) can be accomplished by increasing n from 10 to 12, that is increasing by 20%.

4. ASSIGNMENT 4

4.1 Assignment 4 Problems

4.2-1 For each of the following baseband signals: (i) $m(t) = \cos 1000t$; (ii) $m(t) = 2\cos 1000t + \cos 2000t$; (iii) $m(t) = \cos 1000t \cos 3000t$:

(a) Sketch the spectrum of $m(t)$.

(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos 10{,}000t$.

(c) Identify the upper sideband (USB) and the lower sideband (LSB) spectra.

(d) Identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC, USB, and LSB spectra. Explain the nature of frequency shifting in each case.

4.2-2 Repeat Prob. 4.2-1 [parts **(a)**, **(b)**, and **(c)** only] if: **(i)** $m(t) = \text{sinc}\,(100t)$; **(ii)** $m(t) = e^{-|t|}$; **(iii)** $m(t) = e^{-|t-1|}$. Observe that $e^{-|t-1|}$ is $e^{-|t|}$ delayed by 1 second. For the last case you need to consider both the amplitude and the phase spectra.

4.2-3 Repeat Prob. 4.2-1 [parts **(a)**, **(b)**, and **(c)** only] for $m(t) = e^{-|t|}$ if the carrier is $\cos(10{,}000t - \pi/4)$. *Hint*: Use Eq. (3.36).

4.2-4 You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos\omega_c t$, where $m(t)$ is a signal band-limited to B Hz. Figure P4.2-4 shows a DSB-SC modulator available in the stock room. The carrier generator available generates not $\cos\omega_c t$, but $\cos^3\omega_c t$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.

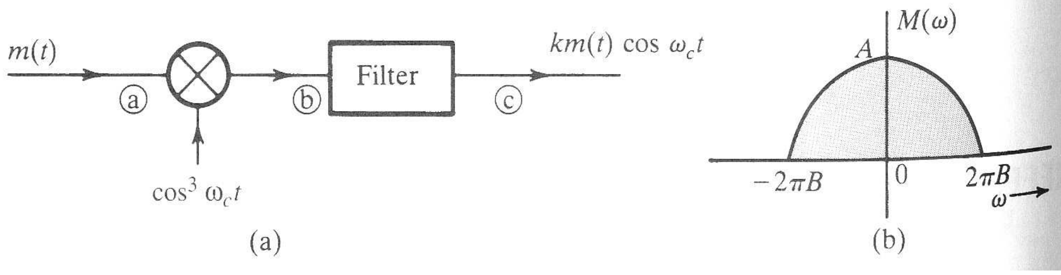
(a) What kind of filter is required in Fig. P4.2-4?

(b) Determine the signal spectra at points b and c, and indicate the frequency bands occupied by these spectra.

(c) What is the minimum usable value of ω_c?

(d) Would this scheme work if the carrier generator output were $\cos^2\omega_c t$? Explain.

(e) Would this scheme work if the carrier generator output were $\cos^n\omega_c t$ for any integer $n \geq 2$?



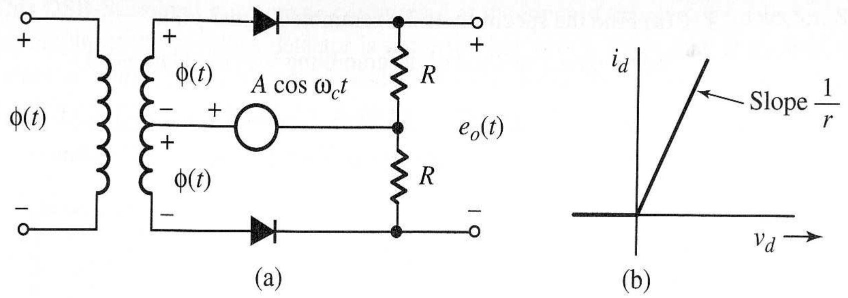


Figure P4.2-6

4.2-6 In Fig. P4.2-6, the input $\phi(t) = m(t)$, and the amplitude $A \gg |\phi(t)|$. The two diodes are identical with a resistance r ohms in the conducting mode and infinite resistance in the cutoff mode. Show that the output $e_o(t)$ is given by

$$e_o(t) = \frac{2R}{R+r} w(t)\, m(t)$$

where $w(t)$ is the switching periodic signal shown in Fig. 2.22a with period $2\pi/W_c$ seconds.

(a) Hence, show that this circuit can be used as a DSB-SC modulator.

(b) How would you use this circuit as a synchronous demodulator for DSB-SC signals.

4.2-7 In Fig. P4.2-6, if $\phi(t) = \sin(\omega_c t + \theta)$, and the output $e_0(t)$ is passed through a low-pass filter, then show that this circuit can be used as a phase detector, that is, a circuit that measures the phase difference between two sinusoids of the same frequency (ω_c). *Hint*: show that the filter output is a dc signal proportional to $\sin\theta$.

4.2-8 Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Fig. P4.2-8. The signal at point b is the multiplexed signal, which now modulates a carrier of frequency 20,000 rad/s. The modulated signal at point c is transmitted over a channel.

(a) Sketch signal spectra at points a, b, and c.

(b) What must be the bandwidth of the channel?

(c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c.

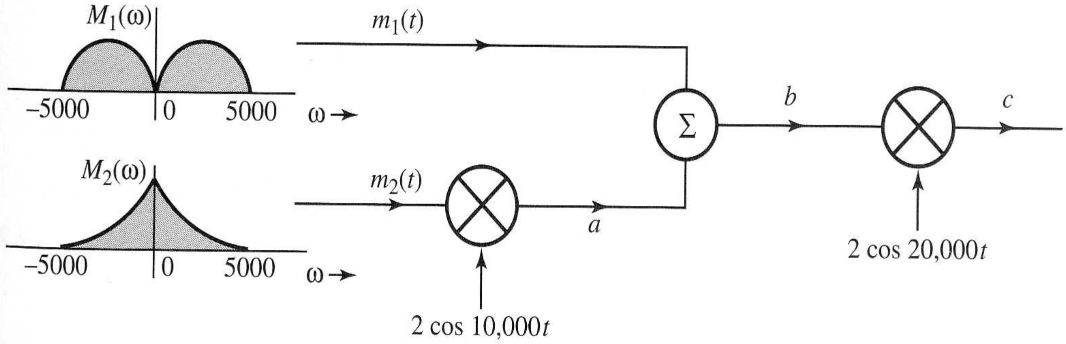


Figure P4.2-8

(a) Find the spectrum of the scrambled signal $y(t)$.

(b) Suggest a method of descrambling $y(t)$ to obtain $m(t)$.

A slightly modified version of this scrambler was first used commercially on the 25-mile radio-telephone circuit connecting Los Angeles and Santa Catalina island.

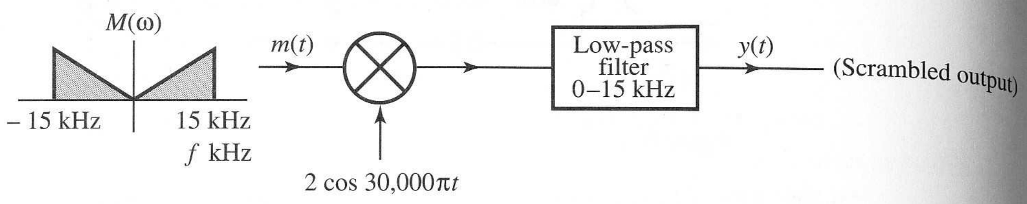


Figure P4.2-9

4.2-10 A DSB-SC signal is given by $m(t)\cos(2\pi)10^6 t$. The carrier frequency of this signal, 1 MHz, is to be changed to 400 kHz. The only equipment available is one ring modulator, a bandpass filter centered at the frequency of 400 kHz, and one sine wave generator whose frequency can be varied from 150 to 210 kHz. Show how you can obtain the desired signal $cm(t)\cos(2\pi \times 400 \times 10^6 t)$ from $m(t)\cos(2\pi)10^6 t$. Determine the value of c.

4.3-1 Figure P4.3-1 shows a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the AM signal $[A + m(t)]\cos\omega_c t$ regardless of the value of A.

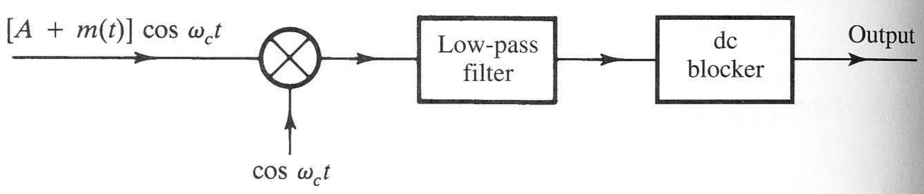


Figure P4.3-1

4.3-2 Sketch the AM signal $[A+m(t)]\cos\omega_c t$ for the periodic triangle signal $m(t)$ shown in Fig. P4.3-2 corresponding to the modulation index: **(a)** $\mu = 0.5$; **(b)** $\mu = 1$; **(c)** $\mu = 2$; **(d)** $\mu = \infty$. How do you interpret the case $\mu = \infty$?

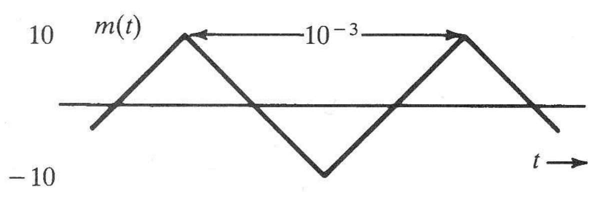


Figure P4.3-2

4.3-3 For the AM signal in Prob. 4.3-2 with $\mu = 0.8$:

(a) Find the amplitude and power of the carrier.

(b) Find the sideband power and the power efficiency η.

4.3-4 **(a)** Sketch the DSB-SC signal corresponding to $m(t) = \cos 2\pi t$.

(b) This DSB-SC signal $m(t) \cos \omega_c t$ is applied at the input of an envelope detector. Show that the output of the envelope detector is not $m(t)$, but $|m(t)|$. Show that, in general, if an AM signal $[A + m(t)] \cos \omega_c t$ is envelope-detected, the output is $|A + m(t)|$. Hence, show that the condition for recovering $m(t)$ from the envelope detector is $A + m(t) > 0$ for all t.

4.3-5 Show that any scheme that can be used to generate DSB-SC can also generate AM. Is the converse true? Explain.

4.3-6 Show that any scheme that can be used to demodulate DSB-SC can also demodulate AM. Is the converse true? Explain.

4.3-7 In the text, the power efficiency of AM for a sinusoidal $m(t)$ was found. Carry out a similar analysis when $m(t)$ is a random binary signal as shown in Fig. P4.3-7 and $\mu = 1$. Sketch the AM signal with $\mu = 1$. Find the sideband's power and the total power (power of the AM signal) as well as their ratio (the power efficiency η).

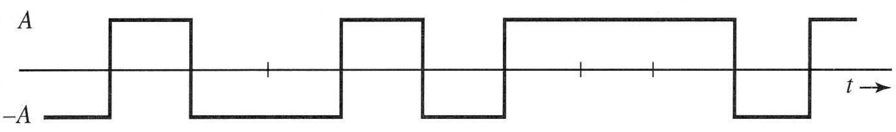


Figure P4.3-7

4.3-8 In the early days of radio, AM signals were demodulated by a crystal detector followed by a low-pass filter and a dc blocker, as shown in Fig. P4.3-8. Assume a crystal detector to be basically a squaring device. Determine the signals at points a, b, c, and d. Point out the distortion term in the output $y(t)$. Show that if $A \gg |m(t)|$, the distortion is small.

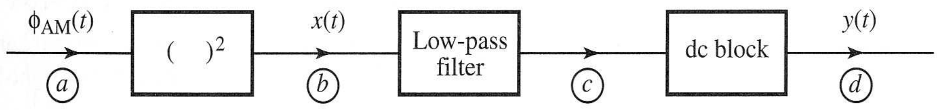


Figure P4.3-8

4.4-1 In a QAM system (Fig. 4.14), the locally generated carrier has a frequency error $\Delta\omega$ and a phase error δ; that is, the receiver carrier is $\cos[(\omega_c + \Delta\omega)t + \delta]$ or $\sin[(\omega_c + \Delta\omega)t + \delta]$. Show that the output of the upper receiver branch is

$$m_1(t) \cos[(\Delta\omega)t + \delta] - m_2(t) \sin[(\Delta\omega)t + \delta]$$

instead of $m_1(t)$, and the output of the lower receiver branch is

$$m_1(t) \sin[(\Delta\omega)t + \delta] + m_2(t) \cos[(\Delta\omega)t + \delta]$$

instead of $m_2(t)$.

4.5-1 A modulating signal $m(t)$ is given by:

(a) $m(t) = \cos 100t$

(b) $m(t) = \cos 100t + 2\cos 300t$

(c) $m(t) = \cos 100t \cos 500t$

In each case:

(i) Sketch the spectrum of $m(t)$.

(ii) Find and sketch the spectrum of the DSB-SC signal $2m(t)\cos 1000t$.

(iii) From the spectrum obtained in **(ii)**, suppress the LSB spectrum to obtain the USB spectrum.

(iv) Knowing the USB spectrum in **(ii)**, write the expression $\varphi_{\text{USB}}(t)$ for the USB signal.

(v) Repeat **(iii)** and **(iv)** to obtain the LSB signal $\varphi_{\text{LSB}}(t)$.

4.5-2 For the signals in Prob. 4.5-1, determine $\varphi_{\text{LSB}}(t)$ and $\varphi_{\text{USB}}(t)$ using Eq. (4.17) if the carrier frequency $\omega_c = 1000$. *Hint*: If $m(t)$ is a sinusoid, its Hilbert transform $m_h(t)$ is the sinusoid $m(t)$ phase-delayed by $\pi/2$ rad.

4.5-3 Find $\varphi_{\text{LSB}}(t)$ and $\varphi_{\text{USB}}(t)$ for the modulating signal $m(t) = B\,\text{sinc}\,(2\pi Bt)$ with $B = 1000$ and carrier frequency $\omega_c = 10{,}000\pi$. Follow these do-it-yourself steps:

(a) Sketch spectra of $m(t)$ and the corresponding DSB-SC signal $2m(t)\cos\omega_c t$.

(b) To find the LSB spectrum, suppress the USB in the DSB-SC spectrum found in **(a)**.

(c) Find the LSB signal $\varphi_{\text{LSB}}(t)$, which is the inverse Fourier transform of the LSB spectrum found in part **(b)**. Follow a similar procedure to find $\varphi_{\text{USB}}(t)$.

4.5-4 If $m_h(t)$ is the Hilbert transform of $m(t)$, then show that the Hilbert transform of $m_h(t)$ is $-m(t)$. (This shows that the inverse Hilbert transform operation is identical to the direct Hilbert transform operation with a negative sign.) Show also that the energies of $m(t)$ and $m_h(t)$ are identical. *Hint*: The Hilbert transform of $m(t)$ is obtained by passing $m(t)$ through a transfer function $H(\omega)$, whose amplitude and phase responses are shown in Fig. 4.17. The Hilbert transform of the Hilbert transform of $m(t)$ is obtained by passing $m(t)$ through $H(\omega)$ in cascade with $H(\omega)$.

4.5-5 An LSB signal is demodulated synchronously, as shown in Fig. P4.5-5. Unfortunately, the local carrier is not $2\cos\omega_c t$ as required, but is $2\cos[(\omega_c + \Delta\omega)t + \delta]$. Show that:

(a) When $\delta = 0$, the output $y(t)$ is the signal $m(t)$ with all its spectral components shifted (offset) by $\Delta\omega$. *Hint*: Observe that the output $y(t)$ is identical to the right-hand side of Eq. (4.17a) with ω_c replaced with $\Delta\omega$.

(b) When $\Delta\omega = 0$, the output is the signal $m(t)$ with phases of all its spectral components shifted by δ. *Hint*: Show that the output spectrum $Y(\omega) = M(\omega)e^{j\delta}$ for $\omega \geq 0$, and equal to $M(\omega)e^{-j\delta}$ when $\omega < 0$.

In each of these cases, explain the nature of distortion. *Hint*: For **(a)**, demodulation consists of shifting an LSB spectrum to the left and right by $\omega_c + \Delta\omega$, and low-pass filtering the result. For part **(b)**, use the expression (4.17b) for $\varphi_{\text{LSB}}(t)$ and multiply it by the local carrier $2\cos(\omega_c t + \delta)$, and low-pass filter the result.

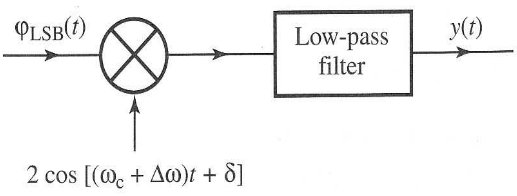


Figure P4.5-5

4.5-6 An USB signal is generated by using the phase-shift method (Fig. 4.20). If the input to this system is $m_h(t)$ instead of $m(t)$, what will be the output? Is this signal still an SSB signal with bandwidth equal to that of $m(t)$? Can this signal be demodulated [to get back $m(t)$]? If so, how?

4.6-1 A vestigial filter $H_i(\omega)$ shown in the transmitter of Fig. 4.22 has a transfer function as shown in Fig. P4.6-1. The carrier frequency is $f_c = 10$ kHz and the baseband signal bandwidth is 4 kHz. Find the corresponding transfer function of the equalizer filter $H_o(\omega)$ shown in the receiver of Fig. 4.22.

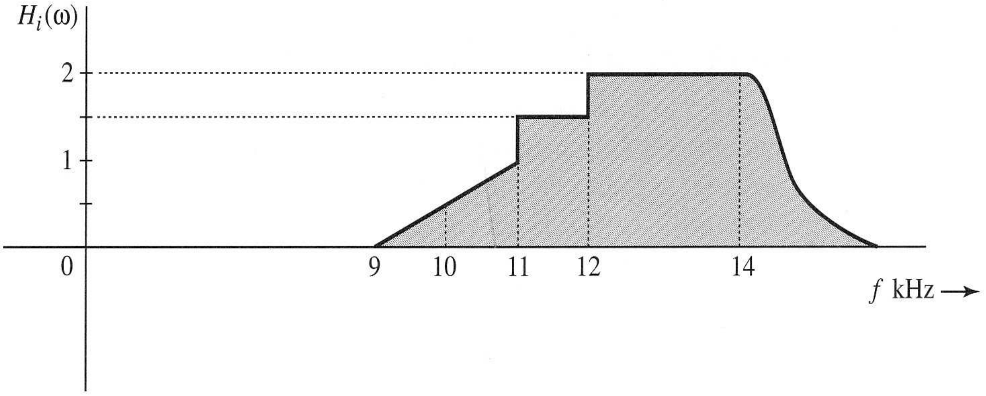


Figure P4.6-1

4.8-1 A transmitter transmits an AM signal with a carrier frequency of 1500 kHz. When an inexpensive radio receiver (which has a poor selectivity in its RF-stage bandpass filter) is tuned to 1500 kHz, the signal is heard loud and clear. This same signal is also heard (not as strong) at another dial setting. State, with reasons, at what frequency you will hear this station. The IF frequency is 455 kHz.

4.8-2 Consider a superheterodyne receiver designed to receive the frequency band of 1 to 30 MHz with IF frequency 8 MHz. What is the range of frequencies generated by the local oscillator for this receiver? An incoming signal with carrier frequency 10-MHz is received at the 10 MHz setting. At this setting of the receiver we also get interference from a signal with some other carrier frequency if the receiver RF stage bandpass filter has poor selectivity. What is the carrier frequency of the interfering signal?

4.2 Assignment 4 Solutions

- **4.2-1**: ,
  - For $m(t) = \cos 1000t$,

    $$\psi_{DSB-SC}(t) = m(t) \cdot \cos 10,000t = \cos 1000t \, \cos 10,000t = \frac{1}{2}[\cos 9000t + \cos 11,000t]$$

    where the first term is LSB part, the second term is USB part.
  - For $m(t) = 2\cos 1000t + \cos 2000t$,

    $$\begin{aligned}\psi_{DSB-SC}(t) &= m(t) \cdot \cos 10,000t = [2\cos 1000t + \cos 2000t] \, \cos 10,000t \\ &= \cos 9000t + \cos 11,000t + \frac{1}{2}[\cos 8000t + \cos 12,000t] \\ &= \left[\cos 9000t + \frac{1}{2}\cos 8000t\right] + \left[\cos 11,000t + \frac{1}{2}\cos 12,000t\right]\end{aligned}$$

    where the first bracket includes LSB part, and the second USB part.
  - For $m(t) = \cos 1000t \cdot \cos 3000t$,

    $$\begin{aligned}\psi_{DSB-SC}(t) &= m(t) \cdot \cos 10,000t = 0.5 \cdot [\cos 2000t + \cos 4000t] \, \cos 10,000t \\ &= \frac{1}{2}[\cos 8000t + \cos 12,000t] + \frac{1}{2}[\cos 6000t + \cos 14,000t] \\ &= \frac{1}{2}[\cos 8000t + \cos 6000t] + \frac{1}{2}[\cos 12,000t + \cos 14,000t]\end{aligned}$$

    where the first bracket includes LSB part, and the second USB part.

- **4.2-2**: The relevant plots are shown in Fig. S4.2-2.

- **4.2-3**: The relevant plots are shown in Fig. S4.2-3.

- **4.2-4**: (a) The signal at point b is

  $$g_a(t) = m(t)\cos^3 \omega_c t = m(t)\left[\frac{3}{4}\cos \omega_c t + \frac{1}{4}\cos 3\omega_c t\right]$$

  The term $\frac{3}{4}\cos \omega_c t$ is the desired modulated signal, whose spectrum is centered at $\pm\omega_c$. The remaining term $\frac{1}{4}\cos 3\omega_c t$ is the unwanted term, which represents the modulated signal with carrier frequency $3\omega_c$ with spectrum centered at $\pm 3\omega_c$, as shown in Fig. S4.2-4. The bandpass filter centered at $\pm\omega_c$ allows to pass the desired term $\frac{3}{4}\cos \omega_c t$, but suppresses the unwanted term $\frac{1}{4}\cos 3\omega_c t$. Hence, this system works as desired with the output $\frac{3}{4}\cos \omega_c t$.

  (b) Fig. S4.2-4 shows the spectra at point b and c.

  (c) The minimum usable value of ω_c is $2\pi B$ in order to avoid spectral folding at dc.

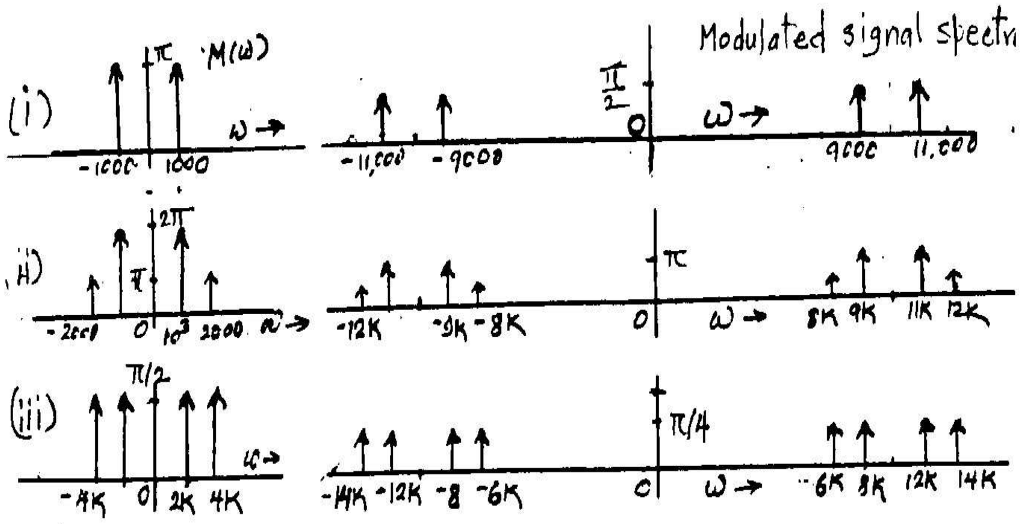


Fig. S4.2-1

Fig. 4.1: Solution for 4.2-1.

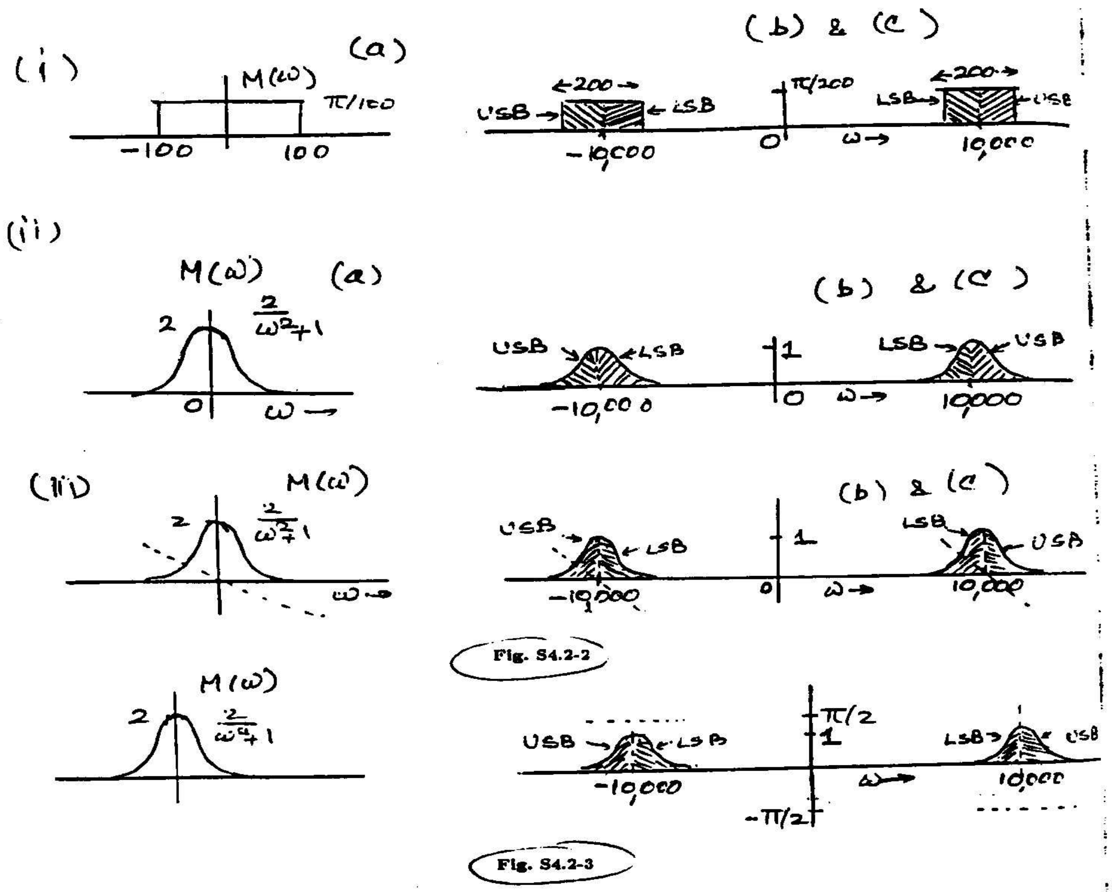


Fig. 4.2: Solution for 4.2-3.

(d)

$$m(t)\cos^2\omega_c t = \frac{m(t)}{2}[1+\cos 2\omega_c t] = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t$$

This signal at point b consists of the baseband signal $\frac{1}{2}m(t)$ and a modulated signal $\frac{1}{2}m(t)\cos 2\omega_c t$, which has a carrier frequency $2\omega_c$, not the desired value ω_c. both the components will be suppressed by the filter, whose center frequency is ω_c. Hence, this system will not do the desired job.

(e) The reader may verify that the identity for $\cos n\omega_c t$ contains a term $\cos\omega_c t$ when n is odd. This is not true when n is even. Hence, the system works for a carrier $\cos^n \omega_c t$ only when n is odd.

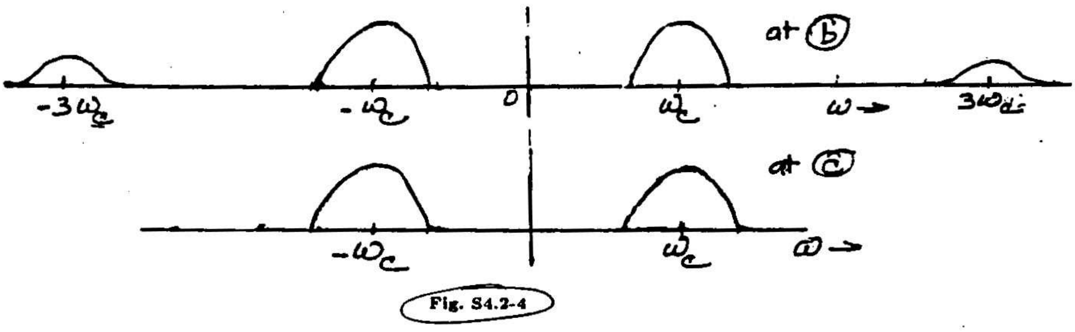


Fig. 4.3: Solution for 4.2-4.

- **4.2-6** The resistance of each diode is r ohms while conducting, and ∞ when off. When the carrier $A\cos\omega_c t$ is prositive, the diodes conduct (during the entire positive half cycle), and when the carrier is negative, the diodes are open (during the entire negative half cycle). Thus, during the positive half cycle, the voltage $R/(R+r)\phi(t)$ appears across each of the resistor R. During the negative half cycle, the output voltage is zero. Therefore, the diodes act as a gate in the circuit that is basically a voltage divider with a gain $2R/(R+r)$. The output is therefore,

$$e_o(t) = \frac{2R}{R+r}w(t)m(t)$$

The period of $w(t)$ is $T_0 = 2\pi/\omega_c$. Hence, from Eq. (2.75),

$$w(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \cdots\right]$$

The output $e_o(t)$ is

$$e_o(t) = \frac{2R}{R+r}w(t)m(t) = \frac{2R}{R+r}m(t)\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \cdots\right)\right]$$

(a) If we pass the output $e_o(t)$ through a bandpass filter (centered at ω_c), the filter suppresses the signal $m(t)$ and $m(t)\cos n\omega_c t$ for all $n \neq 1$, leaving only the modulated term $\frac{4R}{\pi(R+r)}m(t)\cos\omega_c t$ intact. Hence, the system acts as a modulator.

(b) The same circuit can be used as a demodulator if we use a bandpass filter at the output. In this case, the input is $\phi(t) = m(t)\cos\omega_c t$ and the output is $\frac{4R}{\pi(R+r)}m(t)$.

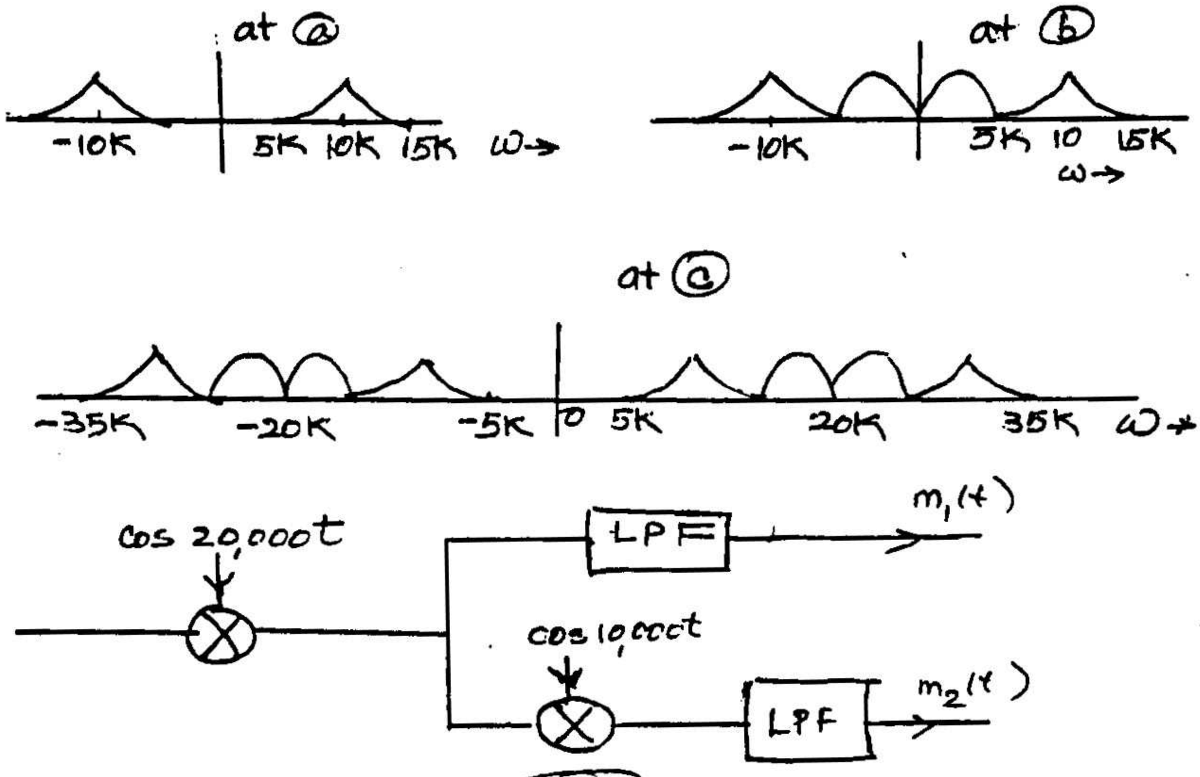


Fig. S4.2-8

4.2-8 **(a)** Fig. S4.2-8 shows the signals at points a. b. and c.
(b) From the spectrum at point c, it is clear that the channel bandwidth must be at least 30,000 rad/s (from 5000 to 35,000 rad/s.).
(c) Fig. S4.2-8 shows the receiver to recover $m_1(t)$ and $m_2(t)$ from the received modulated signal.

- **4.3-1** $g_a(t) = [A + m(t)]\cos\omega_c t$. Hence,

$$g_b(t) = [A + m(t)]\cos^2\omega_c t = \frac{1}{2}[A + m(t)] + \frac{1}{2}[A + m(t)]\cos 2\omega_c t$$

The first term is a lowpass signal because its spectrum is centered at $\omega = 0$. The lowpass filter allows this term to pass, but suppresses the second term, whose spectrum is centered at $\pm 2\omega_c$. Hence, the output of the lowpass filter is

$$y(t) = A + m(t)$$

When this signal is passed through a DC block, the DC term A is suppressed yielding the output $m(t)$. This shows that the system can demodulate AM signal regardless of the value of A. This is a synchronous or coherent demodulation.

- **4.3-2** (a)

$$\mu = 0.5 = \frac{m_p}{A} = \frac{10}{A} \rightarrow A = 20$$

(b)

$$\mu = 1.0 = \frac{m_p}{A} = \frac{10}{A} \rightarrow A = 10$$

(c)

$$\mu = 2.0 = \frac{m_p}{A} = \frac{10}{A} \rightarrow A = 5$$

(d)

$$\mu = \infty = \frac{m_p}{A} = \frac{10}{A} \rightarrow A = 0$$

This means that $\mu = \infty$ represents the DSB-SC case.

4.3-3 **(a)** According to Eq. (4.10a), the carrier amplitude is $A = m_p/\mu = 10/0.8 = 12.8$. The carrier power is $P_c = A^2/2 = 78.125$.

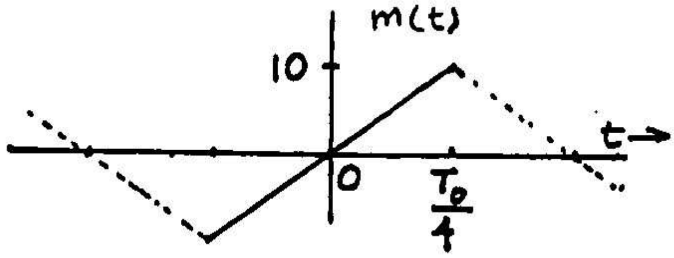


Fig. S4.3-3

(b) The sideband power is $\widetilde{m^2(t)}/2$. Because of symmetry of amplitude values every quarter cycle, the power of $m(t)$ may be computed by averaging the signal energy over a quarter cycle only. Over a quarter cycle $m(t)$ can be represented as $m(t) = 40t/T_0$ (see Fig. S4.3-3). Hence,

$$\widetilde{m^2(t)} = \frac{1}{T_0/4}\int_0^{T_0/4}\left[\frac{40t}{T_0}\right]^2 dt = 33.34$$

The sideband power is

$$P_s = \frac{\widetilde{m^2(t)}}{2} = 16.67$$

The efficiency is

$$\eta = \frac{P_s}{P_c + P_s} = \frac{16.67}{78.125 + 16.67} \times 100 = 19.66\%$$

4.3-4 From Fig. S4.3-4 it is clear that the envelope of the signal $m(t)\cos\omega_c t$ is $|m(t)|$. The signal $[A + m(t)]\cos\omega_c t$ is identical to $m(t)\cos\omega_c t$ with $m(t)$ replaced by $A + m(t)$. Hence, using the previous argument, it is clear that its envelope is $|A + m(t)|$. Now, if $A + m(t) > 0$ for all t, then $A + m(t) = |A + m(t)|$. Therefore, the condition for demodulating AM signal using envelope detector is $A + m(t) > 0$ for all t.

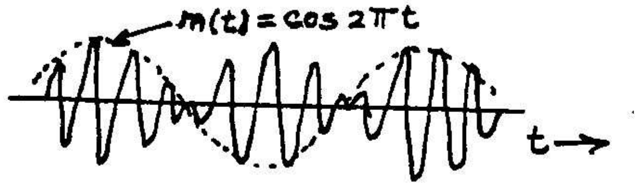


Fig. S4.3-4

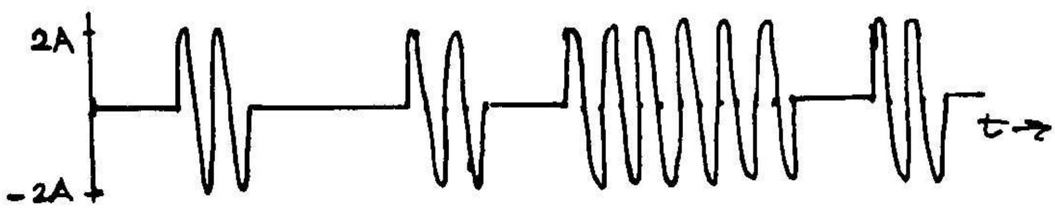


Fig. S4.3-7

4.3-7 Observe that $m^2(t) = A^2$ for all t. Hence, the time average of $m^2(t)$ is also A^2. Thus

$$\widetilde{m^2(t)} = A^2 \qquad P_s = \frac{\widetilde{m^2(t)}}{2} = \frac{A^2}{2}$$

The carrier amplitude is $A = m_p/\mu = m_p = A$. Hence $P_c = A^2/2$. The total power is $P_t = P_c + P_s = A^2$. The power efficiency is

$$\eta = \frac{P_s}{P_t} \times 100 = \frac{A^2/2}{A^2} \times 100 = 0.5$$

The AM signal for $\mu = 1$ is shown in Fig. S4.3-7.

4.3-8 The signal at point a is $[A + m(t)]\cos\omega_c t$. The signal at point b is

$$x(t) = [A + m(t)]^2 \cos^2\omega_c t = \frac{A^2 + 2Am(t) + m^2(t)}{2}(1 + \cos 2\omega_c t)$$

The lowpass filter suppresses the term containing $\cos 2\omega_c t$. Hence, the signal at point c is

$$w(t) = \frac{A^2 + 2Am(t) + m^2(t)}{2} = \frac{A^2}{2}\left[1 + \frac{2m(t)}{A} + \left(\frac{m(t)}{A}\right)^2\right]$$

Usually, $m(t)/A \ll 1$ for most of the time. Only when $m(t)$ is near its peak, this condition is violated. Hence, the output at point d is

$$y(t) \approx \frac{A^2}{2} + Am(t)$$

A blocking capacitor will suppress the dc term $A^2/2$, yielding the output $Am(t)$. From the signal $w(t)$, we see that the distortion component is $m^2(t)/2$.

- **4.4-1** In Fig. 4.14, when the carrier is $\cos[(\Delta\omega)t+\delta]$ or $\sin[(\Delta\omega)t+\delta]$, we have

$$\begin{aligned} x_1(t) &= 2[m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t]\cos[(\omega_c+\Delta\omega)t+\delta] \\ &= 2m_1(t)\cos\omega_c t\cos[(\omega_c+\Delta\omega)t+\delta] + 2m_2(t)\sin\omega_c t\cos[(\omega_c+\Delta\omega)t+\delta] \\ &= m_1(t)\{\cos[(\Delta\omega)t+\delta]+\cos[(2\omega_c+\Delta\omega)t+\delta]\} \\ &\quad +m_2(t)\{\sin[(2\omega_c+\Delta\omega)t+\delta]-\sin[(\Delta\omega)t+\delta]\} \end{aligned}$$

Similarly,

$$\begin{aligned} x_2(t) &= m_1(t)\{\sin[(2\omega_c+\Delta\omega)t+\delta]+\sin[(\Delta\omega)t+\delta] \\ &\quad +m_2(t)\{\cos[(\Delta\omega)t+\delta]-\cos[(2\omega_c+\Delta\omega)t+\delta]\} \end{aligned}$$

After $x_1(t)$ and $x_2(t)$ are passed through lowpass filter, the outputs are

$$m_1'(t) = m_1(t)\cos[(\Delta\omega)t+\delta] - m_2(t)\sin[(\Delta\omega)t+\delta]$$

$$m_2'(t) = m_1(t)\sin[(\Delta\omega)t+\delta] + m_2(t)\cos[(\Delta\omega)t+\delta]$$

- **4.5-1** To generate a DSB-SC signal from $m(t)$, we multiply $m(t)$ with $\cos\omega_c t$. However, to generate the SSB signals of the same relative magnitude, it is convenient to multiply $m(t)$ with $2\cos\omega_c t$. This also avoids the nuisance of the fractions 1/2, and yields the DSB-SC spectrum $M(\omega-\omega_c)+M(\omega+\omega_c)$. We suppress the USB spectrum (above ω_c and below $-\omega_c$) to obtain the LSB spectrum. Similarly, to obtain the USB spectrum, we suppress the LSB spectrum (between $-\omega_c$ and ω_c) from the DSB-SC spectrum. Fig.S4.5-1 a,b, and c show the three cases.

  (a) From Fig.a, we can express $\psi_{LSB}(t) = \cos 900t$ and $\psi_{USB}(t) = \cos 1100t$.

  (b) From Fig.b, we can express $\psi_{LSB}(t) = 2\cos 700t + \cos 900t$ and $\psi_{USB}(t) = \cos 1100t + 2\cos 1300t$.

  (b) From Fig.c, we can express $\psi_{LSB}(t) = 0.5[\cos 400t + \cos 600t$ and $\psi_{USB}(t) = 0.5[\cos 1400t + 2\cos 1600t]$.

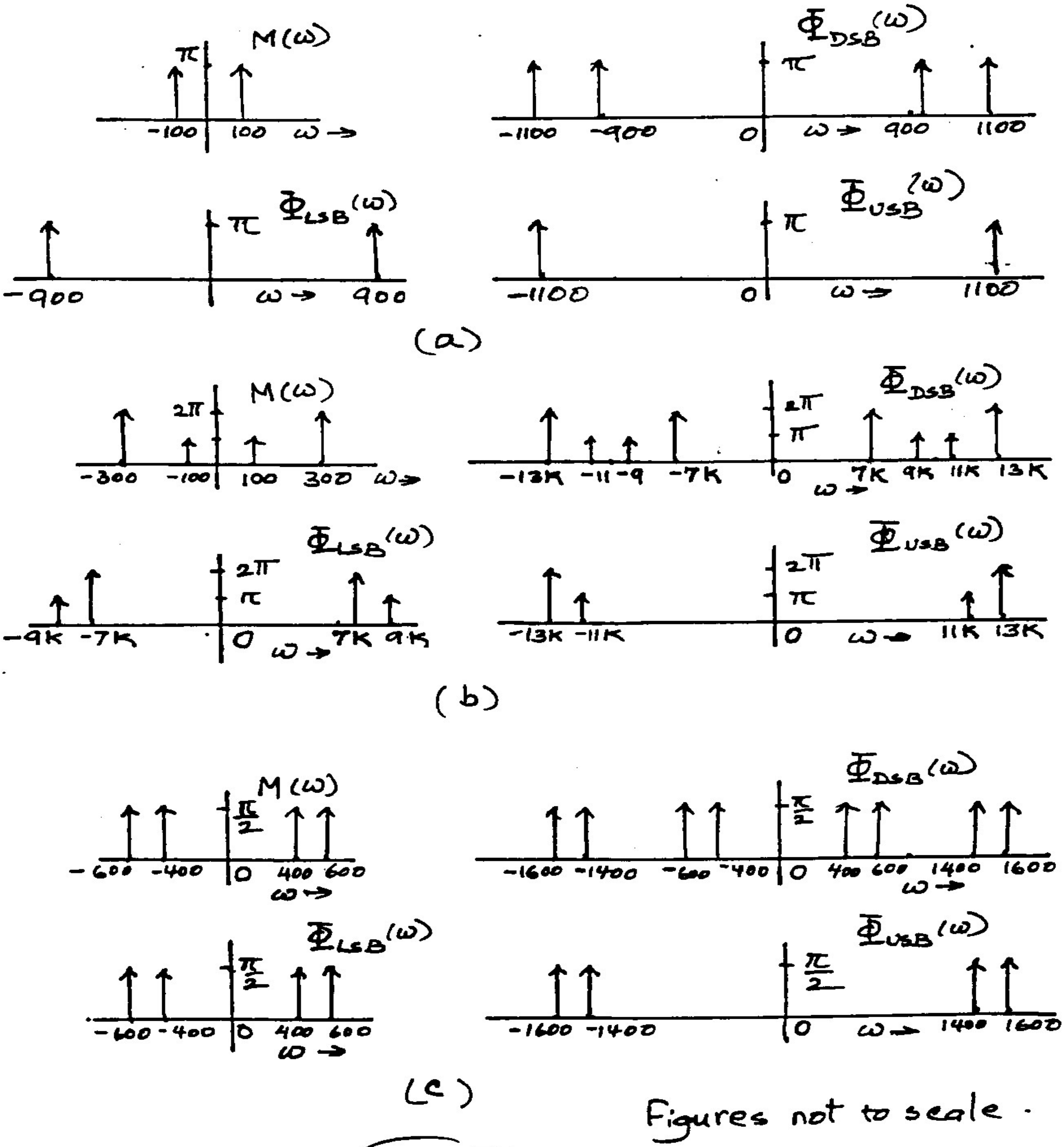


Fig. S4.5-1

4.5-2

$$\varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t \quad \text{and} \quad \varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

(a) $m(t) = \cos 100t$ and $m_h(t) = \sin 100t$. Hence,

$$\varphi_{LSB}(t) = \cos 100t \cos 1000t + \sin 100t \sin 1000t = \cos(1000 - 100)t = \cos 900t$$

$$\varphi_{USB}(t) = \cos 100t \cos 1000t - \sin 100t \sin 1000t = \cos(1000 + 100)t = \cos 1100t$$

(b) $m(t) = \cos 100t + 2\cos 300t$ and $m_h(t) = \sin 100t + 2\sin 300t$. Hence,

$$\varphi_{LSB}(t) = (\cos 100t + 2\cos 300t)\cos 1000t + (\sin 100t + 2\sin 300t)\sin 1000t = \cos 900t + 2\cos 700t$$

$$\varphi_{USB}(t) = (\cos 100t + 2\cos 300t)\cos 1000t - (\sin 100t + 2\sin 300t)\sin 1000t = \cos 1100t + 2\cos 1300t$$

(c) $m(t) = \cos 100t \cos 500t = 0.5\cos 400t + 0.5\cos 600t$ and $m_h(t) = 0.5\sin 400t + 0.5\sin 600t$. Hence,

$$\varphi_{LSB}(t) = (0.5\cos 400t + 0.5\cos 600t)\cos 1000t + (0.5\sin 400t + 0.5\sin 600t)\sin 1000t = 0.5\cos 400t + 0.5\cos 600t$$

$$\varphi_{USB}(t) = (0.5\cos 400t + 0.5\cos 600t)\cos 1000t - (0.5\sin 400t + 0.5\sin 600t)\sin 1000t = 0.5\cos 1400t + 0.5\cos 1600t$$

- **4.5-3** (a) Fig. S4.5-3a shows the spectrum of $m(t)$ and Fig.S4.5-3b shows the corresponding DSB-SC spectrum $2m(t)\cos 10,000\pi t$.

  (b) Fig.S4.5-3c shows the corresponding LSB spectrum obtained by suppressing the USB spectrum.

  (c) Fig.S4.5-3d shows the corresponding USB spectrum obtained by suppressing the LSB spectrum. We now find the inverse Fourier transforms of the LSB and USB spectra from Table 3.1 (pair 18) and the frequency shifting property as

$$\psi_{LSB}(t) = 1000\text{sinc}(1000\pi t)\cos 9000\pi t$$

$$\psi_{USB}(t) = 1000\text{sinc}(1000\pi t)\cos 11000\pi t$$

- **4.5-5** The incoming SSB signal at the receiver is given by [Eq. (4.17b)]

$$\psi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

  Let the local carrier be $\cos[(\omega_c + \Delta\omega)t + \delta]$. The product of the incoming signal and the local carrier is $e_d(t)$, given by

$$e_d(t) = \psi_{LSB}(t)\cos[(\omega_c + \Delta\omega)t + \delta] = 2[m(t)\cos\omega_c t + m_h(t)\sin\omega_c t]\cos[(\omega_c + \Delta\omega)t + \delta]$$

  The lowpass filter suppresses the sum frequency component centered at the frequency $(2\omega_c + \Delta\omega)$, and passes only the difference frequency component centered at the frequency

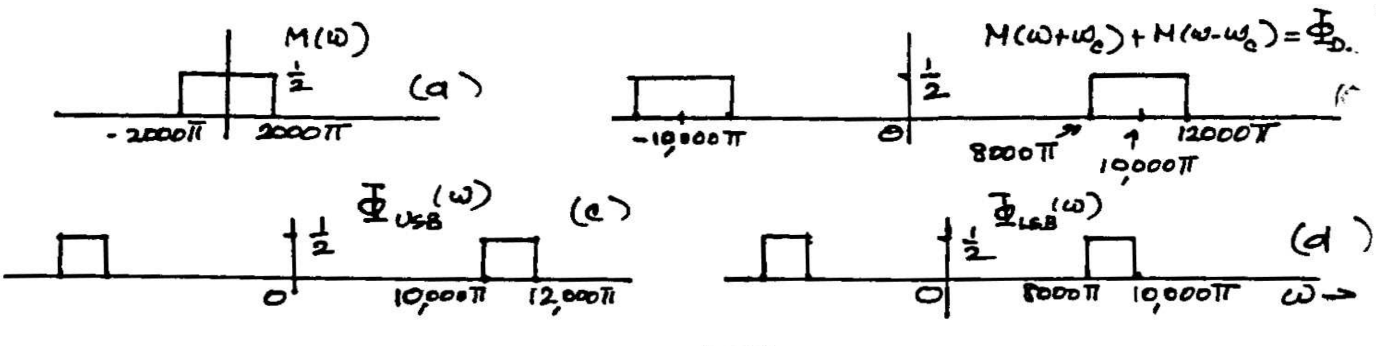


Fig. S4.5-3

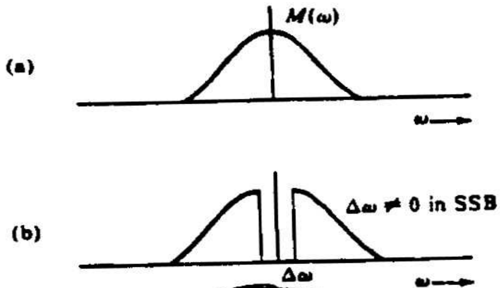


Fig. S4.5-5

$\Delta\omega$. Hence, the filter output $e_o(t)$ is given by

$$e_o(t) = m(t)\cos[(\Delta\omega)t + \delta] - m_h(t)\sin[(\Delta\omega)t + \delta]$$

Observe that if both $\Delta\omega$ and δ are zero, the output is given by

$$e_o(t) = m(t)$$

as expected. If only $\delta = 0$, then the output is given by

$$e_o(t) = m(t)\cos(\Delta\omega)t - m_h(t)\cos(\Delta\omega)t$$

This is an USB signal corresponding to a carrier frequency $\Delta\omega$ as shown in Fig. S5.5-5b. This spectrum is the same as the spectrum $M(\omega)$ with each frequency component shifted by a frequency $\Delta\omega$. This changes the sound of an audio signal slightly. For voice signals, the frequency shift within ±20Hz is considered tolerable. Most US systems, however, restrict the shift to ±2Hz.

(b) When only $\Delta\omega = 0$, the lowpass filter output is

$$e_o(t) = m(t)\cos\delta - m_h(t)\sin\delta$$

We now show that this is a phase distortion, where each frequency component of $M(\omega)$ is shifted in phase by amount δ. The Fourier transform of this equation yields

$$E_o(\omega) = M(\omega)\cos\delta - M_h(\omega)\sin\delta$$

But from Eq.(4.14b)

$$M_h(\omega) = -jsgn(\omega)M(\omega) = \begin{cases} -jM(\omega) & \omega > 0 \\ M(\omega) & \omega < 0 \end{cases}$$

and

$$E_o(\omega) = \begin{cases} M(\omega)e^{j\delta} & \omega > 0 \\ M(\omega)e^{-j\delta} & \omega < 0 \end{cases}$$

It follows that the amplitude spectrum of $e_o(t)$ is $M(\omega)$. The same as that for $m(t)$. But the phase of each component is shifted by δ. Phase distortion generally is not a serious problem with voice signals, because the human ear is somewhat insensitive to phase distortion. Such distortion may change the quality of speech, but the voice is still intelligible. In video signals and data transmissions, however, phase distortion may be intolerable.

- **4.5-6** We showed in Prob. 4.5-4 that the Hilbert transform of $m_h(t)$ is $-m(t)$. Hence, if $m_h(t)$ (instead of $m(t)$) is applied at the input in Fig.4.20, the USB output is

$$y(t) = m_h(t)\cos\omega_c t - m(t)\sin\omega_c t = m(t)\cos\left(\omega_c t + \frac{\pi}{2}\right) + m_h(t)\sin\left(\omega_c t + \frac{\pi}{2}\right)$$

  Thus, if we apply $m_h(t)$ at the input of the Fig.4.20, the USB output is an LSB signal corresponding to $m(t)$. The carrier also acquires a phase shift $\pi/2$. Similarly, we can show that if we apply $m_h(t)$ at the input of the Fig.4.20, the LSB output would be an USB signal corresponding to $m(t)$ (with a carrier phase shifted by $\pi/2$).

- **4.6-1** From Eq. (4.20)

$$H_o(\omega) = \frac{1}{H_i(\omega + \omega_c) + H_i(\omega - \omega_c)} \qquad |\omega| \leq 2\pi B$$

  Fig.S4.6-1a shows $H_i(\omega - \omega_c)$ and $H_i(\omega + \omega_c)$. Fig. S4.6-1b shows the reciprocal, which is $H_o(\omega)$.

- **4.8-1** A station can be heard at its allocated frequency 1500 kHz as well as at its image frequency. The two frequencies are $2f_{IF}$ Hz apart. In the present case, $f_{IF} = 455$ kHz. hence, the image frequency is $2 \times 455 = 910$ kHz apart. Therefore, the station will also be heard if the receiver is tuned to frequency 1500-910=590 kHz. The reason for this is as follows.When the receiver is tuned to 590 kHz, the local oscillator frequency is $f_{LO} = 599 + 455 = 1045$ kHz. Now this frequency f_{LO} is multiplied with the incoming signal of frequency $f_c = 1500$ kHz. The output yields the two modulated signals whose carrier frequencies are the sum and difference frequencies, which are 1500+1045=2545 kHz and 1500-1045=455 kHz. The sum carrier is suppressed, but the difference carrier passes through, and the station is received.

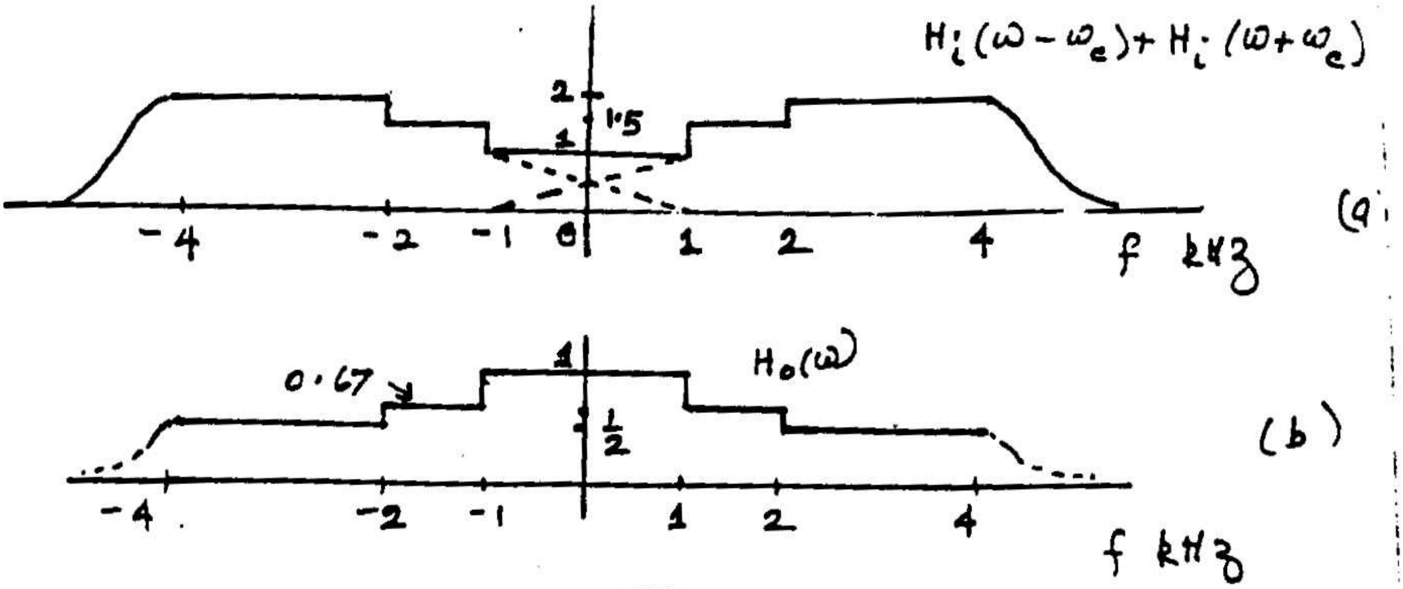


Fig. S4.8-1

- **4.8-2** The local oscillator generates frequencies in the range 1+8=9 Mhz. When the receiver setting is 10Mhz, $f_{LO} = 10+8 = 18$ Mhz. Now, if there is a station at $18+8 = 26$ Mhz, it will beat (mix) with $f_{LO} = 18$ Mhz to produce two signals centered at 26+18=44 Mhz and at 26-18=8 Mhz. The sum component is suppressed by the IF filter, but the difference component, which is centered at 8 Mhz, passes through the IF filter.

5. ASSIGNMENT 5

5.1 Assignment 5 Problems

5.1-1 Sketch $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$ for the modulating signal $m(t)$ shown in Fig. P5.1-1, given $\omega_c = 10^8$, $k_f = 10^5$, and $k_p = 25$.

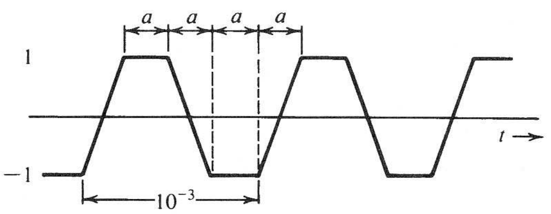


Figure P5.1-1

5.1-2 A baseband signal $m(t)$ is the periodic sawtooth signal shown in Fig. P5.1-2. Sketch $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$ for this signal $m(t)$ if $\omega_c = 2\pi \times 10^6$, $k_f = 2000\pi$, and $k_p = \pi/2$. Explain why it is necessary to use $k_p < \pi$ in this case.

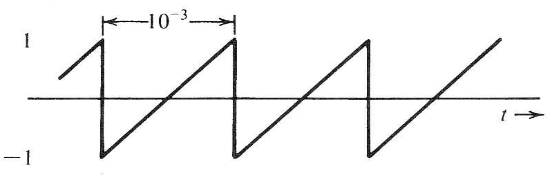


Figure P5.1-2

5.1-3 Over an interval $|t| \le 1$, an angle modulated signal is given by

$$\varphi_{EM}(t) = 10 \cos 13,000t$$

It is known that the carrier frequency $\omega_c = 10,000$.

(a) If this were a PM signal with $k_p = 1000$, determine $m(t)$ over the interval $|t| \le 1$.

(b) If this were an FM signal with $k_f = 1000$, determine $m(t)$ over the interval $|t| \le 1$.

5.2-1 For a modulating signal

$$m(t) = 2 \cos 100t + 18 \cos 2000\pi t$$

(a) Write expressions (do not sketch) for $\varphi_{PM}(t)$ and $\varphi_{FM}(t)$ when $A = 10$, $\omega_c = 10^6$, $k_f = 1000\pi$, and $k_p = 1$. For determining $\varphi_{FM}(t)$, use the indefinite integral of $m(t)$, that is, take the value of the integral at $t = -\infty$ to be 0.

(b) Estimate the bandwidths of $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$.

5.2-2 An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^6$ is described by the equation

$$\varphi_{\text{EM}}(t) = 10 \cos(\omega_c t + 0.1 \sin 2000\pi t)$$

(a) Find the power of the modulated signal.

(b) Find the frequency deviation Δf.

(c) Find the phase deviation $\Delta\phi$.

(d) Estimate the bandwidth of $\varphi_{\text{EM}}(t)$.

5.2-3 Repeat Prob. 5.2-2 if

$$\varphi_{\text{EM}}(t) = 5 \cos(\omega_c t + 20 \sin 1000\pi t + 10 \sin 2000\pi t)$$

5.2-4 Estimate the bandwidth for $\varphi_{\text{PM}}(t)$ and $\varphi_{\text{FM}}(t)$ in Prob. 5.1-1. Assume the bandwidth of $m(t)$ in Fig. P5.1-1 to be the third-harmonic frequency of $m(t)$.

5.2-5 Estimate the bandwidth of $\varphi_{\text{PM}}(t)$ and $\varphi_{\text{FM}}(t)$ in Prob. 5.1-2. Assume the bandwidth of $m(t)$ to be the fifth harmonic frequency of $m(t)$.

5.2-6 Given $m(t) = \sin 2000\pi t$, $k_f = 200,000\pi$, and $k_p = 10$.

(a) Estimate the bandwidths of $\varphi_{\text{FM}}(t)$ and $\varphi_{\text{PM}}(t)$.

(b) Repeat part **(a)** if the message signal amplitude is doubled.

(c) Repeat part **(a)** if the message signal frequency is doubled.

(d) Comment on the sensitivity of FM and PM bandwidths to the spectrum of $m(t)$.

5.2-7 Given $m(t) = e^{-t^2}$, $f_c = 10^4$ Hz, $k_f = 6000\pi$, and $k_p = 8000\pi$.

(a) Find Δf, the frequency deviation for FM and PM.

(b) Estimate the bandwidths of the FM and PM waves. *Hint*: Find $M(\omega)$ and observe the rapid decay of this spectrum. Its 3-dB bandwidth is even smaller than 1 Hz ($B \ll \Delta f$).

5.3-1 Design (only the block diagram) an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 98.1 MHz and $\Delta f = 75$ kHz. A narrow-band FM generator is available at a carrier frequency of 100 kHz and a frequency deviation $\Delta f = 10$ Hz. The stock room also has an oscillator with an adjustable frequency in the range of 10 to 11 MHz. There are also plenty of frequency doublers, triplers, and quintuplers.

5.3-2 Design (only the block diagram) an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 96 MHz and $\Delta f = 20$ kHz. A narrow-band FM generator with $f_c = 200$ kHz and adjustable Δf in the range of 9 to 10 Hz is available. The stock room also has an oscillator with adjustable frequency in the range of 9 to 10 MHz. There is a bandpass filter with any center frequency, and only frequency doublers are available.

5.4-1 Show that when $m(t)$ has no jump discontinuities, an FM demodulator followed by an integrator (Fig. P5.4-1a) acts as a PM demodulator, and a PM demodulator followed by a differentiator (Fig. P5.4-1b) serves as an FM demodulator even if $m(t)$ has jump discontinuities. *Hint*: For an input $A \cos[\omega_c t + \psi(t)]$, the output of an ideal FM demodulator is $\dot{\psi}(t)$ and that of an ideal PM demodulator is $\psi(t)$.

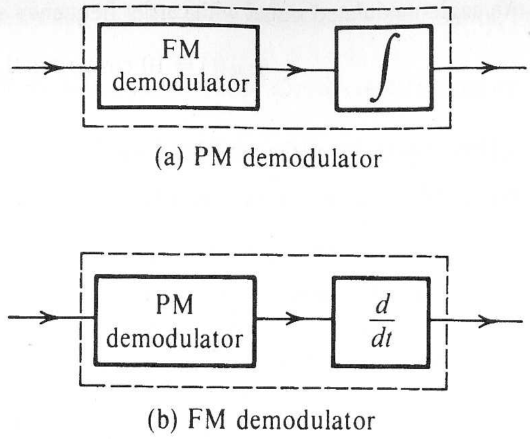


Figure P5.4-1

5.4-2 A periodic square wave $m(t)$ (Fig. P5.4-2a) frequency-modulates a carrier of frequency f_c = 10 kHz with Δf = 1 kHz. The carrier amplitude is A. The resulting FM signal is demodulated, as shown in Fig. P5.4-2b by the method discussed in Sec. 5.4 (Fig. 5.11). Sketch the waveforms at points b, c, d, and e.

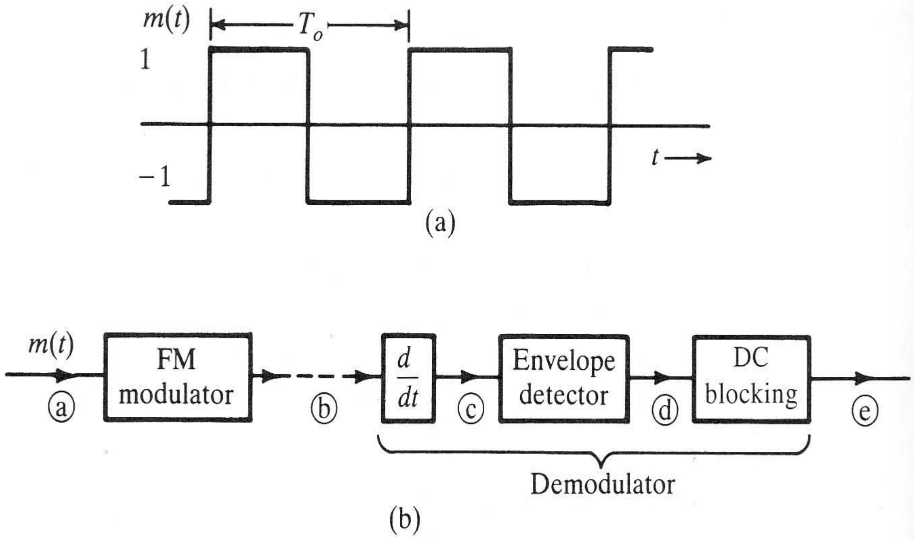


Figure P5.4-2

5.4-3 Using small-error analysis, show that a first-order loop [$H(s) = 1$] cannot track an incoming signal whose instantaneous frequency is varying linearly with time [$\theta_i(t) = kt^2$]. This signal can be tracked within a constant phase if $H(s) = (s + a)/s$. It can be tracked with a zero phase error if $H(s) = (s^2 + as + b)/s^2$.

5.2 Assignment 5 Solutions

- **5.1-1**: In this case, $f_c = 10$ MHz, $m_p = 1$ and $m'_p = 8000$.

  For FM:
  $\Delta f = k_f m_p/2\pi = 2\pi \times 10^5/2\pi = 10^5$ Hz. Also, $f_c = 10^7$. Hence, $(f_i)_{\max} = 10^7 + 10^5 = 10.1$ MHz, and $(f_i)_{\min} = 10^7 - 10^5 = 9.9$ MHz. The carrier frequency increases linearly from 9.9 MHz to 10.1 MHz over a quarter (rising) cycle of duration a seconds. For the next a seconds, when $m(t) = 1$, the carrier frequency remains at 10.1 MHz. Over the next quarter (the falling) cycle of duration a, the carrier frequency decreases linearly from 10.1 MHz to 9.9 MHz, and over the last quarter cycle, when $m(t) = -1$, the carrier frequency remains at 9.9 MHz. This cycles repeat periodically with the period $4a$ seconds as shown in Fig.1.

  For PM:
  $\Delta f = k_p m'_p/2\pi = 50\pi \times 8000/2\pi = 2 \times 10^5$ Hz. Also, $(f_i)_{\max} = 10^7 + 2 \times 10^5 = 10.2$ MHz, and $(f_i)_{\min} = 10^7 - 2 \times 10^5 = 9.8$ MHz. Fig. 1b shows $\dot{m}(t)$. We conclude that the frequency remains at 10.2 MHz over the (rising) quarter cycle, where $\dot{m}(t) = 8000$. For the next a second, $\dot{m}(t) = 0$, and the carrier frequency remains at 10 MHz. Over the next a seconds, where $\dot{m}(t) = -8000$, the carrier frequency remains at 9.8 MHz. Over the last quarter cycle, $\dot{m}(t) = 0$ again, and the carrier frequency remains at 10 MHz. This cycles repeat periodically with the period $4a$ seconds as shown in Fig.1.

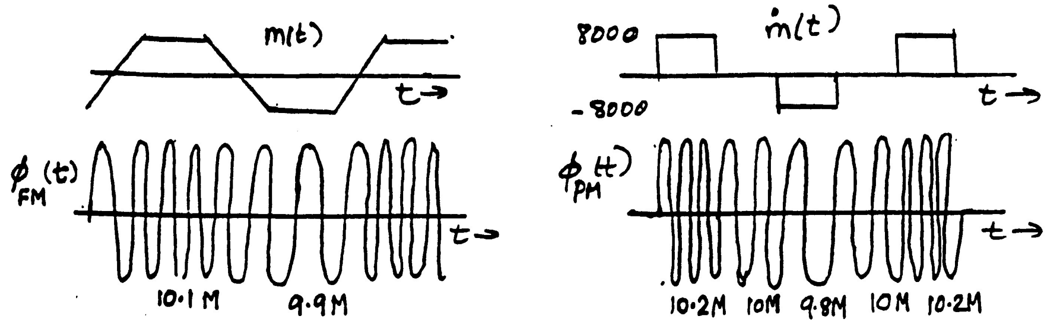


Fig. 5.1: Solution for 5.1-1.

- **5.1-2**: In this case, $f_c = 1$ MHz, $m_p = 1$ and $m'_p = 2000$. For FM:
  $\Delta f = k_f m_p/2\pi = 20{,}000\pi/2\pi = 10^4$ Hz. Also, $f_c = 1$ MHz. Hence, $(f_i)_{\max} = 10^6 + 10^4 = 1.01$ MHz, and $(f_i)_{\min} = 10^6 - 10^4 = 0.99$ MHz. The carrier frequency increases linearly from 0.99 MHz to 1.01 MHz over the cycle (over the interval $-10^{-3}/2 < t < 10^{-3}/2$). Then instantaneously, the carrier frequency falls to 0.99MHz and starts rising linearly to 10.01 MHz over the next cycle. This cycle repeats periodically with period 10^{-3} as shown in Fig.2

For PM:

Here, because $m(t)$ has jump discountinuities, we shall use a direct approach. For convenience, we select the origin for $m(t)$ as shown in Fig.2. Over the interval $-10^{-3}/2 < t < 10^{-3}/2$, we can express the message signal as $m(t) = 2000t$. Hence,

$$\phi_{PM}(t) = \cos\left[2\pi(10)^6\, t + \frac{\pi}{2}m(t)\right] = \cos[2\pi(10)^6\, t + 1000\pi t] = \cos[2\pi(10^5 + 500)\, t]$$

At the discontinuity, the amount of jump is $m_d = 2$. Hence, the phase discontinuity is $k_p m_d = \pi$. Therefore, the carrier frequency is constant throughout at $10^6 + 500$ Hz. But at the points of discontinuity, there is a phase discontinuity of π radians as shown in Fig.2. In this case, we must maintain $k_p < \pi$ because there is a discontinuity of the amount 2. For $k_p > \pi$, the phase discontinuity will be higher than 2π given rise to ambiguity in demodulation.

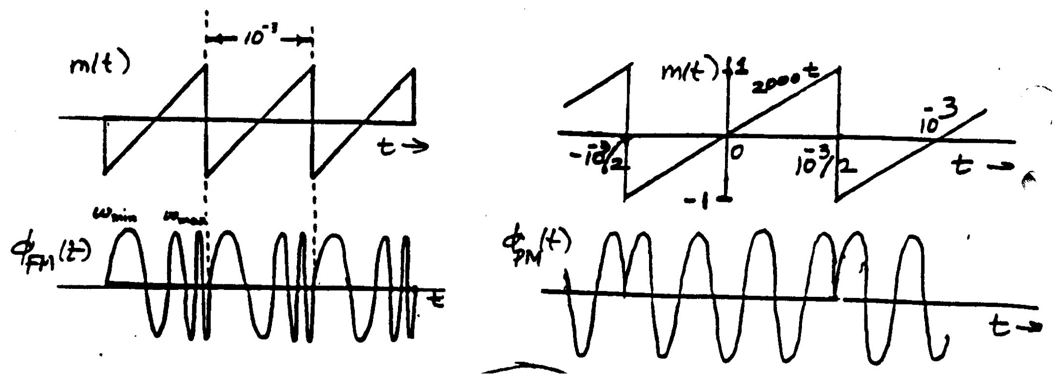


Fig. 5.2: Solution for 5.1-2.

- **5.1-3**: (a)

$$\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)] = 10\cos[10,000t + k_p m(t)]$$

We are given that $\phi_{PM}(t) = 10\cos(13,000t)$ with $k_p = 1000$. clearly, $m(t) = 3t$ over the interval $|t| \leq 1$.

(b)

$$\phi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_0^t m(\alpha)\, d\alpha\right] = 10\cos\left[10,000t + k_f \int_0^t m(\alpha)\, d\alpha\right]$$

Therefore,

$$k_f \int_0^t m(\alpha)\, d\alpha = 1000 \int_0^t m(\alpha)\, d\alpha = 3000t$$

Hence,

$$3t = \int_0^t m(\alpha)\, d\alpha \quad \rightarrow \quad m(t) = 3$$

- **5.2-1**: In this case, $k_f = 1000\pi$ and $k_p = 1$. For

$$m(t) = 2\cos 100t + 18\cos 2000\pi t$$

and

$$\dot{m}(t) = -200\sin 100t - 36,000\pi \sin 2000\pi t$$

Therefore, $m_p = 20$ and $m'_p = 36,000\pi + 200$. Also the baseband signal BW $B = 2000\pi/2\pi = 1kHz$.

For FM: $\Delta f = k_f m_p/2\pi = 10,000$ and $B_{FM} = 2(\Delta f + B) = 2(20,000 + 1000) = 42kHz$.

For PM: $\Delta f = k_f m'_p/2\pi = 18,000 + 100/\pi Hz$, and $B_{PM} = 2(\Delta f + B) = 2(18,031.83 + 1000) = 38.06366kHz$.

- **5.2-2**: $\phi_{EM}(t) = 10\cos(\omega_c t + 0.1\sin 2000\pi t)$. Here, the baseband signal bandwidth $B = 2000\pi/2\pi = 1000Hz$. Also,

$$\omega_i(t) = \omega_c + 200\pi \cos 2000\pi t$$

Therefore, $\Delta\omega = 200\pi$ and $\Delta f = 100Hz$ and $B_{EM} = 2(\Delta f + B) = 2(100 + 1000) = 2.2kHz$

- **5.2-3**: $\phi_{EM}(t) = 5\cos(\omega_c t + 20\sin 1000\pi t + 10\sin 2000\pi t)$. Here, the baseband signal bandwidth $B = 2000\pi/2\pi = 1000Hz$. Also,

$$\omega_i(t) = \omega_c + 20,000\pi \cos 1000\pi t + 20,000\pi \cos 2000\pi t$$

Therefore, $\Delta\omega = 20,000\pi + 20,000\pi = 40,000\pi$ and $\Delta f = 20kHz$ and $B_{EM} = 2(\Delta f + B) = 2(20,000 + 1000) = 42kHz$

- 5.2-4: The baseband signal bandwidth $B = 3 \times 1000 = 3000Hz$

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{10^5 \times 1}{2\pi} = 15.951kHz$ and $B_{FM} = 2(\Delta f + B) = 37.831kHz$.

For PM: $\Delta f = \frac{k_p m'_p}{2\pi} = \frac{25 \times 8000}{2\pi} = 31.831kHz$ and $B_{FM} = 2(\Delta f + B) = 66.662kHz$.

- 5.2-5: The baseband signal bandwidth $B = 5 \times 1000 = 5000Hz$

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{2000\pi \times 1}{2\pi} = 1kHz$ and $B_{FM} = 2(\Delta f + B) = 2(2 + 5) = 14kHz$.

For PM: To find B PM, we observe from Fig. 2 that $\phi_{FM}(t)$ is essentially a sequence of sinusoidal pulses of width $T = 10^{-3}$ seconds and of frequency $f_c = 1$ MHz. Such a pulse and its spectrum are depicted in Figs. 3.22c and d, respectively. The bandwidth of the pulse, as seen from Fig. 3.22d, is $4\pi/T$ rad/s or $2/T$ Hz. Hence, $B_{PM} = 2kHz$.

- 5.2-6: (a)

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{200,000\pi \times 1}{2\pi} = 100kHz$ and the baseband signal bandwidth $B = \frac{2000\pi}{2\pi} = 1kHz$. Therefore, $B_{FM} = 2(\Delta f + B) = 202kHz$.

For PM: $\Delta f = \frac{k_p m'_p}{2\pi} = \frac{10\times 2000\pi}{2\pi} = 10kHz$ and $B_{FM} = 2(\Delta f + B) = 2(10+1) = 22kHz$.

(b) $m(t) = 2\sin 2000\pi t$, and $B = 2000\pi/2\pi = 1kHz$. Also, $m_p = 2$ and $m'_p = 4000\pi$.

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{200{,}000\pi\times 2}{2\pi} = 200kHz$ and the baseband signal bandwidth $B = \frac{2000\pi}{2\pi} = 1kHz$. Therefore, $B_{FM} = 2(\Delta f + B) = 2(200+1) = 402kHz$.

For PM: $\Delta f = \frac{k_p m'_p}{2\pi} = \frac{10\times 4000\pi}{2\pi} = 20kHz$ and $B_{FM} = 2(\Delta f + B) = 2(20+1) = 42kHz$.

(c) $m(t) = \sin 4000\pi t$, and $B = 4000\pi/2\pi = 2kHz$. Also, $m_p = 1$ and $m'_p = 4000\pi$.

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{200{,}000\pi\times 1}{2\pi} = 100kHz$ and, $B_{FM} = 2(\Delta f + B) = 2(100+2) = 204kHz$.

For PM: $\Delta f = \frac{k_p m'_p}{2\pi} = \frac{10\times 4000\pi}{2\pi} = 20kHz$ and $B_{FM} = 2(\Delta f + B) = 2(20+2) = 44kHz$.

(d) Doubling the amplitude of $m(t)$ roughly doubles the bandwidth of both FM and PM. Doubling the frequency of $m(t)$ (expanding the spectrum $M(\omega)$ by a factor of 2) has hardly any effect on the FM bandwidth. However, it roughly doubles the bandwidth of PM, indicating that PM spectrum is sensitive to the shape of the baseband spectrum. FM spectrum is relatively insensitive to the nature of the spectrum $M(\omega)$.

- 5.2-7: From pair 22 (Table 3.1), we obtain

$$e^{-t^2} \Rightarrow \sqrt{\pi}e^{-\omega^2/4}$$

The spectrum $M(\omega) = \sqrt{\pi}e^{-\omega^2/4}$ is a Gaussian pulse, which decays rapidly. Its 3 dB bandwidth is 1.178 rad/s=0.187 Hz. This is an extremely small bandwidth compared to Δf.

Also, $\dot{m}(t) = -2te^{-t^2/2}$. The spectrum of $\dot{m}(t)$ is $M'(\omega) = j\omega M(\omega) = j\sqrt{\pi}\omega e^{-\omega^2/4}$. This spectrum also decays rapidly away from the origin, and its bandwidth can also be assumed o be negligible compared to Δf.

For FM: $\Delta f = \frac{k_f m_p}{2\pi} = \frac{6000\pi\times 1}{2\pi} = 3kHz$ and, $B_{FM} \approx 2(\Delta f) = 2\times 3 = 6kHz$.

For PM: To find m'_p, we set the derivative of $\dot{m}(t) = -2te^{-t^2/2}$ equal to zero. This yields

$$\ddot{m}(t) = -2e^{-t^2/2} + 4t^2e^{-t^2/2} = 0 \quad \Rightarrow \quad t = \frac{1}{\sqrt{2}}$$

and $m'_p = \dot{m}(1/\sqrt{2}) = 0.858$, and $\Delta f = \frac{k_p m'_p}{2\pi} = \frac{8000\pi\times 0.858}{2\pi} = 3.432kHz$ and $B_{PM} \approx 2(\Delta f) = 2(3.432) = 6.864kHz$.

- 5.3-1: The block diagram of the design is shown in Fig. 5.3

- 5.3-2: The block diagram of the design is shown in Fig. 5.4

- 5.4-1: (a)

$$\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$

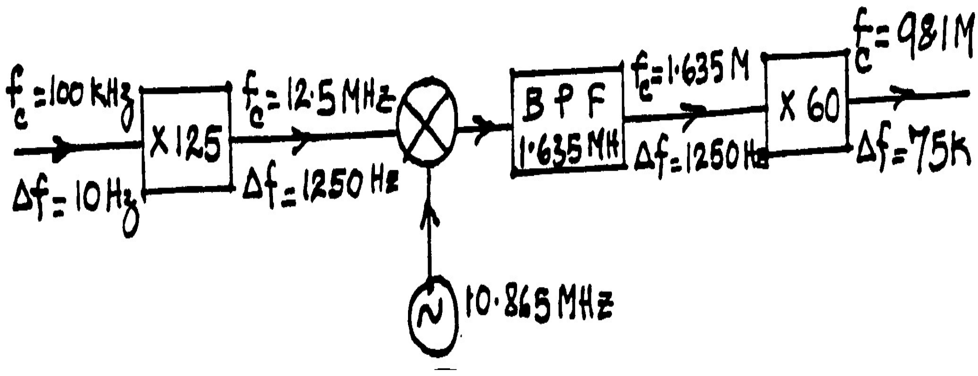


Fig. 5.3: Solution for 5.3-1.

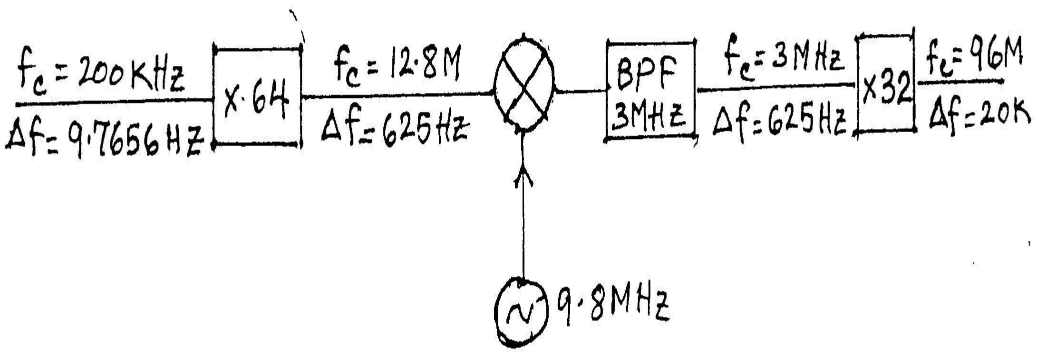


Fig. 5.4: Solution for 5.3-2.

When this $\phi_{PM}(t)$ is passed through an ideal FM demodulator. The output is $k_p\dot{m}(t)$. This signal, when passed through an ideal integrator, yields $k_p m(t)$. Hence, FM demodulator followed by an ideal integrator acts as a PM demodulator. However, if $m(t)$ has a discontinuity, $\dot{m}(t) = \infty$ at the point(s) of discontinuity, and the system will fail.

(b)

$$\phi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_0^t m(\alpha)\, d\alpha\right]$$

when this signal $\phi_{FM}(t)$ is passed through an ideal PM demodulator, the output is $k_f \int_0^t m(\alpha)\, d\alpha$. When this signal is passed through an ideal differentiator, the output is $k_f m(t)$. Hence, PM demodulator followed by an ideal differentiator, acts as FM demodulator regardless of whether $m(t)$ has jump discontinuities or not.

- 5.4-2: Fig. 5 shows the waveforms at points b, c, d and e. The figure is self explanatory.

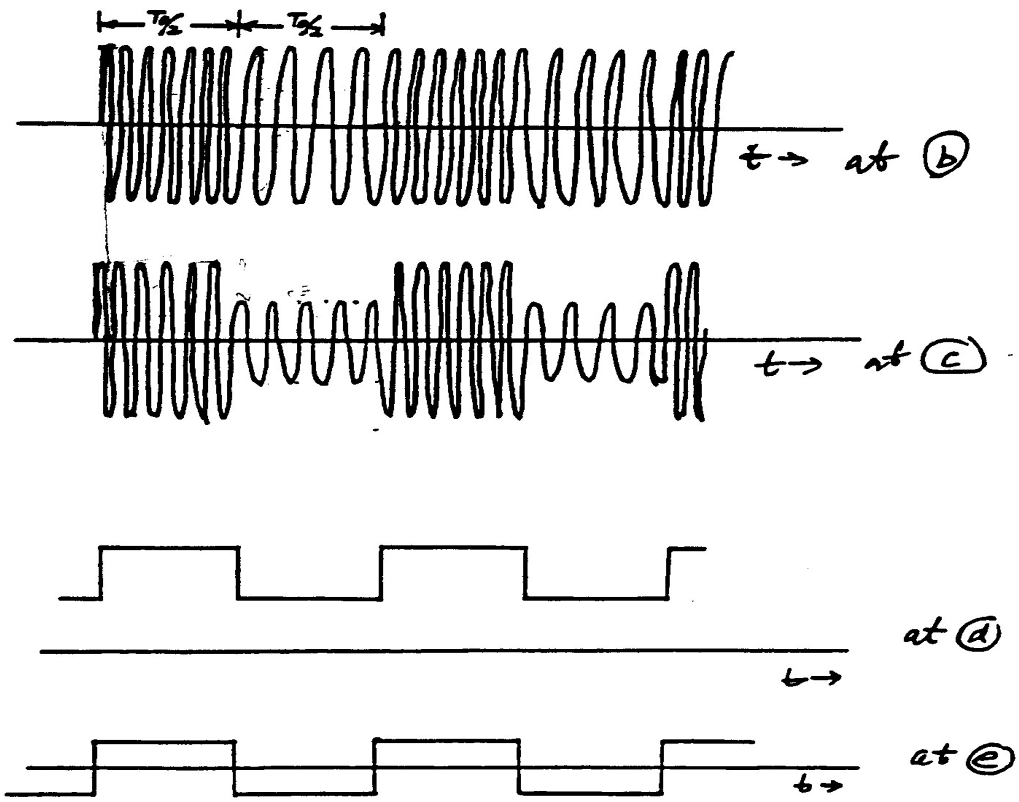


Fig. 5.5: Solution for 5.4-2.

6. ASSIGNMENT 6

6.1 Assignment 6 Problems

- **11.1-4**: Determine $\overline{X(t)}$ and $R_X(t_1, t_2)$ for the random process in Prob. 11.1-1, and determine whether this is a wide-sense stationary process.

- **11.1-8**: Repeat Prob. 11.1-7 for the random process

$$x(t) = a\cos(\omega_c t + \Theta)$$

where ω_c is a constant, a and Θ are independent RVs uniformly distributed in the ranges (-1,1) and $(0, 2\pi)$ respectively.

- **11.2-3**: Show that if the PSD of a random process $X(t)$ is band-limited to B Hz, and if

$$R_X\left(\frac{n}{2B}\right) = \begin{cases} 1 & n = 0 \\ 0 & n = \pm 1, \pm 2, \pm 3, \cdots \end{cases}$$

then $X(t)$ is a white bandlimited process; that is, $S_X(\omega) = k\text{rect}(\omega/4\pi B)$. Hint: Using the interpolation formula, reconstruct $R_X(\tau)$.

11.5-1 A white process of PSD $\mathcal{N}/2$ is transmitted through a bandpass filter $H(\omega)$ (Fig. P11.5-1). Represent the filter output $n(t)$ in terms of quadrature components, and determine $S_{n_c}(\omega)$, $S_{n_s}(\omega)$, $\overline{n_c^2}$, $\overline{n_s^2}$, and $\overline{n^2}$ when the center frequency used in this representation is 100 kHz (that is, $\omega_c = 200\pi \times 10^3$).

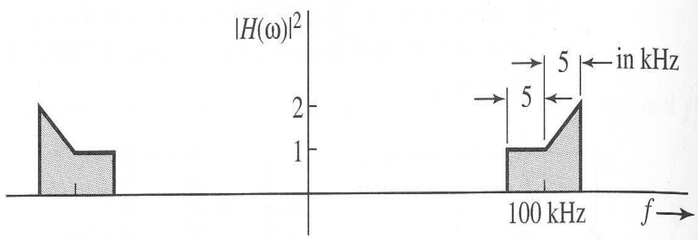


Figure P11.5-1

11.5-2 Repeat Prob. 11.5-1 if the center frequency ω_c used in the representation is not a true center frequency. Consider three cases: **(a)** $f_c = 105$ kHz; **(b)** $f_c = 95$ kHz; **(c)** $f_c = 90$ kHz.

11.5-3 A random process $x(t)$ with the PSD shown in Fig. P11.5-3a is passed through a bandpass filter (Fig. 11.5-3b). Determine the PSDs and mean square values of the quadrature components of the output process. Assume the center frequency in the representation to be 0.5 MHz.

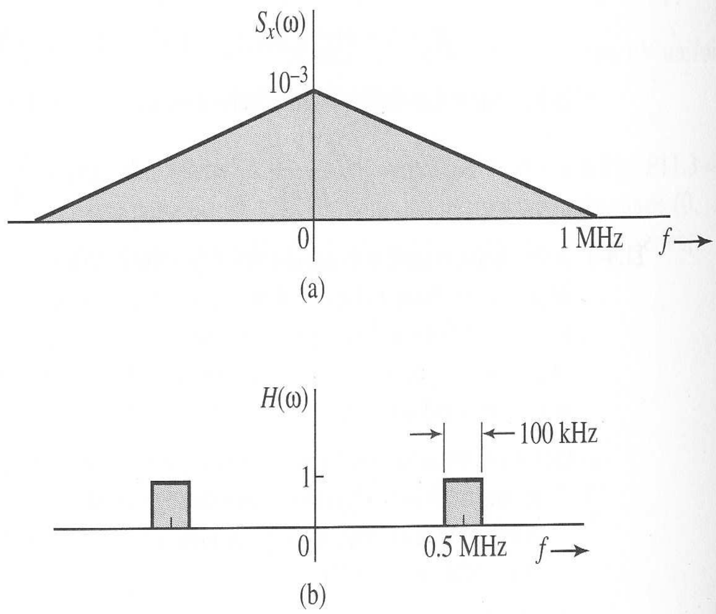


Figure P11.5-3

- **12.1-1**: A certain telephone channel has $H_c(\omega) \approx 10^{-3}$ over the signal band. The message signal PSD is $S_m(\omega) = \beta \text{rect}(\omega/2\alpha)$, with $\alpha = 8000\pi$. The channel noise PSD is $S_n(\omega) = 10^{-8}$. If the output SNR at the receiver is required to be at least 30 dB, what is the minimum transmitted power required? Calculate the value of β corresponding to this power.

- **12.2-1**: For a DSB-SC system with a channel noise PSD of $S_n(\omega) = 10^{-10}$ and a baseband signal of bandwidth 4 kHz, the receiver output SNR is required to be at least 30 dB. The receiver is as shown in Fig.12.3.

  a. What must be the signal power S_i received at the receiver input?

  b. What is the receiver output noise power N_0?

  c. What is the minimum transmitted power S_T if the channel transfer function is $H_c(\omega) = 10^{-4}$ over the transmission band?

- **12.2-2**: Repeat Prob. 12.2-1 for SSB-SC.

- **12.2-3**: Determine the output SNR of each of the two quadrature multiplexed channels and compare the results with those of DSB-SC and SSB-SC.

- **12.2-4**: Assume $[m(t)]_{\max} = -[m(t)]_{\min} = m_p$.

  a. Show that for AM

  $$m_p = \mu A$$

  b. Show that the output SNR for AM [Eq. (12.14)] can be expressed as

  $$\frac{S_o}{N_o} = \frac{\mu^2}{K^2 + \mu^2}\gamma$$

  where $k^2 = m_p^2/\overline{m^2}$.

  c. Using the result in part (2), show that for tone modulation with $\mu = 1$, $S_o/N_o = \gamma/3$.

  d. Show that if S_T and S_T' are the AM and DSB-SC transmitted powers, respectively, required to attain a given output SNR, then

  $$S_T \approx k^2 S_T' \quad \text{for} \quad \mu = 1 \quad \text{and} \quad k^2 >> 1$$

- **12.3-1**: For an FM communication system with $\beta = 2$ and white channel noise with PSD $S_n(\omega) = 10^{-10}$, the output SNR is found to be 28 dB. The baseband signal $m(t)$ is gaussian and band-limited to 15 kHz, and 3σ loading is used. The demodulator constant $\alpha = 10^{-4}$. This means that the FM demodulator output is $\alpha\dot{\psi}$ when the input is $A\cos(\omega_c t + \psi(t))$. In the present case, the signal at the demodulator output is $\alpha k_f m(t)$. The output noise is also multiplied by α.

a. Determine the received signal power S_i.

b. Determine the output signal power S_o.

c. Determine the output noise power N_o.

6.2 Assignment 6 Solutions

- **11.1-4**:

  Since $x(t) = a\cos(\omega t + \theta)$

$$\begin{aligned} E[x(t)] &= E[a\cos(\omega t+\theta)] = E[a]\cos(\omega t+\theta) = \cos(\omega t+\theta)\int_{-A}^{A} a p_a(a)\,da \\ &= [\cos(\omega t+\theta)/(2A)]\int_{-A}^{A} a\,da = 0 \end{aligned}$$

$$\begin{aligned} R_X(t_1,t_2) &= E[a^2\cos(\omega t_1+\theta)\cos(\omega t_2+\theta)] = \cos(\omega t_1+\theta)\cos(\omega t_2+\theta)E[a^2] \\ &= \cos(\omega t_1+\theta)\cos(\omega t_2+\theta)\int_{-A}^{A}\frac{a^2}{2A}\,da = \frac{A^3}{3}\cos(\omega t_1+\theta)\cos(\omega t_2+\theta) \quad (6.1) \end{aligned}$$

- **11.1-8**:

  $x(t) = a\cos(\omega_c t+\theta), E[a] = 0$ and $E[a^2] = \frac{1}{3}$

  (b)

$$E[x(t)] = E[a\cos(\omega_c t+\theta)] = E[a]\cdot E[\cos(\omega_c t+\theta)] = 0$$

  (c)

$$\begin{aligned} R_X(t_1,t_2) &= E[a^2]\cdot E[\cos(\omega_c t_1+\theta)\cos(\omega_c t_2+\theta)] = \frac{1}{3}E[\cos\omega_c(t_1-t_2)+\cos\omega_c(t_1+t_2)+2\theta] \\ &= \frac{1}{3}\cos\omega_c(t_1-t_2) + \frac{1}{2\pi}\int_0^{2\pi}\cos[\omega_c(t_1-t_2)+2\theta]\,d\theta = \frac{1}{3}\cos\omega_c(t_1-t_2) \end{aligned}$$

  (d) The process is W.S.S

- **11.2-3**: $R_X(\tau) = 0$ for $\tau = \pm\frac{n}{2B}$ and its Fourier transform $S_X(\omega)$ is bandlimited to B Hz. Hence, $R_X(\tau) = 0$ is a waveform bandlimited to B Hz and according to Eq.6.10b

$$R_X(\tau) = \sum_{n=-\infty}^{\infty} R_X\left(\frac{n}{2B}\right)\text{sinc}\,(2\pi B\tau - n).$$

  Since $R_X\left(\frac{n}{2B} = 0\right.$ for all n except $n = 0$, therefore,

$$R_X(\tau) = R_X(0)\text{sinc}(2\pi B\tau)$$

  and

$$S_X(\omega) = \frac{R_X(0)}{2B}\,\text{rect}\left(\frac{\omega}{4\pi B}\right)$$

  Hence, $x(t)$ is a white process bandlimited to B Hz.

11.5-1 $n(t) = n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$

The PSD of $n_c(t)$ and $n_s(t)$ are identical. They are shown in Fig. S11.5-1. Also, $\overline{n^2}$ is the area under $S_n(\omega)$, and is given by $\overline{n^2} = 2\left[\frac{\mathcal{N}}{2}\times 10^4 + \frac{10^4}{2}\left(\frac{\mathcal{N}}{2}\cdot\frac{1}{2}\right)\right] = 1.25\times 10^4\mathcal{N}$

$\overline{n_c^2}\left(\text{or } \overline{n_s^2}\right)$ is the area under $S_{n_c}(\omega)$, and is given by $\overline{n_c^2} = \overline{n_s^2} = 2\left[5000\mathcal{N} + \frac{\mathcal{N}}{2}\cdot\frac{1}{2}\times 5000\right] = 1.25\times 10^4\mathcal{N}$

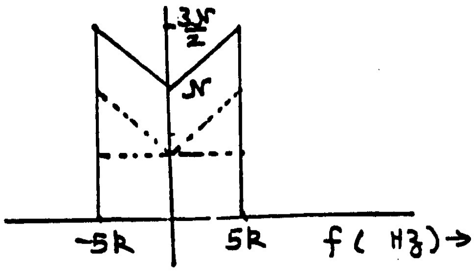


Fig. S11.5-1

11.5-2 We follow a procedure similar to that of the solution of Prob. 11.5-1 except that the center frequencies are different. For the 3 center frequencies $S_{n_c}(\omega)\left[\text{or } S_{n_s}(\omega)\right]$ are shown in Fig. S11.5-2. In all the three cases, the area under $S_{n_c}(\omega)$ is the same, viz., $1.25\times 10^4\mathcal{N}$. Thus in all 3 cases

$\overline{n_c^2} = \overline{n_s^2} = 1.25\times 10^4\mathcal{N}$

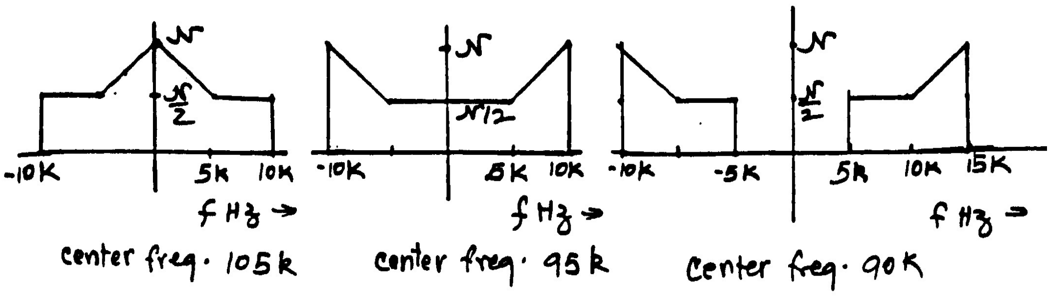


Fig. S11.5-2

11.5-3 $\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)} = 2\left[\frac{1}{2}\times 10^{-3}\times 100\times 10^3\right] = 100$

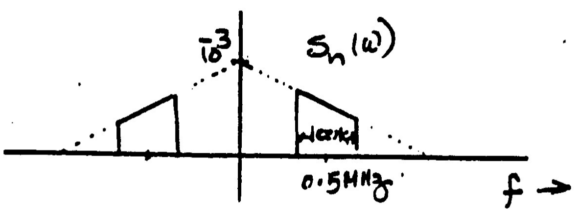


Fig. S11.5-3

- **12.2-1**: since $N_0/2 = 10^{-10}$, we have $N_0 = 2 \times 10^{-10}$.

  (a)
  $$30dB = 1000 = \frac{S_0}{\sigma_o^2} = \gamma = \frac{S_i}{N_0 B} = \frac{S_i}{2 \times 10^{-10} \times 4000}$$
  therefore, $S_i = 8 \times 10^{-4}$.

  (b) From Eq.(12.7), the receiver output noise power σ_o^2 is given as
  $$\sigma_o^2 = N_0 B = 2 \times 10^{-10} \times 4000 = 8 \times 10^{-7}$$

  (c) Since $S_i = |H_c(\omega)|^2 S_T = 10^{-8} S_T$, therefore, $S_T = S_i/10^{-8} = 8 \times 10^4$.

12.1-1 $\frac{S_o}{N_o} = \gamma = \frac{S_i}{\mathcal{N}B}$, $\mathcal{N} = 2 \times S_n(\omega) = 2 \times 10^{-8}$, $B = \frac{\alpha}{2\pi} = 4000$ Hz.

$$\gamma = 1000 = \frac{S_i}{2 \times 10^{-8} \times 4000} \Rightarrow S_i = 0.08$$

Also, $H_c(\omega) = 10^{-3}$. Hence, $S_T = \frac{S_i}{|H_c(\omega)|^2} = 8 \times 10^4$

$$S_T = \frac{1}{2\pi}\beta[2 \times 8000\pi] = 8 \times 10^4 \Rightarrow \beta = 10$$

12.2-2 (a) $\frac{S_o}{N_o} = 1000 = \frac{S_i}{\mathcal{N}B} = \frac{S_i}{10^{-10} \times 4000} \Rightarrow S_i = 4 \times 10^{-4}$

(b) $N_o = \mathcal{N}B = 10^{-10} \times 8000 = 4 \times 10^{-7}$

(c) $S_i = |H_c(\omega)|^2 S_T = 10^{-8} S_T = 4 \times 10^{-4} \Rightarrow S_T = 4 \times 10^4$

12.2-3 Let the signals $m_1(t)$ and $m_2(t)$ be transmitted over the same band by carriers of the same frequency (ω_c), but in phase quadrature. The two transmitted signals are $\sqrt{2}[m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t]$

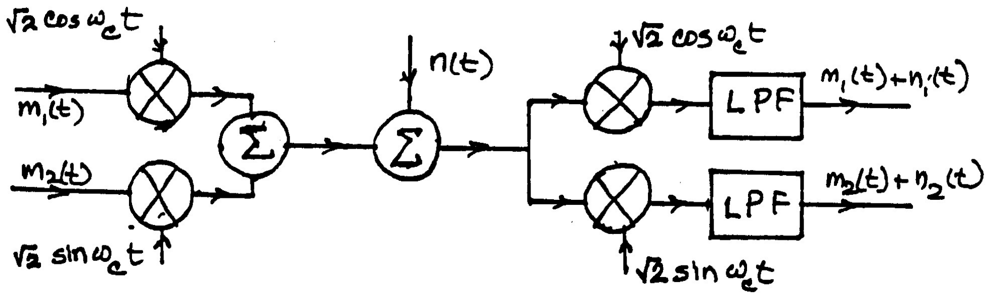


Fig. S12.2-3

The bandpass noise over the channel is $n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$. Hence, the received signal is

$$\left[\sqrt{2}m_1(t) + n_c(t)\right]\cos\omega_c t + \left[\sqrt{2}m_2(t) + n_s(t)\right]\sin\omega_c t$$

Eliminating the high frequency terms, we get the output of the upper lowpass filter as $m_1(t) + \frac{1}{\sqrt{2}}n_c(t)$

Similarly, the output of the lower demodulator is $m_2(t) + \frac{1}{\sqrt{2}}n_s(t)$

These are similar to the outputs obtained for DSB-SC on page 535. Hence, we have $\frac{S_o}{N_o} = \gamma$ for both QAM channels.

12.2-4 (a) $\mu = \frac{-[m(t)]\min}{A} = \frac{m_p}{A}$ Hence, $m_p = \mu A$

(b) $$\frac{S_o}{N_o} = \frac{\overline{m^2}}{A^2 + \overline{m^2}}\cdot\gamma = \frac{\overline{m^2}}{\frac{m_p^2}{\mu^2} + \overline{m^2}}\cdot\gamma = \frac{\mu^2}{\kappa^2 + \mu^2}\gamma \qquad \text{where } \kappa^2 = \frac{m_p^2}{\overline{m^2}}$$

(c) For tone modulation $\kappa^2 = \frac{m_p^2}{m_p^2/2} = 2$ and for $\mu = 1$, $\frac{S_o}{N_o} = \frac{1}{2+1}\gamma = \frac{\gamma}{3}$

(d) Ratio $\frac{S_T}{S_T'} = \frac{A^2 + \overline{m^2}}{\overline{m^2}} = \frac{m_p^2 + \overline{m^2}}{\overline{m^2}} = \frac{m_p^2}{\overline{m^2}} + 1 \approx \kappa$ if $\kappa^2 >> 1$

12.3-1 $\frac{S_o}{N_o} = 28\text{dB} = 631.$ Hence,

$$\frac{S_o}{N_o} = 631 = 3\beta^2\gamma\frac{\overline{m^2(t)}}{m_p^2}$$
$$= 3(2)^2\gamma\frac{\sigma_m^2}{(3\sigma_m)^2}$$

Therefore, $\gamma = \frac{631\times 9}{12} = 473.25$

(a) Also, $\gamma = \frac{S_i}{\mathcal{N}B} \Rightarrow S_i = \gamma\mathcal{N}B = 473.25\times 2\times 10^{-10}\times 15000 = 1.4197\times 10^{-3}$

(b) $\beta = \frac{\Delta\omega}{2\pi B} = \frac{k_f m_p}{2\pi B} \Rightarrow 2 = \frac{k_f(3\sigma_m)}{30{,}000\pi} \Rightarrow k_f\sigma_m = 20{,}000\pi$

$$S_o = \alpha^2 k_f^2\overline{m^2(t)} = \alpha^2 k_f^2\sigma_m^2 = (10^{-4})^2(20{,}000\pi)^2 = 4\pi^2$$

(c) $N_o = \frac{S_o}{631} = 0.0199$