Chapter 1

Optical Signal Fundamentals

1.1. BASIC THEORIES

There are three theories that are widely used to describe the behavior of optical signals. Each of them better explain certain phenomena.

1.1.1. Quantum Theory

Optical Signal is consists of discrete units called photons. The energy in a photon $E_g = h\nu$, where h is the planck's constant 6.6256 $\times 10^{-34}$ J.s na ν is the frequency.

Ex: Find the energy of a photon travelling with 200 THz frequency.

1.1.2. Electromagnetic Theory

Optical signal is an electromagnetic signal. It has electric and magnetic fields that are orthogonal to each other. Typically, the frequency of this EM wave is extremely high (in the order of THz). Therefore, it is more convenient to measure it in terms of wavelength. The relationship is given by,

$$c = \nu\lambda$$

where, c - speed of light, ν - frequency and λ - wavelength

Ex: Find the ν is λ = 1550 nm. Ans: 193.5 THz

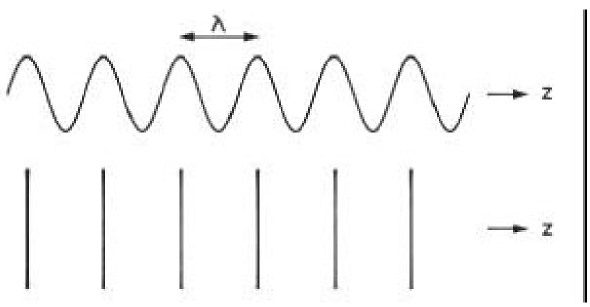

Figure 1.1. Travelling Wavefront and the Wavelength λ

50 nm - 400 nm	Ultra Violet
400 nm - 700 nm	Visible Spectrum
800 nm - 1600 nm	Near Infrared
1700 - 100,000 nm	Far Infrared

Frequency Bands and Their Names

Near infrared band is used in optical communications, especially the window at 1550 nm is used because the attenuation in Silica (fiber) is the lowest at this wavelength.

1.1.3. Ray Theory

According to ray theory, light travels in a straight line abiding to the laws of geometrical optics. This gives us an easy tool to analyze the behavior of optical signal *when the physical dimension of the associated objects is much larger than the wavelength of the optical signal*. For example with prisms and lenses. We will use ray theory to get some results quickly[1].

1.2. WAVE THEORY AND POLARIZATION

General electromagnetic wave,

$$\overline{\mathbf{E}} = E_x \cos(\omega t - kz + \phi_x)\hat{\mathbf{i}} + E_y \cos(\omega t - kz + \phi_y)\hat{\mathbf{j}}$$

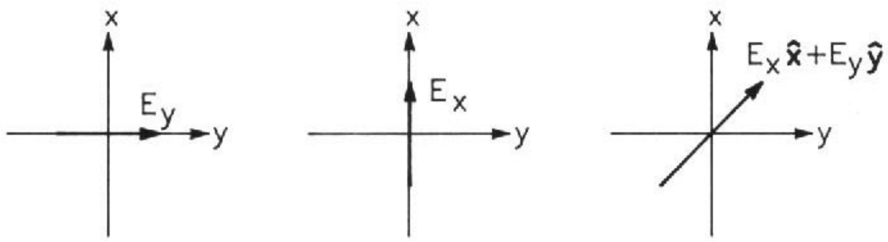

Figure 1.2. Horizontally polarized Wave (left), Vertically polarized wave (middle) and the general polarization as the vector addition of these two

1.2.1. Elliptically Polarized Light

The vector addition of the two components with a phase shift $\phi = |\phi_x - \phi_y| \neq 0$ will be in general elliptically polarized. see Fig. 1.3.

- For a fixed point z, the tip of the E-field vector rotates periodically in the xy plane tracing out an ellipse
- At a fixed time t, the locus of the tip of the E-field vector follows a helical trajectory in space having periodicity λ

[1]However, accuracy of this approach deteriorates in case of fibers, especially single mode fibers. Because, in this case, the fiber core radius (typically 9 μm) is comparable to the optical wavelength.

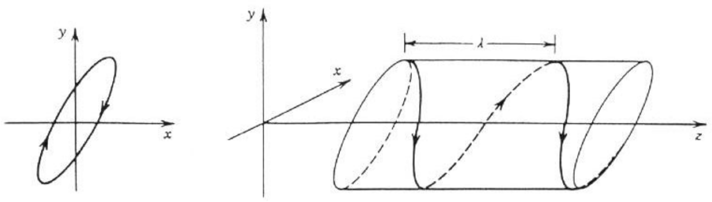

Figure 1.3. Elliptically polarized wave

1.2.2. Linearly Polarized Light

In the following cases light will be linearly polarized:

- One of the components E_x or $E_y = 0$
- If $\phi = 0$ or π.

then the angle is determined by the magnitudes of E_y and E_x. See Fig. 1.4.

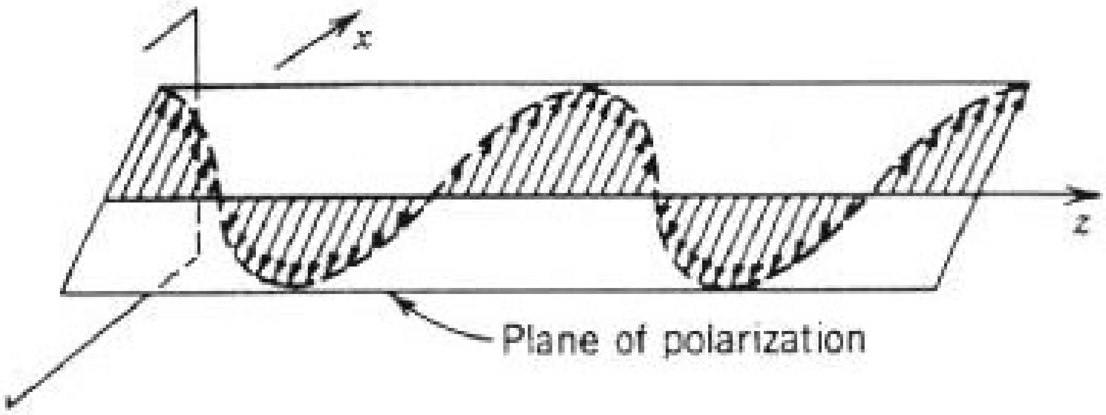

Figure 1.4. Linearly Polarized Wave

1.2.3. Circularly polarized light

- if $\phi = \pi/2$ and $E_x = E_y = E$ then, $E^2 = E_x^2 + E_y^2$.

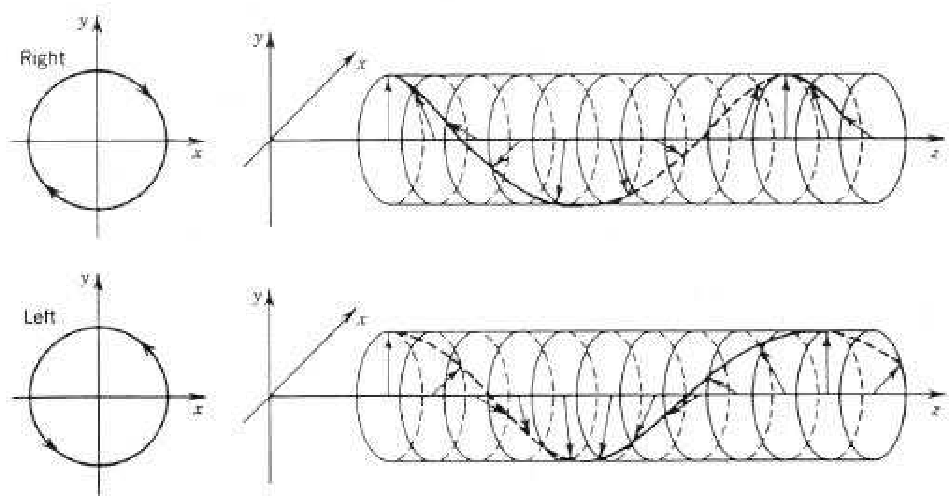

Figure 1.5. Right hand circular polarization (RHCP, $\phi = \pi/2$, up) and left hand circular polarization (LHCP, $\phi = -\pi/2$, down)

Chapter 2

Optical Fiber

2.1. FIBER OPTIC TRANSMISSION SYSTEMS

A fiber optic transmitter and receiver, connected by fiber optic cable - offer a wide range of benefits not offered by traditional copper wire or coaxial cable. These include:

- The ability to carry much more information and deliver it with greater fidelity than either copper wire or coaxial cable. Fiber optic cable can support much higher data rates, and at greater distances, than coaxial cable, making it ideal for transmission of serial digital data.
- The fiber is totally immune to virtually all kinds of interference, including lightning, and will not conduct electricity. It can therefore come in direct contact with high voltage electrical equipment and power lines. It will also not create ground loops of any kind.
- As the basic fiber is made of glass, it will not corrode and is unaffected by most chemicals. It can be buried directly in most kinds of soil or exposed to most corrosive atmospheres in chemical plants without significant concern.
- Since the only carrier in the fiber is light, there is no possibility of a spark from a broken fiber. Even in the most explosive of atmospheres, there is no fire hazard, and no danger of electrical shock to personnel repairing broken fibers.
- Fiber optic cables are virtually unaffected by outdoor atmospheric conditions, allowing them to be lashed directly to telephone poles or existing electrical cables without concern for extraneous signal pickup.
- A fiber optic cable, even one that contains many fibers, is usually much smaller and lighter in weight than a wire or coaxial cable with similar information carrying capacity. It is easier to handle and install, and uses less duct space. (It can frequently be installed without ducts.)
- Fiber optic cable is ideal for secure communications systems because it is very difficult to tap but very easy to monitor. In addition, there is absolutely no electrical radiation from a fiber. How are fiber optic cables able to provide all of these advantages?

2.2. OPTICAL FIBER

Fiber optic cable functions as a ”light guide,” guiding the light introduced at one end of the cable through to the other end. The light source can either be a light-emitting diode (LED)) or a laser.

The light source is pulsed on and off, and a light-sensitive receiver on the other end of the cable converts the pulses back into the digital ones and zeros of the original signal.

Even laser light shining through a fiber optic cable is subject to loss of strength, primarily through dispersion and scattering of the light, within the cable itself. The faster the laser fluctuates, the greater the risk of dispersion. Light strengtheners, called repeaters, may be necessary to refresh the signal in certain applications.

While fiber optic cable itself has become cheaper over time - a equivalent length of copper cable cost less per foot but not in capacity. Fiber optic cable connectors and the equipment needed to install them are still more expensive than their copper counterparts.

2.3. SINGLE MODE FIBER

Single Mode cable is a single stand of glass fiber with a diameter of 8.3 to 10 microns that has one mode of transmission. Single Mode Fiber with a relatively narrow diameter, through which only one mode will propagate. Carries higher bandwidth than multimode fiber, but requires a light source with a narrow spectral width. Synonyms monomode optical fiber, single-mode fiber, single-mode optical waveguide, unimode fiber.

Single-mode fiber gives you a higher transmission rate and up to 50 times more distance than multimode, but it also costs more. Single-mode fiber has a much smaller core than multimode. The small core and single lightwave virtually eliminate any distortion that could result from overlapping light pulses, providing the least signal attenuation and the highest transmission speeds of any fiber cable type.

Single-mode optical fiber is an optical fiber in which only the lowest order bound mode can propagate at the wavelength of interest typically 1300 to 1320nm.

2.4. MULTIMODE FIBER

Multimode cable is made of of glass fibers, with a common diameters in the 50-to-100 micron range for the light carry component (the most common size is 62.5). POF is a newer plastic-based cable which promises performance similar to glass cable on very short runs, but at a lower cost.

Multimode fiber gives you high bandwidth at high speeds over medium distances. Light waves are dispersed into numerous paths, or modes, as they travel through the cable's core typically 850 or 1300nm. Typical multimode fiber core diameters are 50, 62.5, and 100 micrometers. However, in long cable runs (greater than 3000 feet [914.4 ml), multiple paths of light can cause signal distortion at the receiving end, resulting in an unclear and incomplete data transmission.

2.5. FIBER PARAMETERS

	Step Index Fiber	**Graded Index Fiber**
Refractive Index Profile	$n_1\,; r \le a$ $n_2\,; r > a$	$n_1\sqrt{1-2\Delta(r/a)^\alpha}\,; r \le a$ $n_2\,; r > a$
Numerical Aperture	$\sqrt{n_1^2 - n_2^2}$	$\sqrt{n(r)^2 - n_2^2}\,; r \le a$
Normalized Frequency (V)	$\frac{2\pi a}{\lambda}(NA)$	$\frac{2\pi a}{\lambda}(NA)$
Cut-off Value of the normalized frequency	2.405	$2.405\sqrt{1+2/\alpha}$
Number of Modes (M)	$V^2/2$	$\frac{V^2\alpha}{2(\alpha+2)}$
Modal Dispersion $\Delta T_{mod}/L$	$\frac{n_1^2\Delta}{cn_2}$	$\frac{n_1\Delta^2}{8c}$ (when $\alpha = 2(1-\Delta)$)

n_1	Core refractive index
n_2	Cladding refractive index ($n_1 > n_2$)
a	Core radius
r	Varying radius
α	Profile parameter
NA	Numerical Aperture
L	Total length of the optical fiber (typically in km)
Δ	$\frac{n_1^2-n_2^2}{2n_1^2} \approx 1 - \frac{n_2}{n_1}$

2.5.1. Note on Number of Modes

Only for large number of modes ($V >> 2.4$), the number of modes can be given by $V^2/2$. Under this condition the ratio between power travelling in the cladding and in the core is given by,

$$\frac{P_{cladding}}{P_{total}} \approx \frac{4}{3\sqrt{M}}$$

If V is close to 2.4, then exact solution to modal equations should be used to find the number of modes (Fig. 2.19 and Fig. 2.22 in the textbook)

2.6. DISPERSION IN FIBER

Temporal dispersion is the major effect that limits the bit rate in fiber optic communication system. For no ISI, the bit rate for a non return to zero (NZR) system is related to the total dispersion by,

$$B_{NRZ} \le \frac{0.7}{\Delta T}$$

For a return to zero (RZ) system, the bit rate is,

$$B_{RZ} \le \frac{0.35}{\Delta T}$$

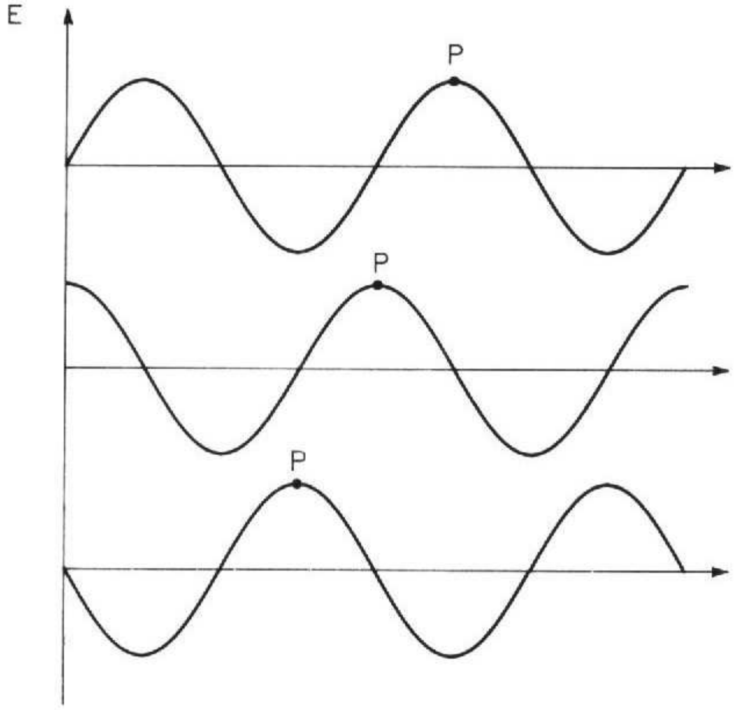

Figure 2.1. Constant Phase Point P that Travels along βz axis in Space; (1) t = 0; (2) t = T/4; (3) t = T/2

There are several dispersion mechanisms exist in optical fibers. Depending on the propagation conditions, some of these are dominant. Not all dispersions are significant under all conditions.

2.7. GROUP AND PHASE VELOCITIES

Electric filed of an electromagnetic plane wave propagating in z direction is give by,

$$\mathbf{E} = E_o cos(\omega t - \beta z)$$

where, $\beta = 2\pi/\lambda \; m^{-1}$ is the wave number and $\omega = 2\pi\nu = 2\pi c/\lambda$ rad/m is the angular frequency. E_o is the peak amplitude.

2.7.1. Phase Velocity

The phase velocity is defined only when there is a single electromagnetic wave. Corresponding to Fig. 2.1, P is a point of constant phase or the wavefront. At point P, $(\omega t - \beta z)$ = constant at any time. Therefore, the phase velocity

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$$

Note, $v_p = c = 3 \times 10^8$ m/s is the speed of light in free space.

2.7.2. Group Velocity

This is defined by the slope of the electromagnetic wave group. When there are number of electromagnetic waves with slightly differing frequencies travel together, the group velocity is important. This is the velocity at which the actual energy travels with a *group* of electromagnetic waves. Group velocity v_g is defined by,

$$v_g = \frac{dz}{dt} = \frac{d\omega}{d\beta}$$

In practice, no optical source emits a single frequency. The optical signal emitted always have a group of frequencies and occupies a finite spectrum. The bandwidth of this output spectrum is called the *line width* of the optical source. Therefore, the group velocity is the more realistic velocity in optical communications.

2.7.3. Group Velocity Index n_g

The refractive index n of the medium is actually a function of wavelength; $n = n(\lambda) = n(\omega)$. Since $\omega = 2\pi\nu = 2\pi c/\lambda$, Therefore, the actual propagation constant β is

$$\beta(\omega) = \frac{n(\omega)\omega}{c}$$

$$\frac{d\beta}{d\omega} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}$$

$$v_g = \frac{d\omega}{d\beta} = \frac{1}{d\beta/d\omega} = \frac{c}{(n + \omega dn/d\omega)}$$

Define the group refractive index n_g as,

$$n_g = n + \omega\frac{dn}{d\omega}$$

where the mode index n at the operating wavelength is given by [AGRAWAL]

$$n = n_2 + b(n_1 - n_2)$$

where b is the normalized propagation constant. Therefore, the group velocity is

$$v_g = \frac{c}{n_g}$$

The group refractive index defines the velocity in the medium n under realistic conditions.

2.7.4. Group Velocity Dispersion

Group Velocity dispersion = Material Dispersion + Waveguide Dispersion

Group Velocity Dispersion is also known as **Chromatic Dispersion** or **Intra Modal Dispersion**.
Define group delay as the inverse of the group velocity,

$$\tau_g = \frac{1}{v_g} = \frac{d\beta}{d\omega}$$

The average delay of the signal to travel through the distance L is $T = L/v_g = L\tau_g$

$$\Delta T = \frac{dT}{d\omega}\Delta\omega = L\frac{d\tau_g}{d\omega}\Delta\omega = L\frac{d^2\beta}{d\omega^2}(\Delta\omega) = L\beta_2(\Delta\omega) \quad (2.1)$$

Define the group velocity dispersion parameter β_2,

$$\beta_2 = \frac{d^2\beta}{d\omega^2}$$

Now, $\omega = \frac{2\pi c}{\lambda}$. Therefore,

$$\frac{d\omega}{d\lambda} = \frac{-2\pi c}{\lambda^2}$$

$$\Delta\omega = \frac{-2\pi c}{\lambda^2}\Delta\lambda \tag{2.2}$$

Group velocity dispersion,

$$D_{GVD} = \frac{d\tau_g}{d\lambda} = \frac{d}{d\lambda}\left(\frac{d\beta}{d\omega}\right) = \frac{d}{d\omega}\left(\frac{d\beta}{d\omega}\right)\frac{d\omega}{d\lambda} = \frac{-2\pi c}{\lambda^2}\frac{d^2\beta}{d\beta^2} = \frac{-2\pi c}{\lambda^2}\beta_2 \tag{2.3}$$

Comparing equations (2.2) and (2.3), $D_{GVD} = \frac{\Delta\lambda}{\Delta\omega}\beta_2$
By substitution of this in (2.1),

$$\Delta T_{GVD} = \beta_2(\Delta\omega)L = D_{GVD}(\Delta\lambda)L \tag{2.4}$$

Where, L is the distance in km, $(\Delta\lambda)$ is the linewidth of the optical source in nm and, D_{GVD} is the group velocity dispersion in ps/nm/km.

Group velocity dispersion D_{GVD} can be re-written as,

$$D_{GVD} = \frac{-2\pi c}{\lambda^2}\beta_2 = \frac{-2\pi}{\lambda^2}\frac{d}{d\omega}\frac{c}{v_g} = \frac{-2\pi}{\lambda^2}\frac{dn_g}{d\omega} = \frac{-2\pi}{\lambda^2}\frac{d\left(n + \omega\frac{dn}{d\omega}\right)}{d\omega} = \frac{-2\pi}{\lambda^2}\frac{dn}{d\omega} - \frac{2\pi}{\lambda^2}\omega\frac{d^2n}{d\omega^2}$$

Therefore,

$$D_{GVD} = D_{mat} + D_{WG}$$

where the material dispersion D_{mat} is,

$$D_{mat} = -\frac{2\pi}{\lambda^2}\frac{dn_{2g}}{d\omega} = \frac{1}{c}\frac{dn_{2g}}{d\lambda}$$

Waveguide dispersion D_{WG} is,

$$D_{WG} = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}\frac{Vd^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb)}{dV}\right]$$

2.7.5. Waveguide dispersion:

In single mode fiber, about 20 % energy travels in the cladding. This signal will have a different velocity than the signal travels in the core because $n_2 < n_1$. This phenomena pave way to waveguide dispersion. Waveguide dispersion can be written in terms of fiber parameters as,

$$\Delta T_{wg} = -\frac{n_1 - n_2}{\lambda c}V\frac{d^2Vb}{dV^2}L \tag{2.5}$$

where $b = \frac{\beta/k - n_2}{n_1 - n_2}$. This dispersion will be dominant in single mode fibers and not significant in multimode fibers.

2.7.6. Material dispersion:

All optical sources have a finite line width $\Delta\lambda$. Because $n = n(\lambda)$ each wavelength will travel at a slightly different velocity. As a result, there will be material dispersion. Material dispersion parameter: D_{mat} ps/nm/km. Material dispersion exists in all fibers, and will be high if a wide line width source (LED) is used. Material dispersion i given by,

$$\Delta T_{mat} = D_{mat}(\Delta\lambda)L \tag{2.6}$$

In standard silica fiber, at 1310 nm, waveguide and material dispersions will cancel out each other. This is called the *zero dispersion wavelength*.

Although material dispersion can **not** be modified much, waveguide dispersion can be either *shifted* or *optimized* to achieve

1. Dispersion Shifted fiber that has zero dispersion at 1550 nm or
2. Dispersion flattened fiber that has low dispersion for a wide wavelength range.

2.7.7. Polarization Mode Dispersion

Even in single mode fibers, there are two independent, degenerate linearly polarized (LP) propagation modes exists. (Horizontally and Vertically Polarized)

In perfectly symmetrical fibers, the propagation constants for these two modes are identical $\beta_x = \beta_y$.

In actual fibers $\beta_x \neq \beta_y$. Each mode propagate with a different phase velocity. (Remember, $\beta = \frac{2\pi n}{\lambda}$).

Define: Birefringence $B_f = n_y - n_x$

Beat Length

$$L_p = \frac{2\pi}{k(n_y - n_x)} = \frac{\lambda}{(n_y - n_x)}$$

If the group velocities of of the two orthogonal polarizations are μ_{gx} and μ_{gy}, then the differential time delay ΔT_{pol} over a distance L is,

$$\Delta T_{pol} = |L/\mu_{gx} - L/\mu_{gy}| \approx D_{PMD}\sqrt{L}$$

Since PMD randomly vary along the fiber, only a statistical measure can be given. D_{PMD} is typically 0.1 to 1.0 $ps\sqrt{km}$.

2.7.8. Total Dispersion

The total dispersion depends on number of factors and determine the final bit rate.

For Multi Mode Fibers:

$$\Delta T_{Total} = \sqrt{\Delta T_{mat}^2 + \Delta T_{mod}^2}$$

For Single Mode Fibers:

$$\Delta T_{Total} = \sqrt{\Delta T_{GVD}^2 + \Delta T_{pol}^2}$$

where, the group velocity dispersion is:

$$\Delta T_{GVD} = \Delta T_{mat} + \Delta T_{wg}$$

The polarization mode dispersion is typically smaller than the group velocity dispersion. Then, for single mode fibers:

$$\Delta T_{Total} \approx \Delta T_{GVD}$$

Chapter 3

Optical Sources

The optical source should best suit the channel and modulating signal characteristics. The channel can be, optical fiber, diffuse wireless or point to point wireless in a communication system.

3.1. CONSIDERATIONS

- Suitable physical dimensions
- Suitable radiation pattern (beam width)
- Linearity and large dynamic range (output power proportional to driving current)
- Ability to be directly modulated at high speeds (fast response time)
- Adequate output power to overcome channel losses
- Narrow spectral width (or line width)
- Thermal stability
- Reliability (LED better than laser)
- Cost considerations
- Direct modulation considerations
- Driving circuit considerations, (impedance matching etc) for analog systems
- Conversion efficiency

3.2. THE LIGHT EMITTING DIODE

3.2.1. Basic Physics

Electron energy in semiconductors fall into two distinct bands, *valence band* (lower energy level) and *conduction band* (higher energy level). By external energy supply (thermal, electrical) and electron can be made to jump to conduction band creating a hole in the valance band.

In an **intrinsic semiconductor** there are equal number of electrons and holes. By adding pentavalent (Gp-V) donor impurities (Ex: Arsenic, As) we can create an **n-type extrinsic semiconductor** that will have excess electrons in the conduction band. By adding pentavalent (Gp-III) acceptor impurities we can create a **p-type extrinsic semiconductor** that will have excess holes in the valence band.

Electrons in an n-type material are **majority carriers** and holes in an n-type material are **minority carriers** and vice versa.

If the momentum of the holes in the valance band and the momentum of the electrons in the conduction band are the same in a specific temperature, then it is called **direct bandgap semiconductor**.

When an electron jumps from a higher energy state (E_2) to a lower energy state (E_1) **(recombination)** the difference in energy $E_g = E_2 - E_1$ is released either as a photon of energy $E_g = h\nu$ (radiative recombination) as heat or phonons (lattice vibration). Both these are non-radiative recombinations.

3.2.2. Basic LED operation

In a semiconductor light source, a PN junction (that consists of semiconductor materials with suitable bandgap energy) acts as the active or recombination region. When the PN junction is forward biased electrons are supplied externally. Then, electrons and holes recombine either radiatively (emitting photons) or non-radiatively (emitting heat or phonons). This is simple LED operation. In a LASER, the photon is further processed in a resonance cavity to achieve a coherent, highly directional optical beam with a narrow line width.

For fiber-optics, the LED should have a high radiance (light intensity), fast response time and a high quantum efficiency. There are, double or single hetero-structure devices, surface emitting (diffused radiation) and edge emitting (more directional) LEDs. The emitted wavelength depends on band gap energy of the semiconductor material.

3.2.3. Semiconductor Materials

Semiconductor materials are selected to emit the desired wavelength. First generation sources were of GaAlAs that emit at 700 - 900 nm window. Later InGaAsP sources were devised they can be tuned to emit anywhere from 1200 - 1600 nm range, fitting into the currently most widely used windows (Fig. 1.3, Keiser).

When an electron jumps from a higher energy state (E_2) to a lower energy state (E_1) the difference in energy $E_g = E_2 - E_1$ is released as a photon. E_g is called the bandgap energy. The emission wavelength depends on the bandgap energy.

$$E_g = h\nu = hc/\lambda \tag{3.1}$$

$$\lambda(\mu m) = \frac{1.24}{E_g(eV)} \tag{3.2}$$

Wavelength is tuned by varying the ratio between alloys, x and y ($In_{1-x}Ga_xAs_yP_{1-y}$). From empirical Fig 4.13 [Keiser] for lattice matched configurations $y = 2.2x$ with $0 \leq x \leq 0.47$,

$$E_g = 1.35 - 0.72y + 0.12y^2 \tag{3.3}$$

Find the emission wavelength when x = 0.2.

Empirical formula for $Ga_{1-x}Al_xAs$ is,

$$E_g = 1.424 + 1.266x + 0.266x^2 \tag{3.4}$$

Find the emission wavelength when x = 0.2.

3.2.4. Line Width of an LED

Emitted wavelength is related to photon energy by,

$$\lambda = \frac{hc}{E_g} \tag{3.5}$$

By differentiating,

$$\frac{d\lambda}{dE_g} = -\frac{hc}{E_g^2} \tag{3.6}$$

Assuming $\Delta\lambda$ is small,

$$\Delta\lambda = \left|\frac{hc}{E_g^2}\right| \Delta E_g \tag{3.7}$$

From semiconductor physics, $\Delta E_g = \Delta(h\nu) \approx 3k_B T$,

$$|\Delta\lambda| = \lambda^2 \frac{3k_B T}{hc} \tag{3.8}$$

These are typical values and the exact value depends on the LED structure.

Similarly, the change in wavelength due to temperature change is,

$$\Delta\lambda = \left|\frac{hc}{E_g^2}\right| \left(\frac{dE_g}{dT}\right) \Delta T \tag{3.9}$$

3.2.5. LED Rate Equation

The injected carriers will decay exponentially,

$$n = n_o e^{-t/\tau} \tag{3.10}$$

Decay rate

$$\frac{dn}{dt} = \frac{n_o e^{-t/\tau}}{-\tau} = -\frac{n}{\tau} \tag{3.11}$$

Rate of change = supply rate - decay rate

$$\frac{dn}{dt} = \frac{I}{q} - \frac{n}{\tau} \tag{3.12}$$

At steady state, $dn/dt = 0$. Steady state electron density at the active region $n = I\tau/q \propto I$

3.2.6. Quantum Efficiency of LED:

Internal quantum efficiency

$$\eta_{int} = \frac{R_r}{R_r + R_{nr}}$$

where R_r is the radiative recombination rate and R_{nr} is the non-radiative recombination rate.

For exponential decay of excess carriers, the radiative recombination lifetime is $\tau_r = n/R_r$ and the no-radiative recombination lifetime is $\tau_{nr} = n/R_{nr}$.

If the current injected into the LED is I, then the total number of recombinations per second is, $R_r + R_{nr} = n/\tau = I/q$ where, q is the charge of an electron. That is, $R_r = \eta_{int} I/q$.

Note that $1/\tau = 1/\tau_r + 1/\tau_{nr}$

Since R_r is the total number of photons generated per second, the optical power generated internally to the LED is

$$P_{int} = R_r h\nu = \frac{\eta_{int} I(h\nu)}{q} = \frac{\eta_{int} hcI}{q\lambda} = \frac{1.24\eta_{int} I}{\lambda(\mu m)}$$

Ex: Find the internal quantum efficiency when, $\tau_r = 30$ ns, $\tau_{nr} = 100$ ns, $I = 40$ mA and $\lambda = 1310$ nm. What is the internal power P_{int}? (Ans: 29.2 mW)

3.2.7. Fresnel Reflection

Whenever there is an index mismatch and light travels from one medium (n_1) to a different medium (n_2), only a fraction of the incoming energy will pass through. The power that enters the second medium (n_2) depends on the *Fresnel Transmissivity*

$$T = T(0) = \frac{4n_1n_2}{(n_1+n_2)^2}$$

The *Fresnels reflectivity* R is defined as (referring to power)[1],

$$R = R(0) = \left(\frac{n_1 - n}{n_1 + n}\right)^2 = r^2 \tag{3.13}$$

Note that $R + T = 1$. Fresnels loss = $-10Log(T)$.

3.2.8. External Efficiency

This depends on the optical power escapes the LED. From Fig 4.15 [Keiser], there is a cone of emission. The power that escapes the LED medium (n_2) depends on the *Fresnel Transmissivity* $T(0)$.

The external efficiency is given by integrating $T(0)$ over the cone of emission.

$$\eta_{ext} = \frac{1}{4\pi}\int_0^{\phi_c} T(0)2\pi sin(\phi)d\phi \approx \frac{1}{n(n+1)^2}$$

.

3.2.9. Coupling Efficiency

This is the ratio between the power coupled into the fiber P_F and the power emitted from the light source P_s.

$$\eta = \frac{P_F}{P_s}$$

For surface emitting lambertian sources the output power $B(\theta) = B_o cos(\theta)$. Here, B_o is the radiance along normal to the radiating surface.

Considering a source smaller than ($r_s \leq a$), and in close proximity to the fiber core, the power coupled to a step index fiber from the LED is:

$$\eta_c = \frac{\int_0^{\theta_a} B_o cos\theta sin\theta d\theta}{\int_0^{\pi/2} B_o cos\theta sin\theta d\theta}$$

$$\eta_c = \frac{\int_0^{\theta_a} sin2\theta d\theta}{\int_0^{\pi/2} sin2\theta d\theta} = \frac{[-cos2\theta/2]_0^{\theta_a}}{[-cos2\theta/2]_0^{\pi/2}} = sin^2(\theta_a) = (NA)^2$$

$$P_{LED,step} = P_s(NA)^2 \tag{3.14}$$

For $r_s > a$, power coupled is:

$$P_{LED,step} = (\frac{a}{r_s})^2 P_s(NA)^2 \tag{3.15}$$

[1] Note reflection coefficient $r = \sqrt{R}$ refers to the amplitude. There should be no confusion between R and r

If the refractive index of the medium in between the LED and the fiber (air) is n, then add Fresnel loss $(1-R)$. Therefore $P_{coupled} = (1-R)P_{emitted}$.

Combining all together, the power coupled to the fiber is,

$$P_{coupled} = \frac{(1-R)}{n(n+1)^2}(\frac{a}{r_s})^2(NA)^2\eta_{int}\frac{1.24I}{\lambda}$$

for $r_s > a$.

3.2.10. Frequency Response of an LED

The modulation (frequency) response depends on

- the injected carrier lifetime τ and
- parasitic capacitance

Typically the LED is a first order low pass filter

$$P(\omega) = \frac{P_o}{\sqrt{1+(\omega\tau)^2}} \quad (3.16)$$

$P(\omega) \propto I(\omega)$, Electrical power $\propto I^2(\omega)$

Electrical 3-dB BW occurs when electrical power goes to half, that is when

$$\frac{I^2(\omega_e)}{I^2(0)} = \frac{1}{2} \longrightarrow \frac{P(\omega_e)}{P(0)} = \frac{1}{\sqrt{2}} \longrightarrow \omega_e = \frac{1}{\tau}$$

Optical 3-dB BW occurs when optical power goes half, that is when

$$\frac{P(\omega_o)}{P(0)} = \frac{1}{2} \longrightarrow \omega_o = \frac{\sqrt{3}}{\tau}$$

In order to support an electrical bandwidth of B Hz, the optical side should have a bandwidth of $\sqrt{3}B$.

3.2.11. Optical Loss and Electrical Loss

Note that, optical power $P(\omega) \propto I(\omega)$ while electrical power $\propto I^2(\omega)$. Therefore, optical power loss in a fiber link is $= P_{in}/P_{out}$. However, electrical power loss $= I^2_{in}/I^2_{out} = P^2_{in}/P^2_{out}$. As a result, electrical power loss is the square of the optical power loss (in the linear scale). It follows that loss/gain in the optical domain will appear twice in the electrical domain in the log scale. This observation is especially significant in analog systems.

Electrical loss (dB)= 2$\times$ Optical Loss (dB).

3.3. LASER DIODE

LASER: Light Amplification by Stimulated Emission

3.3.1. Stimulated Emission

Stimulated emission is the basis for obtaining photon amplification.

Basic LED operation When an electron jumps from a higher energy state (E_2) to a lower energy state (E_1) the difference in energy $\Delta E = E_2 - E_1$ is released either

- as a photon of energy $h\nu$ (radiative recombination)
- or as heat (non-radiative recombination)

Absorption an atom in the ground state might absorb a photon emitted by another atom, thus making a transition to an excited state.

Spontaneous Emission random emission of a photon, which enables the atom to relax to the ground state

Stimulated Emission An atom in an excited state might be stimulated to emit a photon by another incident photon. In this case, both photons will have,

1. identical energy $\rightarrow$ identical wavelength $\rightarrow$ narrow line width
2. identical direction $\rightarrow$ spatial coherence $\rightarrow$ narrow beam width
3. identical phase and$\rightarrow$ temporal coherence
4. identical polarization

When there are more atoms in the conduction band than the valance band, it is called the **population inversion**. This non-equilibrium state usually happens when we have three or more energy levels.

From the Einstein relations, it can be shown [senior] that for systems in thermal equilibrium (like incandescent lamp),

$$\frac{SpontaneousEmission}{StimulatedEmission} = exp(h\nu/k_BT) - 1 \tag{3.17}$$

This does not hold for laser (with population inversion)

3.4. FABRY PEROT RESONATOR CAVITY

3.4.1. Lasing Condition

To determine the lasing condition and the resonant frequencies, we express the electromagnetic wave propagating the longitudinal direction as,

$$E(z,t) = E(z)e^{j(\omega t - \beta z)}$$

where $E(z)$ is the field intensity. Steady state conditions for laser oscillation are achieved when the gain in the amplifying medium matches the total losses. For simplicity, all the losses in the medium can be included in a single loss coefficient per unit length as $\overline{\alpha}\ m^{-1}$. Reflectivities of the mirrors are R_1 and R_2. Cavity length is L. Hence the exponentially increasing *fractional loss* $= R_1R_2exp(-2\overline{\alpha}L)$

It is found that the increase in beam intensity resulting from stimulated emission is exponential too [senior]. Therefore, if the gain coefficient per unit length produced by stimulated emission is $\overline{g}\ m^{-1}$, the fractional round trip gain is *fractional gain* $= exp(2\overline{g}L)$ Hence,

$$exp(2\overline{g}L) = R_1R_2exp(-2\overline{\alpha}L)$$

Therefore, the threshold gain can be written as,

$$\overline{g_{th}} = \overline{\alpha} + \frac{1}{2L}ln\frac{1}{R_1R_2}$$

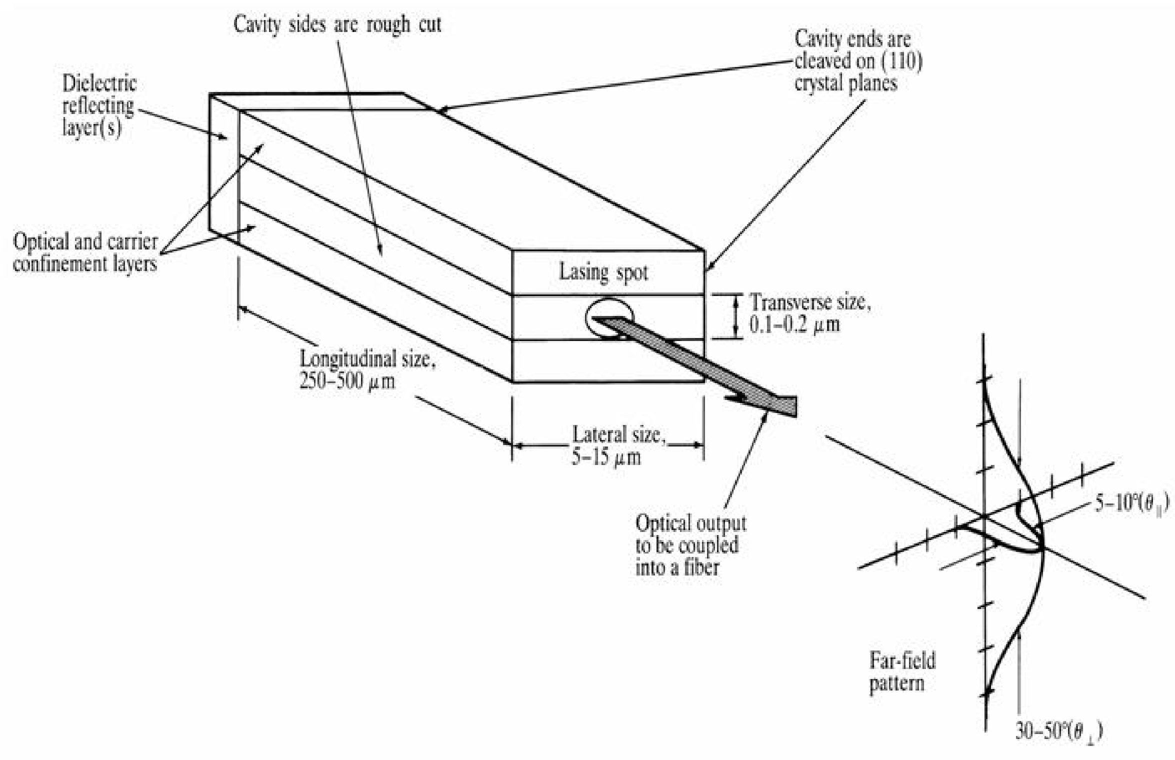

Figure 3.1. The Fabry Perot Laser Cavity

For lasers with strong carrier confinement, the threshold current density J_{th} is given by, $g_{th} = \beta J_{th}$ [2]. Therefore,

$$\overline{I_{th}} = \frac{Lw}{\beta}\left[\overline{\alpha} + \frac{1}{2L}ln\frac{1}{R_1R_2}\right] \tag{3.18}$$

Ex: L = 0.25 mm, w = 0.1 mm, β = 21 X 10(-3) Acm^{-3}. $\overline{\alpha} = 10cm^{-1}$, n for GaAs is 3.6. Find the threshold current. (Ans 663 mA)

3.4.2. Lasing Modes

Cavity length L must be an integer m number of half wavelengths for a standing wave pattern,

$$2\beta_m L = 2\pi m$$

where $\beta_m = k_m n = 2\pi n/\lambda_m$ is the propagation condition in medium n corresponding to the m^{th} mode. Substituting,

$$\frac{2nL}{\lambda_m} = m$$

Since $c = \nu\lambda$

[2] $I = J \times L \times w = J \times theareaoftheopticalcavity$ for a laser and LED

$$m = \frac{2Ln}{c}\nu_m$$

$$m - 1 = \frac{2Ln}{c}\nu_{m-1}$$

Subtracting,

$$\Delta\nu = \nu_m - \nu_{m-1} = \frac{c}{2Ln}$$

Since $\Delta\nu/\nu = \Delta\lambda/\lambda$,

$$\Delta\lambda = \frac{\lambda^2}{2Ln}$$

This is the spectral separation between the stable modes in a Fabry Perot cavity.

The relationship between gain and frequency can be assumed to have the Gaussian form

$$g(\lambda) = g(0)exp\left[-\frac{(\lambda - \lambda_0)^2}{2\sigma_\lambda^2}\right] \tag{3.19}$$

where λ_0 is the wavelength at the center (with the highest gain of $g(0)$), σ_λ is the factor that controls the width of the gain envelope. This is related to the RMS line width of the laser. $g(0)$ is the maximum gain that is proportional to the population inversion.

3.4.3. Laser Rate Equations

The total carrier population inside a semiconductor laser diode is determined by three processes: carrier injection, spontaneous recombination and stimulated emission. For a PN junction with a carrier confinement region of depth d, the laser *rate equations* are given by the following. These two equations govern the dynamic nature of the laser during time varying injected current

$$\frac{d\Phi}{dt} = CN\Phi + R_{sp} - \frac{\Phi}{\tau_{ph}} \tag{3.20}$$

Rate of change of photons = Stimulated emissions + spontaneous emission - Photon loss

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_{sp}} - CN\Phi \tag{3.21}$$

Rate of change of electrons = injection - spontaneous recombination - stimulated emission

N	Number of electrons
Φ	Number of Photons
C	Einsteins Coefficient
τ_{ph}	Photon lifetime
R_{sp}	Rate of spontaneous emission
τ_{sp}	spontaneous recombination lifetime
J	Injection current density
q	Electron Charge

The rate of change $d\phi/dt > 0$ for stimulated emission to start, that is

$$CN - 1/\tau_{ph} \geq 0$$

This condition will be satisfied for $N > N_{th}$, where the threshold point is given by $N = N_{th}$. The value for N_{th}

is obtained by setting the rate of change to zero. Therefore, from Equation (3.21) neglecting R_{sp},

$$\frac{N_{th}}{\tau_{sp}} = \frac{J_{th}}{qd} \tag{3.22}$$

This expression defines a value for the threshold current density J_{th} above which the stimulated emission will be predominant.

Above the threshold point however, the electron density does not significantly increase and remains at N_{th}. Therefore, at steady state condition above threshold, by substitution of (3.22) in (3.20) and (3.21),

$$0 = CN_{th}\Phi_s + R_{sp} - \frac{\Phi_s}{\tau_{ph}}$$

$$0 = \frac{J}{qd} - \frac{N_{th}}{\tau_{sp}} - CN_{th}\Phi_s$$

where Φ_s is the steady state photon density. Adding these two equations and substituting from (3.22) yield,

$$0 = R_{sp} - \frac{\Phi_s}{\tau_{ph}} + \frac{J}{qd} - \frac{N_{th}}{\tau_{sp}}$$

$$\Phi_s = \frac{\tau_{ph}}{qd}(J - J_{th}) + \tau_{ph}R_{sp} \tag{3.23}$$

The first term is the number of photons emitted through stimulation and the second term is the spontaneous emission term (which is often ignored).

3.4.4. External Quantum Efficiency

This is calculated from the straight line portion of the power transfer curve of the laser diode,

$$\eta_{ext} = \frac{q}{E_g}\frac{dP}{dI} = 0.8065\lambda(\mu m)\frac{dP(mW)}{dI(mA)} \tag{3.24}$$

3.4.5. Single Mode Lasers

By having built in frequency selective reflectors, it is possible create positive feedback conditions for only a single mode. In the most widely used **distributed feedback lasers (DFB)** this is achieved by having Bragg grating written in the active region. The Bragg wavelength is given by,

$$\lambda_B = \frac{2n_e\Lambda}{k} \tag{3.25}$$

Typically first order is used (k=1). n_e is the effective refractive index and Λ is the grating period.

3.4.6. Analog Modulation

At this point it is worth to mention that the optical power emitted in to the fiber is constant in a directly modulated *analog* fiber optic link despite the variations in the RF power. The optical power output is only proportional to the DC bias current, which is typically kept constant. With typical modulation depths (say at 0.3), the peak of the modulated optical intensity does not exceed 30 % of the mean value.

Chapter 4

Optical Receiver and Various Noise Sources

The receiver is typically wide band and cost effective compared to laser in fiber optic links. Typically the performance of commercial receivers are adequate for most applications. Let us briefly review the concerns of optical receivers.

Noise, sensitivity at high power levels, and frequency response (speed) are the primary concerns with optical receivers. High bandwidth detectors, though commercially available, come with a penalty of low responsivity. This is because high bandwidth detectors tend to have smaller photosensitive areas which, limit the power conversion efficiency. On the other hand, large area detectors have high junction-capacitance which, limits the bandwidth. Furthermore, even the same photo detector is more nonlinear at higher frequencies than at low frequencies. For example, high power detectors with a maximum photocurrent of 150 mA have only about 295 MHz bandwidth, while high-speed detectors with a 50 GHz bandwidth have only about 1-2 mA photocurrent, as reported in [Charles COX].

4.1. CONSIDERATIONS

- High sensitivity (responsivity)at the desired wavelength and low responsivity elsewhere
- Low noise
- Reasonable cost
- Fast response time (high bandwidth)
- Insensitive to temperature variations
- Compatible physical dimensions
- Long operating life

4.2. PIN AND AVALANCHE PHOTO DIODE

Two type of detectors, namely the *positive-intrinsic-negative* (PIN) and the *avalanche photo diodes* (APD), are mostly used in fiber optic receivers. As the name implies the APD has a self multiplying mechanism so that it has high gain. The tradeoff of having the gain is the ‘excess noise’ due to random nature of the self multiplying process. Compared to short wavelengths (say 800 nm), at high wavelengths (say 1310 and 1550 nm), APD’s have the same excess noise, but they have an order of magnitude lower avalanche gain. Therefore, APD’s have relatively low responsivity at longer wavelengths.

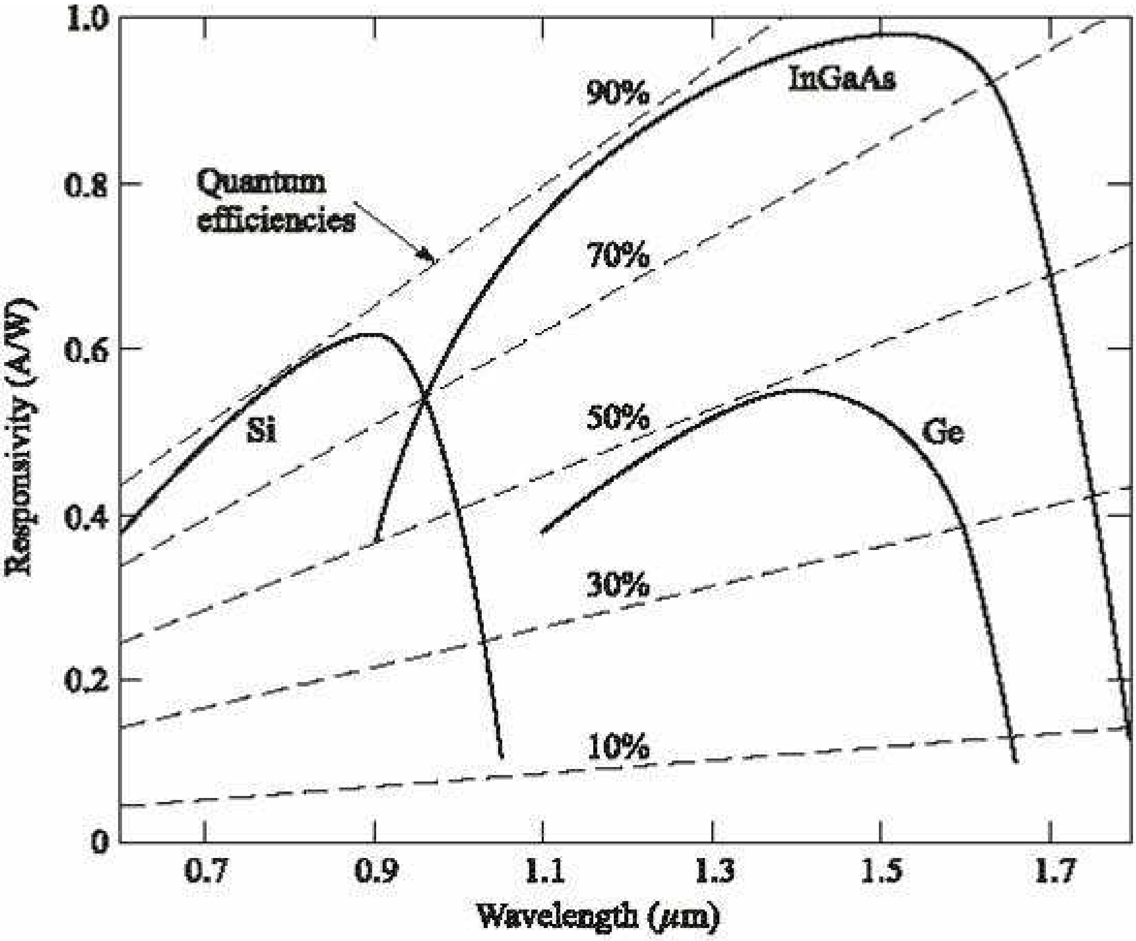

Figure 4.1. Comparison of the responsivity for different PIN photodiodes

4.2.1. PIN Photo Diode

This is the most widely used photodiode. The device consists of a p and n type semiconductor regions separated by an *intrinsic* (pure, actually very lightly n-doped) layer. A photodiode is normally **reverse biased** at optical receivers. Incident photons will supply enough energy for electron-hole recombination that will trigger an external photocurrent.

Incident optical radiation is absorbed in the semiconductor material according to the exponential law,

$$P(x) = P_o(1 - e^{-\alpha_s(\lambda)x}) \tag{4.1}$$

Here $\alpha_s(\lambda)$ is the *absorption coefficient*, P_o is the incident optical power. Note that the absorption coefficient $\alpha_s(\lambda)$ quickly becomes vary large for small λ (Fig. 6.3 - Keiser). This phenomena determines the lower wavelength at which a photodiode has reasonable responsivity.

The incident photons should have enough energy to trigger recombination. This factor defines an upper cut-off wavelength beyond which the responsivity of the photodiode drastically drops (Fig. 4.1). The upper cut-off wavelength depends on the bandgap energy of the semiconductor material. Note that, typically the *responsive-linewidth* of a photodiode is an order of magnitude larger (typ. 500 nm) than the linewidth of an LED.

$$\lambda_c(\mu m) = \frac{1.24}{E_g(eV)} \tag{4.2}$$

If the depletion region has a width w then the total power absorbed is $P(w) = P_o(1 - e^{-\alpha_s(\lambda)w})(1 - R)$, where R is the Fresnel reflectivity. The current generated is:

$$I_p = q(\text{number of electrons}) = \frac{q}{h\nu} P_o(1 - e^{-\alpha_s(\lambda)w})(1 - R) \tag{4.3}$$

Quantum efficiency η is the ratio between number of electrons generated and the number of incident photons,

$$\eta = \frac{I_p/q}{P_o/h\nu} \tag{4.4}$$

Responsivity $\Re$ of the photodiode in mA/mW is defined as,

$$\Re = \frac{I_p}{P_o} = \frac{\eta q}{h\nu} = \frac{q}{h\nu}(1 - e^{-\alpha_s(\lambda)w})(1 - R) \tag{4.5}$$

Fig. 6-4 (Keiser) shows the relationship between $\Re$ and wavelength for some semiconductor material.

4.2.2. Avalanche Photodiode (APD)

- APD achieves high sensitivity by having an internal gain.
- This internal gain is obtained by having a high electric field that energizes photo-generated electrons and holes
- These electrons and holes ionize bound electrons in the valence band upon colliding with them
- This mechanism is known as *impact ionization*
- The newly generated electrons and holes are also accelerated by the high electric field
- They gain enough energy to cause further *impact ionization*
- This phenomena is the *avalanche effect*

The avalanche gain M is defined by,

$$M = \frac{I_M}{I_p}$$

where I_M is the multiplied current and I_p original photocurrent. Therefore, $\Re_{APD} = M\Re_{PIN}$. Note that M is a statistical quantity because of the random nature of avalanche multiplication process.

4.3. A CONVENTION ON NOTATIONS

- The direct current value is denoted by, I_P; capitol main entry and capital suffix.
- The time varying (either randomly or periodically) current with a non-zero mean is denoted by, I_p capitol main entry and small suffix.
- The time varying (either randomly or periodically) current with a zero mean is denoted by, i_p small main entry and small suffix.

- Therefore, the total current I_p is the sum of the DC component I_P and the AC component i_p.

$$I_p = I_P + i_p \tag{4.6}$$

4.4. NOISE IN PHOTONIC RECEIVERS

Signal to noise ration of a photodiode decides its performance. To have a high SNR,

1. The detector should have high responsivity
2. The noise should by minimal

In a typical PIN diode receiver, there are three major noise mechanisms. Namely, shot noise, thermal noise and the dark current noise. All these noise mechanisms are unavoidable. However, their relative importance depends on a particular design.

4.4.1. Quantum (Shot) Noise

Light is composed of photons, which are discrete packets of energy. Thus, the randomness of the arrival time of each photon generates a random noise component at the output current of the photo diode which, is referred to as the quantum or shot noise. The shot noise is proportional to the average value of the optical signal. For a PIN diode, the shot noise power is given by,

$$\left\langle i_Q^2 \right\rangle = 2q\Re P_o B = 2qI_P B \tag{4.7}$$

where, P_o is the optical power at the detector, q is the charge of an electron, B is the bandwidth of interest and $\Re$ is the photo diode responsivity. The detector current, which is denoted by I_p, is responsivity times P_o. That is $I_p = \Re P_o$.

For avalanche photodiodes,

$$\left\langle i_Q^2 \right\rangle = 2qI_P BM^2 F(M) \tag{4.8}$$

where, M is the avalanche noise and $F(M)$ is the excess noise (or noise figure). Both these are unity for PIN diodes.

4.4.2. Thermal Noise

Thermal noise is due to the resistive elements in the receiver amplifier. The thermal noise is independent to the optical signal level but increase with the temperature. The thermal noise power is given by,

$$\left\langle i_T^2 \right\rangle = 4K_B T_o B/R_L \tag{4.9}$$

where, T_o is the absolute temperature in Kelvin and K_B is the Boltzman constant and R_L is the receiver load impedance.

4.4.3. Dark Current Noise

Even in absolute dark, there is a very small current from the photodiode due various leakage effects. There are two, *bulk* and *surface* dark currents The noise power associated with the bulk dark current is given by,

$$\left\langle i_{DB}^2 \right\rangle = 2qI_D M^2 F(M) B \tag{4.10}$$

where I_D is the dark current. Note that this undergoes the avalanche multiplication process. The noise power due to the surface leakage current is,

$$\left\langle i_{DS}^2 \right\rangle = 2qI_L B \tag{4.11}$$

where, I_L is the surface leakage current. Typically, the i_{DS} term is negligible compared to i_{DB}.

Usually the combination of all these noise are specified by the manufacturer and called EIN, i.e. equivalent input noise. For example, a typical value for a DFB laser transmitter and a PIN diode receiver, the total EIN is specified as -125 dBm/Hz. This has to be multiplied with the used bandwidth to obtain actual noise power.

4.4.4. Interferometric Noise (IN)

Interferometric noise can appear in an optical system when the received signal is accompanied by weak delayed replica of itself or other light wave components. These doubly reflected signals mix electrically with the original signal and cause an excess noise. Reflections arise either from discrete reflectors such as splices and connectors or by Rayleigh scattering within the fiber itself.

Basically when the fiber has poor connectors or very long with high optical power, the IN becomes significant. For the fiber lengths less than 20 km the Rayleigh scatter introduced Interferometric noise is negligible. Furthermore, if the number of connectors that have a back reflection factor of -35 dB or better is less than 17, then the discrete reflection effect is also negligible [Shibutani].

4.4.5. Relative Intensity Noise (RIN)

The RIN exists only in *analog systems* when the laser is always on. In this case, the light produced by the laser is not stable in intensity. The basic physical mechanism of a laser is amplification by stimulated emission, which is random in nature. This randomness introduces a noise that increases with the optical power. The noise due to multiple optical reflections (Interferometric noise) and Brillouin scattering also increase with optical power. All these noise processes can be grouped together as relative intensity noise (RIN). A fluctuation in the optical output intensity due to multiple reflections in fiber optic link leads to this optical intensity noise. The noise power due to RIN is given as, where m is the modulation index, P_o is the mean optical power and $s(t)$ is the modulating (electrical) signal.

$$\langle i_{RIN}^2(t)\rangle = P_{RIN}\Re^2 P_o^2 M^2 F(M) B \left[1 + m^2 \left\langle s^2(t)\right\rangle\right] \tag{4.12}$$

Typically, a RIN parameter P_{RIN} is specified for a given laser diode in dBm/Hz, for example -155 dBm/Hz. This expression is more accurate than the widely used expression for the variance of the RIN. Many authors have omitted the second term $m^2\langle s^2(t)\rangle$. This is acceptable because, most of the time m is in the range of 0.1 and $s(t) << 1$ so that, this term is insignificant. However, with higher values of m and $s(t)$ this term is not negligible. We include this term because with nonlinearity compensation schemes, m can be higher. Furthermore, the expression in (4.12) better explains, some empirical results.

4.5. THE SIGNAL TO NOISE RATIO

4.5.1. Digital Systems

The complete signal to noise ratio of a digital fiber optic link considering all these noise processes is given below where, F is the receiver amplifier noise figure.

$$SNR = \frac{M^2\langle i_p^2\rangle}{2qBM^2F(M)(I_p + I_D) + 2qI_LB + 4FK_BT_oB/R_L} \tag{4.13}$$

The dark current I_D is typically in the order of nano-Amps. The detected current I_D is typically in milli-Amps range since the optical power in this applications is in mW range and the responsivity $\Re$ lies between zero and one mA/mW. Therefore, the dark current term is neglected without any loss in accuracy.

Furthermore, experimentally it has been shown that the avalanche noise figure $F(M) \approx M^x$. The parameter x takes the value of 0.3 for Si, 0.7 for GaAs and 1.0 for Ge avalanche photodiodes. Hence, the modified signal to noise ratio due to the receiver noises is given by,

$$SNR \approx \frac{M^2\langle i_p^2\rangle}{2qBM^{x+2}I_p + 4FK_BT_oB/R_L} \tag{4.14}$$

4.5.2. Analog Systems

Analog systems differ from digital systems in following aspects:

- The LASER or LED is always on. Therefore, there is a large mean optical power, say P_o.
- A relatively small ac component is superimposed on top of this mean value.
- There will be RIN in addition to other noise.

Considering direct intensity modulation on the laser diode, the instantaneous optical power output $P(t)$ from the laser in response to input electrical signal $s(t)$ is ($|s(t)| \leq 1$),

$$P(t) = P_o[1 + ms(t)] \tag{4.15}$$

Here m is the optical modulation index, P_o is the mean optical power. Neglecting attenuation in the fiber, detector current $I_p(t)$ is,

$$I_p(t) = \Re MP_o[1 + ms(t)] = I_PM[1 + ms(t)]$$

$$I_p^2(t) = M^2I_P^2[1 + ms(t)]^2$$

The signal power $\langle i_p^2(t)\rangle = M^2m^2I_p^2\langle s^2(t)\rangle$

The complete signal to noise ratio of an analog fiber optic link considering all these noise processes is given below.

$$SNR = \frac{M^2m^2I_P^2\langle s^2(t)\rangle}{2qBM^2F(M)(I_P + I_D) + 2qI_LB + 4FK_BT_oB/R_L + P_{RIN}I_P^2M^2F(M)B\left[1 + m^2E[s^2(t)]\right]} \tag{4.16}$$

This can be approximated to,

$$SNR = \frac{M^2m^2I_P^2\langle s^2(t)\rangle}{2qBM^2F(M)I_P + 4FK_BT_oB/R_L + P_{RIN}I_P^2M^2F(M)B\left[1 + m^2E[s^2(t)]\right]} \tag{4.17}$$

4.5.2.1. Quantitative Discussion

There are several noise terms involved in the expression given. Namely shot, RIN and thermal noises. Thermal noise has a constant variance and depends on the receiver resistance only. This has a white spectrum. The variance of the shot noise is linearly proportional to mean optical power in the fiber. Although the instantaneous optical power in the fiber fluctuates due to RF intensity modulation, the mean optical power does not change unless the DC bias current is changed. Therefore, the shot noise does not change with modulating signal power and constant for a given modulation depth m. However, the RIN changes with RF signal level. This is seen from the expression in (4.12). This is also logical because, the RIN is proportional to the square of the optical power. Since, the instantaneous optical power in the fiber fluctuates at radio frequency, the square of it increases with RF signal level depending on m.

The following additional points are observed from the expression for signal to noise ratio:

1. The higher the bandwidth B of $s(t)$, the lower the SNR because, the wider noise bandwidth in the optical link collects more noise.

2. The higher modulation index m yields better SNR. This is because more power is contained in the side bands compared to the unmodulated carrier. However, nonlinear effects limit m to a lower value ($m < 0.3$).

If the thermal noise at the receiver amplifier is made small enough due to an improved design, then (**??**) becomes,

$$SNR = \frac{M^2 m^2 I_P^2 \langle s^2(t) \rangle}{2qBM^2F(M)I_P + P_{RIN} I_P^2 M^2 F(M) B \left[1 + m^2 E[s^2(t)]\right]} \tag{4.18}$$

From (4.18) we deduce that,

1. In the shot noise limited case
$$SNR = \frac{m^2 I_P \langle s^2(t) \rangle}{2qBF(M)}$$
That is, SNR increases with mean detected current I_P. Mean detected current is proportional to mean optical power P_o. However, large P_o means relatively low m. Therefore, there is an optimum m in the shot noise limited case that will give the highest SNR.

2. In the RIN limited case,
$$SNR = \frac{m^2 \langle s^2(t) \rangle}{P_{RIN} F(M)[1 + m^2 \langle s^2(t) \rangle B} \approx \frac{m^2 \langle s^2(t) \rangle B}{P_{RIN} F(M) B}$$
That is the SNR is *independent* to mean optical power and increases with the RF power. However, when the RF power is large enough ($m^2 E[s^2(t)] > 1$), the SNR saturates.