ELE745 Assignment and Lab Manual

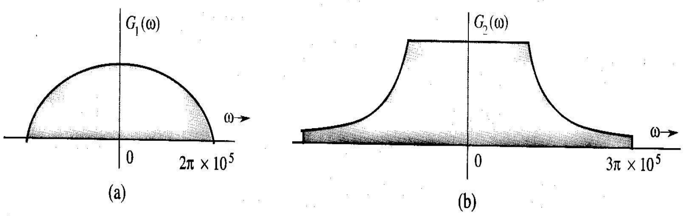
August 22, 2010

CONTENTS

1. ASSIGNMENT 1

1.1 Assignment 1 Problems

1. **Lathi, 6.1-1**
   Following figure shows Fourier spectra of signals $g_1(t)$ and $g_2(t)$. Determine the Nyquist interval and the sampling rate for signals $g_1(t)$, $g_2(t)$, $g_1^2(t)$, $g_2^3(t)$, and $g_1(t)g_2(t)$.



2. **Lathi, 6.1-2**
   Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals:
   (a) $\text{sinc}(100\pi t)$;
   (b) $\text{sinc}^2(100\pi t)$;
   (c) $\text{sinc}(100\pi t) + \text{sinc}(50\pi t)$;
   (d) $\text{sinc}(100\pi t) + 3\text{sinc}^2(60\pi t)$;
   (e) $\text{sinc}(50\pi t)\,\text{sinc}(100\pi t)$.

3. **Lathi, 6.1-4**
   A signal $g(t) = \text{sinc}^2(5\pi t)$ is sampled (using uniformly spaced impulses) at a rate of: (i) 5 Hz; (ii) 10 Hz; (iii) 20 Hz. For each of the three case:
   (a) Sketch the sampled signal;
   (b) Sketch the spectrum of the sampled signal;
   (c) Explain whether you can recover the signal $g(t)$ from the sampled signal;

(d) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the spectrum of the output signal.

4. **Lathi, 6.1-8**

   Prove that a signal cannot be simultaneously time-limited and band-limited. Hint: show that contrary assumption leads to contradiction. Assume a signal simultaneously time-limited and band-limited so that $G(\omega) = 0$ for $|\omega| > 2\pi B$. In this case $G(\omega) = G(\omega)\text{rect}(\omega/4\pi B')$ for $B' > B$. This means that $g(t)$ is equal to $g(t) * 2B'\text{sinc}(2\pi B't)$. Show that the latter cannot be time-limited.

5. **Lathi, 6.2-2**

   A compact disc (CD) records audio signal digitally by using PCM. Assume the audio signal bandwidth to be 15 kHz.

   (a) What is the Nyquist rate?

   (b) If the Nyquist samples are quantized into $L = 65,536$ levels and then binary coded, determine the number of binary digits required to encode a sample.

   (c) Determine the number of binary digits per second (bit/s) required to encode the audio signal.

   (d) For practical reasons, signals are sampled at a rate well above the Nyquist rate. Practical CDs use 44,100 samples per second. If $L = 65,536$, determine the number of bits per second required to encode the signal.

6. **Lathi, 6.2-3**

   A television signal (videl and audio) has a bandwidth of 4.5 MHz. This signal is sampled, quantized, and binary coded to obtain a PCM signal.

   (a) What is the sampling rate if the signal is to be sampled at a rate 20% above the Nyquist rate.

   (b) If the samples are quantized into 1024 levels, determine the number of binary pulses required to encode each sample.

   (c) Determine the binary pulse rate (bits per second) of the binary-coded signal, and the minimum bandwidth required to transmit this signal.

7. **Lathi, 6.2-4**

   Five telemetry signal, each of bandwidth 1 kHz, are to be transmitted simultaneously by binary PCM. The maximum tolerable error in sample amplitudes is 0.2% of the peak signal amplitude. The signals must be sampled at least 20% above the Nyquist rate.

Framing and synchronizing requires an additional 0.5% extra bits. Determine the minimum possible data rate (bits per second) that must be transmitted, and the minimum bandwidth required to transmit this signal.

8. **Lathi, 6.2-6**
   A message signal $m(t)$ is transmitted by binary PCM. If the SNR (signal-to-quantization-noise ratio) is required to be at least 47 dB, determine the minimum value of L required, assuming that $m(t)$ is sinusoidal. Determine the SNR obtained with this minimum L.

9. **Sklar, 1.4**
   Using time averaging, find the average normalized power in the waveform $x(t) = 10\cos 10t + 20\cos 20t$.

10. **Sklar, 1.13**
    Use the sampling property of the unit impulse function to evaluate the following integrals.

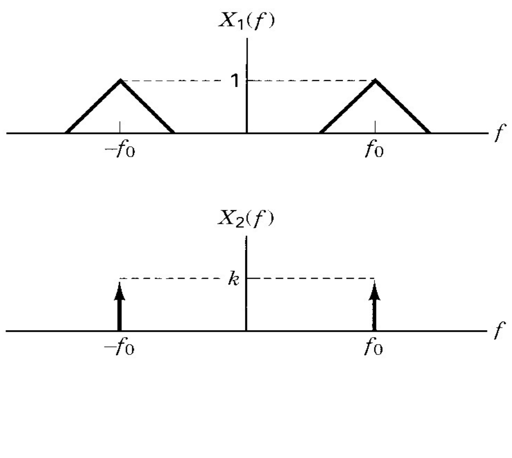
    (a) $\int_{-\infty}^{\infty} \cos 6t\ \delta(t-3)\ dt$

    (b) $\int_{-\infty}^{\infty} 10\delta(t)\ (1+t)^{-1}\ dt$

    (c) $\int_{-\infty}^{\infty} 10\delta(t+4)\ (t^2+6t+1)\ dt$

    (d) $\int_{-\infty}^{\infty} \exp(-t^2)\ \delta(t-2)\ dt$

11. **Sklar, 1.14**
    Find $X_1(f) * X_2(f)$ for the spectra shown below.



12. **Sklar, 2.8**
    Consider an audio signal with spectral components limited to the frequency band 300 to 3300 Hz. Assume that a sampling rate of 8000 samples/s will be used to generate a PCM signal. Assume that the ratio of peak signal power to average quantization noise power at the output needs to be 30 dB.

(a) What is the minimum number of uniform quantization levels needed, and what is the minimum number of bits per sample needed?

(b) Calculate the system bandwidth (as specified by the main spectral lobe of the signal) required for the detection of such a PCM signal.

13. **Sklar, 2.9**

A waveform, $x(t) = 10\cos(1000\pi t + \pi/3) + 20\cos(2000\pi t + \pi/6)$ is to be uniformly sampled for digital transmission.

(a) What is the maximum allowable time interval between sample values that will ensure perfect signal reproduction?

(b) If we want to reproduce 1 hour of this waveform, how many sample values need to be stored?

14. **Sklar, 1.15**

A signal in the frequency range 300 to 3300 Hz is limited to a peak-to-peak swing of 10 V. It is sampled at 8000 samples/s and the samples are quantized to 64 evenly spaced levels. Calculate and compare the bandwidths and ratio of peak signal power to rms quantization noise if the quantized samples are transmitted either as binary pulses or as four-level pulses. Assume that the system bandwidth is defined by the main spectral lobe of the signal.

15. **Sklar, 1.16**

In the compact disc (CD) digital audio system, an analog signal is digitized so that the ratio of the peak-signal power to the peak-quantization noise power is at least 96 dB. The sampling rate is 44.1 kilosamples/s.

(a) How many quantization levels of the analog signal are needed for $(S/N_q)_{\text{peak}} = 96dB$?

(b) How many bits per sample are needed for the number of levels found in part (a)?

(c) What is the data rate in bits/s?

16. **Haykin, 3.2**

In *natural sampling*, an analog signal $g(t)$ is multiplied by a periodic train of rectangular pulse $c(t)$, each of unit area. Given that the pulse repetition frequency of this period train is f_s and the duration of each rectangular pulse is T (with $f_s T << 1$), do the following:

(a) Find the spectrum of the signal $s(t)$ that results from the use of natural sampling; you may assume that time $t = 0$ corresponds to the midpoint of a rectangular pulse in $c(t)$.

(b) Show that the original signal $g(t)$ may be recovered exactly from its naturally sampled version, provided that the conditions embodied in the sampling theorem are satisfied.

17. **Haykin, 3.8**

Twenty-four voice signals are sampled uniformly and then time-division multiplexed. The sampling operation uses flat-top samples with 1 μs duration. The multiplexing operation includes provision for synchronization by adding an extra pulse of sufficient amplitude and also 1 μs duration. The highest frequency component of each voice signal is 3.4 kHz.

(a) Assuming a sampling rate of 8 kHz, calculate the spacing between successive pulses of the multiplexed signal.

(b) Repeat your calculation assuming the use of Nyquist rate sampling.

1.2 Assignment 1 Solutions

1. **Lathi 6.1-1**: The bandwidth of $g_1(t)$ and $g_2(t)$ are 100 kHz and 150 kHz, respectively. Therefore,

   - the Nyquist sampling rates for $g_1(t)$ is 200 kHz, sampling interval $T_s = 1/200k = 5\mu s$
   - the Nyquist sampling rates for $g_2(t)$ is 300 kHz, sampling interval $T_s = 1/300k = 3.33\mu s$.
   - the bandwidth of $g_1^2(t)$ is 200 kHz, $f_{Nyq} = 400$ kHz, $f_{Nyq} = 1/400k = 0.25\mu s$.
   - the bandwidth of $g_2^3(t)$ is 450 kHz, $f_{Nyq} = 900$ kHz, $f_{Nyq} = 1/900k = 1.11\mu s$.
   - the bandwidth of $g_1(t) \cdot g_2(t)$ is 250 kHz, $f_{Nyq} = 500$ kHz, $f_{Nyq} = 1/500k = 2\mu s$.

2. **Lathi 6.1-2**:

   - since
     $$\text{sinc}(100\pi t) \rightarrow 0.01\text{rect}\left(\frac{\omega}{200\pi}\right)$$
     the bandwidth of this signal is 100 π rad/s or 50 Hz. The Nyquist rate is 100 Hz (samples/sec).
   - $$\text{sinc}^2(100\pi t) \rightarrow 0.01\Delta\left(\frac{\omega}{400\pi}\right)$$
     the bandwidth of this signal is 200 π rad/s or 100 Hz. The Nyquist rate is 200 Hz (samples/sec).
   - $$\text{sinc}(100\pi t) + \text{sinc}(50\pi t) \rightarrow 0.01\text{rect}\left(\frac{\omega}{200\pi}\right) + 0.02\text{rect}\left(\frac{\omega}{100\pi}\right)$$
     the bandwidth of the first term on the right-hand side is 50 Hz and the second term is 25 Hz. Clearly the bandwidth of the composite signal is the higher of the two, that is, 100 Hz. The Nyquist rate is 200 Hz (samples/sec).
   - $$\text{sinc}(100\pi t) + 3\text{sinc}^2(60\pi t) \rightarrow 0.01\text{rect}\left(\frac{\omega}{200\pi}\right) + 0.05\Delta\left(\frac{\omega}{240\pi}\right)$$
     the bandwidth of the first term is 50 Hz and that of the second term is 60 Hz. The bandwidth of the sum is the higher of the two, that is, 60 Hz. The Nyquist sampling rate is 120 Hz.

- $$\text{sinc}(50\pi t) \to 0.02\text{rect}\left(\frac{\omega}{100\pi}\right) \qquad \text{sinc}(100\pi t) \to 0.01\text{rect}\left(\frac{\omega}{200\pi}\right)$$

  The two signals have BW 25 Hz and 50 Hz respectively. The spectrum of the product of two signals is $1/(2\pi)$ times the convolution of their spectra. From width property of the convolution, the width of the convoluted signals is the sum of the widths of the signals convolved. Therefore, the BW of the product is 25+50=75 Hz. The Nyquist rate is 150 Hz.

3. **Lathi 6.1-4**: The BW of the signal $g(t)$ is 5 Hz (10π rad/s), since the FT as below:

$$g(t) = \text{sinc}^2(5\pi t) \quad \to \quad G(\omega) = 0.2\Delta\left(\frac{\omega}{20\pi}\right)$$

Therefore, the Nyquist rate is 10 Hz, and the Nyquist interval is $T = 1/10 = 0.1s$.

- When $f_s = 5Hz$, the spectrum $\frac{1}{T}G(\omega)$ repeats every 5 Hz (10π rad/sec). The successive spectra overlap, and the spectrum $G(\omega)$ is not recoverable from $\bar{G}(\omega)$, that is, $g(t)$ cannot be recovered from its samples. If the sampled signal is passed through an ideal lowpass filter of BW 5 Hz, the output spectrum is $\text{rect}(\omega/20\pi)$, and the output signal is $10\text{sinc}(20\pi t)$, which is not the desired signal $\text{sinc}^2(5\pi t)$.
- When $f_s = 10Hz$, the spectrum $\bar{G}(\omega)$consists of back-to-back, nonoverlapping repetition of $\frac{1}{T}G(\omega)$ repeating every 10 Hz. Hence, $G(\omega)$ can be recovered from $\bar{G}(\omega)$ using an ideal lowpass filter of BW 5 Hz (Fig.1(f)), and the output is $10\text{sinc}^2(5\pi t)$.
- in the last case of oversampling ($f_s = 20$ Hz), with empty band between successive cycles. Hence, $G(\omega)$ can be recovered from $\bar{G}(\omega)$ using an ideal lowpass filter or even a practical lowpass filter. The output is $20\text{sinc}^2(5\pi t)$.

4. **Lathi 6.1-8**: assuming a signal $g(t)$ that is simultaneously time-limited and bandlimited. Let $g(\omega) = 0$ for $|\omega| > 2\pi B$. Therefore,

$$g(\omega)\text{rect}\left(\frac{\omega}{4\pi B'}\right) = g(\omega) \quad for \quad B' > B.$$

Therefore, from the time-convolution property

$$g(t) = g(t) * [2B'\text{sinc}(2\pi B't)] = 2B'g(t) * \text{sinc}(2\pi B't).$$

Because $g(t)$ is time-limited, $g(t) = 0$ for $|t| > T$. But $g(t)$ is equal to convolution of $g(t)$ with $\text{sinc}(2\pi B't)$ which is not time-limited. It is impossible to obtain a time-limited signal from the convolution of a time-limited signal with a non-timelimited signal.

5. **Lathi 6.2-2**:
(a): the bandwidth is 15 kHz. The Nyquist rate is 30 kHz.
(b): $65536 = 2^{16}$, so that 16 binary digits are needed to encode each sample.
(c): $30,000 \times 16 = 480,000$ bits/s.
(d): $44,100 \times 16 = 705,600$ bits/s.

6. **Lathi 6.2-3**:
(a): The Nyquist rate is $2 \times 4.5 \times 10^6 = 9$ MHz. The actual sampling rate $=1.2 \times 9 = 10.8$ MHz.
(b): $1024 = 10^{10}$, so that 10 bits or binary pulses are needed to encode each sample.
(c): $10.8 \times 10^6 \times 10 = 108 \times 10^6$ or 108 Mbits/s.

7. **Lathi 6.2-4**:
If m_p is the peak sample amplitude, then

$$\text{quantization error} \leq 0.2\% \times m_p = \frac{m_p}{500}$$

Because the maximum quantization error is

$$\frac{q}{2} = 0.5 \times \frac{2m_p}{L}$$

- it follows that $L \geq 500$. Since L should be a power of 2, we choose $L = 512 = 2^9$. This requires 9-bit binary code per sample. The Nyquist rate is $2 \times 1000 = 2000$ Hz. 20% above this rate is $2 \times 1.2 = 2.4$ kHz. Thus, each signal has 2400 samples/second, and each sample is encoded by 9 bits. Therefore, each signal uses $9 \times 2.4 = 21.6$ kbits/second.
- Five such signals are multiplexed. Hence, we need a total of $5 \times 21.6 = 108$ Kbits/second data bits.
- Framing and synchronization requires additional 0.5% bits, that is $108,000 \times 0.005 = 540$ bits, yielding a total of 108,540 bits/second.
- The minimum transmission bandwidth is $108.54/2 = 54.27$ kHz.

8. **Lathi 6.2-6**:
Let m_p denote the peak amplitude of the sinusoid signal, signal power is $E[m^2(t)] = m_p^2/2$. Let L denote the number of steps, then the stepsize is $q = 2m_p/L$. The noise power is $\sigma^2 = q^2/2 = m_p^2/(3L^2)$. The required SNR is 47 dB=50119, which is

$$\frac{E[m^2(t)]}{\sigma^2} = \frac{3L^2}{2} \geq 50119$$

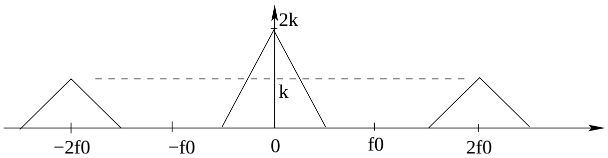
which implies $L \geq 182.8$. Because L is a power of 2, we select $L = 256 = 2^8$. The SNR for this value L is

$$\frac{E[m^2(t)]}{\sigma^2} = \frac{3L^2}{2} = 3(256)^2(0.5) = 98304 = 49.43dB$$

9. **Sklar 1.14**:

$$X_2(f) = k[\delta(f - f_0) + \delta(f + f_0)$$

$$X_1(f) * X_2(f) = X_1(f) * k[\delta(f - f_0) + \delta(f + f_0)$$



10. **Sklar 2.8**:

(a) Let L denote the number of quantization levels. The peak signal power to quantization noise power is

$$\left(\frac{S}{N}\right)_{peak} = 3L^2.$$

We have $10\log_{10}(3L^2) \geq 30$ (dB), and L can be solved as

$$L = \lceil 18.26 \rceil = 19 \quad \text{levels}$$

The number of bits per sample is

$$N = \lceil \log_2 L \rceil = \lceil \log_2 19 \rceil = 5 \quad \text{bits/sample}$$

(b) Let T_b denote the time duration of a bit. Since the sample rate is 8000 samples/s, each sample is represented by 5 bits. Therefore, there are 8000×5 bits each second and

$$T_b = \frac{1}{8000 \times 5} = 25 \quad \mu s$$

the required bandwidth W is

$$W = \frac{1}{T_b} = 40 \quad kHz$$

11. **Sklar 2.9**:

(a) The maximum frequency is $\omega_m = 2\pi f_m = 2000$ and $f_m = 2000/(2\pi) = 318.3$ Hz. Therefore, sampling rate should be

$$f_s \geq 2f_m = 2 \times 318.3 = 636.6 \quad \text{samples/s}$$

The sampling interval should satisfy:

$$T_m = \frac{1}{f_s} \leq 0.00157 \quad s$$

(b) 636.6 samples/s $\times$ 3600 s = 2.29×10^6 samples.

12. **Sklar 2.15**:

(a) Binary case:

$$R = 8000 \text{ samples/s} \times 6 \text{ bits/sample} = 48,000 \text{ bits/s}$$

$$W = \frac{1}{T_b} = R = 48,000 Hz$$

$$\left(\frac{S}{N}\right)_q = 3L^2 = 3(64)^2 = 12,288 \approx 41dB$$

(b) Four-level case:

$$R_s = \frac{48,000 \text{ bits/s}}{2 \text{ bits/symbol}} = 24,000 \text{ symbols/s}$$

$$W = \frac{1}{T} = R_s = 24,000 Hz$$

$$\left(\frac{S}{N}\right)_q = \text{the same as in the binary case} \approx 41dB$$

13. **Sklar 2.16**:

(a) Assuming that the L quantization levels are equally spaced and symmetrical about zero. Then, the maximum possible quantization noise voltage equals 1/2 the q volt interval between any two neighboring levels. Also, the peak quantization noise power, N_q, can be expressed as $(q/2)^2$.

The peak signal power, S, can be designated $(V_{pp}/2)^2$, where $V_{pp} = V_p - (-V_p)$ is the peak-to-peak signal voltage, and V_p is the peak voltage.

since there are L quantization levels and $(L-1)$ intervals (each interval corresponding to q volts), we can write:

$$\left(\frac{S}{N_q}\right)_{peak} = \frac{(V_{pp}/2)^2}{(q/2)^2} = \frac{[q(L-1)/2]^2}{(q/2)^2} \approx \frac{q^2L^2/4}{q^2/4} = L^2$$

Thus, we need to compute how many levels, L, will yield a $(S/N_q)_{peak} = 96$ dB. We therefore write:

$$96dB = 10\log_{10}(S/N_q)_{peak} = 10\log_{10} L^2 = 20\log_{10} L$$

$$L = 10^{96/20} = 63096 \quad \text{levels}$$

(b) The number of bits that corresponding to 63096 levels is

$$l = \lceil \log_2 L \rceil = \lceil \log_2 63096 \rceil = 16 \quad \text{bits/sample}$$

(c) R=16 bits/sample × 44.1 k samples/s = 705,600 bits/s

3: see course notes.

4: (a) For each sampling interval, there are 24+1=25 pulses. Therefore, each pulse occupies:

$$T = \frac{T_s}{25} = \frac{1}{8000 \times 25} = 5\mu s$$

Therefore, the spacing between successive pulses of the multiplexed signal is 5-1=4 μs.

(b) With Nyquist sampling, each pulse occupies:

$$T = \frac{T_s}{25} = \frac{1}{6400 \times 25} = 6.25\mu s$$

Therefore, the spacing between successive pulses of the multiplexed signal is 6.25-1=5.25 μs.

2. ASSIGNMENT 2

2.1 Assignment 2 Problems

1. **Lathi, 7.2-1**

    (a) Find PSDs for polar, on-off, and bipolar signalling, where $p(t)$ is a full-width rectangular pulse, that is, $p(t) = \text{rect}(t/T_b)$?

    (b) Sketch roughly these PSDs and find their bandwidths. For each case, compare the bandwidth of the case where $p(t)$ is a half-width rectangular pulse.

2. **Lathi, 7.2-2**

    (a) A random binary data sequence 100110... is transmitted using a Manchester (split-phase) line code. Sketch the waveform $y(t)$.

    (b) Derive $S_y(\omega)$, the PSD of a Manchester (split-phase) signal in part (a) assuming 1 and 0 equally likely. Roughly sketch this PSD and find its bandwidth.

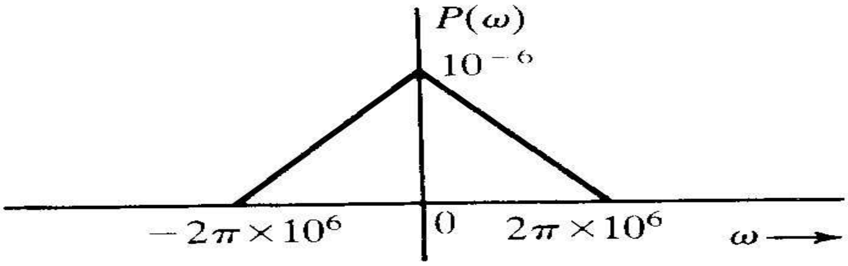
3. **Lathi, 7.2-3**
    Derive the PSD for a binary signal using differential code with half-width rectangular pulses. Determine the PSD $S_y(\omega)$.

4. **Lathi, 7.3-2**
    In a certain telemetry system, there are eight analog measurements, each of bandwidth 2 kHz. Samples of these signals are time-division multiplexed, quantized, and binary coded. The error in sample amplitudes cannot be greater than 1% of the peak amplitude.

    (a) Determine L, the number of quantization levels.

    (b) Find the transmission bandwidth B_T if Nyquist criterion pulses with roll-off factor $r = 0.2$ are used. The sampling rate must be at least 25% above the Nyquist rate.

5. **Lathi, 7.3-4**
    The Fourier transform $P(\omega)$ of the basic pulse $p(t)$ used in a certain binary communication system is shown below:



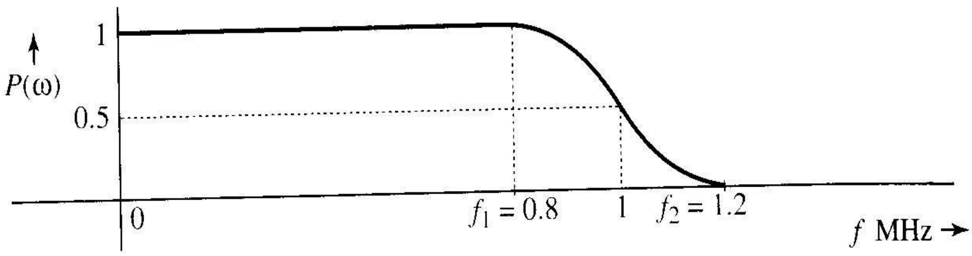
(a) From the shape of $P(\omega)$, explain if this pulse satisfies the Nyquist criterion.

(b) Find $p(t)$ and verify that this pulse does (or does not) satisfy the Nyquist criterion.

(c) If the pulse does satisfy the Nyquist criterion, what is the transmission rate (in bits per second) and what is the roll-off factor?

6. **Lathi, 7.3-5**

A pulse $p(t)$ whose spectrum $P(\omega)$ is shown below satisfies the Nyquist criterion. If $f_1 = 0.8MHz$ and $f_2 = 1.2MHz$, determine the maximum rate at which binary data can be transmitted by this pulse using the Nyquist criterion. What is the roll-off factor?



7. **Lathi, 7.3-6**

Binary data at a rate of 1 Mbits/s is to be transmitted using Nyquist criterion pulses with $P(\omega)$ shown in 7.3-5. The frequencies f_1 and f_2 (in hertz) of this spectrum are adjustable. The channel available for the transmission of this data has a bandwidth of 700 kHz. Determine f_1 and f_2 and the roll-off factor.

8. **Sklar, 3.8**

(a) What is the theoretical minimum system bandwidth needed for a 10-Mbits/s signal using 16-level PAM without ISI?

(b) How large can the filter roll-off factor be if the allowable system bandwidth is 1.375 MHz?

9. **Sklar, 3.10**

   Binary data at 9600 bits/s are transmitted using 8-ary PAM modulation with a system using a raised cosine roll-off filter characteristic. The system has a frequency response out to 2.4 kHz.

   (a) What is the symbol rate?

   (b) What is the roll-off factor of the filter characteristic?

10. **Sklar, 3.11**

    A voice signal in the range 300 to 3300 Hz is sampled at 8000 samples/s. We may transmit these samples directly as PAM pulses or we may first convert each sample to a PCM format and use binary (PCM) waveforms for transmission.

    (a) What is the minimum system bandwidth required for the detection of PAM with no ISI and with a filter roll-off characteristic of $r = 1$?

    (b) Using the same filter roll-off characteristic, what is the minimum bandwidth required for the detection of binary (PCM) waveforms if the samples are quantized to eight levels?

    (c) Repeat part (b) using 128 quantization levels.

2.2 Assignment 2 Solutions

7.2-1 For full width rect pulse $p(t) = \text{rect}\left(\frac{t}{T_b}\right)$

$$P(\omega) = T_b \,\text{sinc}\left(\frac{\omega T_b}{2}\right)$$

For polar signaling [see Eq. (7.12)]

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} = T_b \,\text{sinc}^2\left(\frac{\omega T_b}{2}\right)$$

For on-off case [see Eq. (7.18b)]

$$S_y(\omega) = \frac{|P(\omega)|^2}{4T_b}\left[1 + \frac{2\pi}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi n}{T_b}\right)\right]$$

$$= \frac{T_b}{4}\,\text{sinc}^2\left(\frac{\omega T_b}{2}\right)\left[1 + \frac{2\pi}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi n}{T_b}\right)\right]$$

But $\text{sinc}^2\left(\frac{\omega T_b}{2}\right) = 0$ for $\omega = \frac{2\pi n}{T_b}$ for all $n \neq 0$, and $= 1$ for $n = 0$. Hence,

$$S_y(\omega) = \frac{T_b}{4}\text{sinc}^2\left(\frac{\omega T_b}{2}\right) + \frac{\pi}{2}\delta(\omega)$$

For bipolar case [Eq. (7.20b)]

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b}\sin^2\left(\frac{\omega T_b}{2}\right)$$

$$= T_b\,\text{sinc}^2\left(\frac{\omega T_b}{2}\right)\sin^2\left(\frac{\omega T_b}{2}\right)$$

The PSDs of the three cases are shown in Fig. S7.2-1. From these spectra, we find the bandwidths for all three cases to be R_b Hz.

The bandwidths for the three cases, when half-width pulses are used, are as follows:

Polar and on-off: $2R_b$ Hz; bipolar: R_b Hz.

Clearly, for polar and on-off cases the bandwidth is halved when full-width pulses are used. However, fo the bipolar case, the bandwidth remains unchanged. The pulse shape has only a minor influence in the bipolar case because the term $\sin^2\left(\frac{\omega T_b}{2}\right)$ in $S_y(\omega)$ determines its bandwidth.

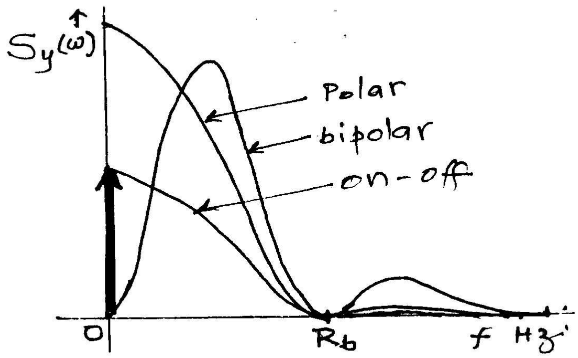


Fig. S7.2-1

.2-2

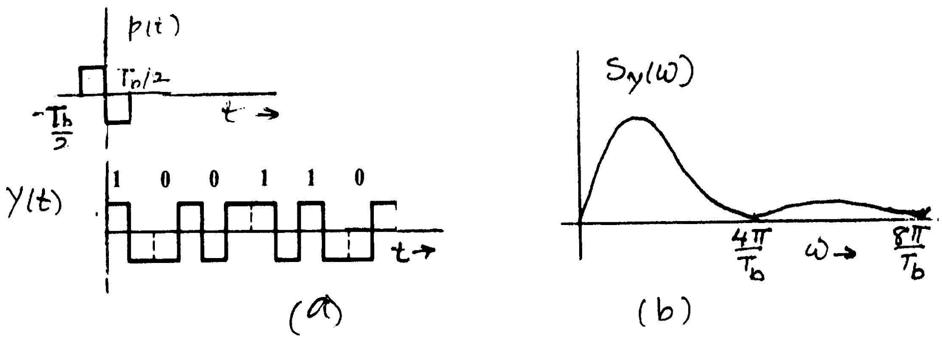


Fig. S7.2-2

$$P(t) = \text{rect}\left(\frac{t + \frac{T_b}{4}}{\frac{T_b}{2}}\right) - \text{rect}\left(\frac{t - \frac{T_b}{4}}{\frac{T_b}{2}}\right)$$

and

$$P(\omega) = \frac{T_b}{2}\text{sinc}\left(\frac{\omega T_b}{4}\right)e^{j\omega T_b/4} + \frac{T_b}{2}\text{sinc}\left(\frac{\omega T_b}{4}\right)e^{-j\omega T_b/4}$$

$$= jT_b\,\text{sinc}\left(\frac{\omega T_b}{4}\right)\sin\left(\frac{\omega T_b}{4}\right)$$

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} = T_b\,\text{sinc}^2\left(\frac{\omega T_b}{4}\right)\sin^2\left(\frac{\omega T_b}{4}\right)$$

From Fig. S7.2-2, it is clear that the bandwidth is $\frac{4\pi}{T_b}$ rad/s or $2R_b$ Hz.

7.2-3 For differential code (Fig. 7.17)

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1)^2 + \frac{N}{2}(-1)^2\right] = 1$$

To compute R_1, we observe that there are four possible 2-bit sequences **11**, **00**, **01**, and **10**, which are equally likely. The product $a_k a_{k+1}$ for the first two combinations is 1 and is −1 for the last two combinations. Hence,

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right] = 0$$

Similarly, we can show that $R_n = 0 \qquad n > 1$ Hence,

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} = \left(\frac{T_b}{4}\right)\text{sinc}^2\left(\frac{\omega T_b}{4}\right)$$

7.3-2 Quantization error $\frac{\Delta V}{2} = \frac{m_p}{L} \leq 0.01 m_p \Rightarrow L \geq 100$

(a) Because L is a power of 2, we select $L = 128 = 2^7$

(b) This requires 7 bit code per sample. Nyquist rate $= 2 \times 2000 = 4$ kHz for each signal. The sampling rate $f_s = 1.25 \times 4000 = 5$ kHZ.

Eight signals require $8 \times 5000 = 40{,}000$ samples/sec.

Bit rate $= 40{,}000 \times 7 = 280$ kbits/s. From Eq. (7.32)

$$B_T = \frac{(1+r)R_b}{2} = \frac{1.2 \times 280 \times 10^3}{2} = 168 \text{ kHz.}$$

7.3-3 **(a)** $B_T = 2R_b \Rightarrow R_b = 1.5$ kbits/s.

(b) $B_T = R_b \Rightarrow R_b = 3$ kbits/s.

(c) $B_T = \frac{1+r}{2} R_b$. Hence, $3000 = \frac{1.25}{2} R_b \Rightarrow R_b = 4.8$ kbits/s.

(d) $B_T = R_b \Rightarrow R_b = 3$ kbits/s.

(e) $B_T = R_b \Rightarrow R_b = 3$ kbits/s.

7.3-4 **(a)** Comparison of $P(\omega)$ with that in Fig. 7.12 shows that this $P(\omega)$ does satisfy the Nyquist criterion with $\omega_b = 2\pi \times 10^6$ and $r = 1$. The excess bandwidth $\omega_x = \pi \times 10^6$.

(b) From Table 3.1, we find

$$p(t) = \text{sinc}^2\left(\pi \times 10^6 t\right)$$

From part (a), we have $\omega_b = 2\pi \times 10^6$ and $R_b = 10^6$. Hence, $T_b = 10^{-6}$. Observe that

$$p(n\,T_b) = 1 \qquad n = 0$$
$$= 0 \qquad n \neq 0$$

Hence $P(t)$ satisfies Eq. (7.36).

(c) the pulse transmission rate is $\frac{1}{T_b} = R_b = 10^6$ bits/s.

7.3-5 In this case $\frac{R_b}{2} = 1$ MHz. Hence, we can transmit data at a rate $R_b = 2$ MHz.

Also, $B_T = 1.2$ MHz. Hence, from Eq. (7.32)

$$1.2 \times 10^6 = \frac{1+r}{2}\left(2 \times 10^6\right) \Rightarrow r = 0.2$$

7.3-6 $f_2 = 700$ kHz. Also, $\frac{R_b}{2} = 500$ kHz and $f_x = 700 - 500 = 200$ kHz.

Hence, $r = \frac{f_x}{R_b/2} = 0.4$ and $f_1 = \frac{R_b}{2} - f_x = 500 - 200 = 300$ kHz.

7.3-7 To obtain the inverse transform of $P(\omega)$, we derive the dual of Eq. (3.35) as follows:

$$g(t-T) \Leftrightarrow G(\omega)e^{-jT\omega} \text{ and } g(t+T) \Leftrightarrow G(\omega)e^{jT\omega}$$

Hence,

$$g(t+T) + g(t-T) \Leftrightarrow 2G(\omega)\cos T\omega \qquad (1)$$

Now, $P(\omega)$ in Eq. (7.34a) can be expressed as

$$P(\omega) = \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi R_b}\right) + \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi R_b}\right)\cos\left(\frac{\omega}{2R_b}\right) \qquad (2)$$

3.8. (Sklar)

(a) 16 level PAM signal ⇒ $2^K = 16$ ⇒ $K = 4$ bits/symbol.

Therefore the symbol rate is

$$R_s = \frac{R}{\log_2 L} = \frac{10 \text{ Mbits/s}}{4 \text{ bits/symbol}} = 2.5 \text{ M symbols/s}$$

Theoretically, the minimum required bandwidth is

$$BW = \frac{R_s}{2} = 1.25 \text{ MHz}.$$

(b) Since $W = \frac{1}{2}(1+\alpha) R_s$

$\therefore$ $1.375 \text{ MHz} = (1+\alpha) \cdot 1.25 \text{ MHz}$

$\therefore$ $\alpha = 0.1$

3.10

(a) For 8-ary PAM each symbol represent $\log_2 8 = 3$ bits.

The symbol rate for 9600 bits/s data flow is.

$$R_s = \frac{9600 \text{ bits/s}}{3 \text{ bits/symbol}} = 3200 \text{ symbol/s}.$$

(2) $W = 2400$ Hz, $R_s = 3200$

$W = \frac{1}{2}(1+\alpha) \cdot R_s$

$2400 = \frac{1}{2}(1+\alpha) \cdot 3200$ ⇒ $\alpha = 0.5$

(Sklar)

3.11 Voice signal ranges 300 - 3300 Hz, $f_s = 8000$ samples/s. P

(a) PAM transmission

$W = \frac{1}{2}(1+\alpha)\cdot R_s$ (where R_s = 8000 pulses/s)

$= \frac{1}{2}(1+1)\cdot 8000$

$= 8$ kHz.

(b) PCM transmission – using 8-level quantization, the bit rate:

$R_b = 8000 \times \log_2 8 = 24$ K bits/s

required bandwidth for ISI free is

$W = \frac{1}{2}(1+\alpha)\cdot R_b$

$= \frac{1}{2}(1+1)\cdot 24\text{ k} = 24$ kHz.

(c) PCM transmission – using 128-level quantization, the bit rate:

$R_b = 8000 \times \log_2 128 = 56$ kbits/s

required bandwidth:

$W = \frac{1}{2}(1+\alpha)\ R_b$

$= \frac{1}{2}(1+1)\cdot 56\text{ k} = 56$ kHz.

3. ASSIGNMENT 3

3.1 Assignment 3 Problems

1. A company has three machines B_1, B_2, and B_3 for making resistors. It has been observed that 80% of resistors produced by B_1 are qualified. The percentage for machines B_2 and B_3 are respectively, 90% and 60%. Each hour, machines B_1, B_2 and B_3 produce 3000, 4000, and 3000 resistors, respectively. All of the resistors are mixed together at random in one bin and packed for shipment.

   (a) What is the probability that the company ships a resistor that is qualified?

   (b) What is the probability that an acceptable resistor comes from machine B_3?

2. **Lathi, 10.1-15**
   A binary source generates digits **1** and **0** randomly with probabilities $P(1) = 0.8$ and $P(0) = 0.2$.

   (a) What is the probability that two 1's and three 0's will occur in a five-digit sequence?

   (b) What is the probability that at least three 1's will occur in a five-digit sequence?

3. **Lathi, 10.1-16**
   In a binary communication channel, the receiver detects binary pulses with an error probability P_e. What is the probability that out of 100 received digits, no more than three digits are in error?

4. **Lathi, 10.2-1**
   For a certain nonsymmetric channel it is given that

$$P_{y|x}(0|1) = 0.1 \quad \text{and} \quad P_{y|x}(1|0) = 0.2$$

   where x is the transmitted digit and y is the received digit. If $P_x(0) = 0.4$, determine $P_y(0)$ and $P_y(1)$.

5. **Lathi, 10.2-5**

   The PDF of a Gaussian RV X is given by

$$f_X(x) = \frac{1}{3\sqrt{2\pi}} e^{-(x-4)^2/18}$$

   Determine (a) $P(X \geq 4)$; (b) $P(X \geq 0)$; (c) $P(X \geq -2)$;

6. **Lathi, 13.5-2**

   Binary data is transmitted by using a pulse $p(t)$ for 0 and a pulse $3p(t)$ for 1. Show that the optimum receiver for this case is a filter matched to $p(t)$ with a detection threshold of $2E_p$. Determine the error probability P_b of this receiver as a function of E_b/N_0 if 0 and 1 are equiprobable.

7. **Sklar 1.6**

   Determine which, if any, of the following functions have the properties of autocorrelation functions. Justify your determinations. [Note: Fourier transform of $R(\tau)$ must be a nonnegative function, why?]

   (a) $x(\tau) = \begin{cases} 1 & \text{for } -1 \leq \tau \leq 1 \\ 0 & \text{otherwise} \end{cases}$

   (b) $x(\tau) = \delta(\tau) + \sin 2\pi f_o \tau$

   (c) $x(\tau) = \exp(|\tau|)$

   (d) $x(\tau) = 1 - \tau \quad \text{for } -1 \leq \tau \leq 0, 0 \text{ elsewhere}$

8. **Sklar 1.7**

   Determine which, if any, of the following functions have the properties of power spectral density functions. Justify your determination.

   (a) $X(f) = \delta(f) + \cos^2(2\pi f)$

   (b) $X(f) = 10 + \delta(f - 10)$

   (c) $X(f) = \exp(-2\pi|f - 10|)$

   (d) $X(f) = \exp[-2\pi(f^2 - 10)]$

9. **Sklar, 3.4**

   Assuming that in a binary digital communication system, the signal component out of the correlator receiver is $a_i(T) = +1$ or -1 V with equal probability. If the Gaussian noise at the correlator output has unit variance, find the probability of a bit error.

10. **Sklar, 3.5**

A bipolar binary signal, $s_i(t)$, is a +1 or -1 V pulse during the interval $(0, T)$. Additive white Gaussian noise having two-sided power spectral density of 10^{-3} W/Hz is added to the signal. If the received signal is detected with a matched filter, determine the maximum bit rate that can be sent with a bit error probability of $P_b \leq 10^{-3}$.

11. **Sklar, 3.7**

A binary communication system transmits signals $s_i(t), i = 1, 2$. The receiver test statistic $z(T) = a_i + n_0$, where the signal component a_i is either $a_1 = +1$ or $a_2 = -1$ and the noise component n_0 is uniformly distributed, yielding the conditional density functions $p(z|s_i)$ given by

$$p(z|s_1) = \begin{cases} \frac{1}{2} & -0.2 \leq z \leq 1.8 \\ 0 & \text{otherwise} \end{cases}$$

and

$$p(z|s_2) = \begin{cases} \frac{1}{2} & -1.8 \leq z \leq 0.2 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability of a bit error, P_b, for the case of equally likely signaling and the use of an optimum decision threshold.

12. **Sklar, 3.14**

Consider that NRZ binary pulses are transmitted along a cable that attenuates signal power by 3 dB (from transmitter to receiver). The pulses are coherently detected at the receiver, and the data rate is 56 kbit/s. Assume Gaussian noise with $N_0 = 10^{-6}$ Watt/Hz. What is the minimum amount of power needed at the transmitter in order to maintain a bit-error probability of $P_b = 10^{-3}$?

13. The purpose of a *radar system* is basically to detect the presence of a target, and to extract useful information about the target. Suppose that in such a system, hypothesis H_0 is that there is no target present, so that the received signal $x(t) = w(t)$, where $w(t)$ is white Gaussian noise with power spectral density $N_0/2$. For hypothesis H_1, a target is present, and $x(t) = w(t) + s(t)$, where $s(t)$ is an echo produced by the target. Assumed that $s(t)$ is completely known and the probability of the existence of a target is 0.5.

    (a) Determine the structure of the optimal receiver.

    (b) Determine the pdf of the decision variable and the optimal decision threshold.

    (c) Evaluate the *probability of false alarm* defined as the probability that the receiver decides a target is present when it is not.

(d) Evaluate the *probability of detection* defined as the probability that the receiver decides a target is present when it is.

14. Two equiprobable messages are transmitted on an AWGN channel with two-sided power spectral density $N_0/2$. The signals are of the form

$$s_1(t) = \sqrt{E}\phi_1(t), \qquad s_2(t) = a\phi_1(t) + \sqrt{E - a^2}\,\phi_2(t)$$

where $-\sqrt{E} \leq a \leq \sqrt{E}$ and $\int_0^T \phi_1(t)\phi_2(t)\,dt = 0$.

(a) Determine the structure of the optimal receiver.

(b) Determine the probability of error of this binary system.

3.2 Assignment 3 Solutions

Problem 1. a: Let A={qualified resistor}, then we can have

$$P[A|B_1] = 0.8 \qquad P[A|B_2] = 0.9 \qquad P[A|B_3] = 0.6 \tag{3.1}$$

The production figures states that 3000+4000+3000=10,000 resistors per hour are produced. The fraction from machine B_1 is $P[B_1] = 3000/10000 = 0.3$. Similarly, $P[B_2] = 0.4$ and $P[B_3] = 0.3$. Applying the law of total probability we have

$$\begin{aligned} P[A] &= P[A|B_1]P[B_1] + P[A|B_2]P[B_2] + P[A|B_3]P[B_3] & (3.2) \\ &= (0.8)(0.3) + (0.9)(0.4) + (0.6)(0.3) = 0.78 & (3.3) \end{aligned}$$

b: applying Bayes' Theorem, we have

$$P[B_3|A] = \frac{P[AB_3]}{P[A]} = \frac{P[A|B_3]P[B_3]}{P[A]} = \frac{0.6 \times 0.3}{0.78} = 0.23. \tag{3.4}$$

Lathi, 10.1-15: A binary source generates digits **1** and **0** randomly with probabilities $P(1) = 0.8$ and $P(0) = 0.2$.

1. What is the probability that two 1's and three 0's will occur in a five-digit sequence?
2. What is the probability that at least three 1's will occur in a five-digit sequence?

Solution: (a) Two 1's and three 0's in a sequence of 5 digits can occur in $\begin{pmatrix} 5 \\ 2 \end{pmatrix} = 10$ ways. The probability one such sequence is

$$P = (0.8)^2 \cdot (0.2)^3 = 0.00512$$

since the event can occur in 10 ways, its probability is

$$10 \times 0.00512 = 0.0512$$

(b) Three 1's occur with probability $\begin{pmatrix} 5 \\ 3 \end{pmatrix} (0.8)^3 \cdot (0.2)^2 = 0.2048$;

Four 1's occur with probability $\begin{pmatrix} 5 \\ 4 \end{pmatrix} (0.8)^4 \cdot (0.2)^1 = 0.4096$;

Five 1's occur with probability $\begin{pmatrix} 5 \\ 5 \end{pmatrix} (0.8)^5 \cdot (0.2)^0 = 0.3277$;

Hence, the probability of at least three 1's occurring is

$$P = 0.2048 + 0.4096 + 0.3277 = 0.9421$$

Lathi, 10.1-16: In a binary communication channel, the receiver detects binary pulses with an error probability P_e. What is the probability that out of 100 received digits, no more than three digits are in error?

Solution: Prob(no more than 3 error) = P(no error)+P(1 error) +P(2 error) +P(3 error), which is

$$\begin{aligned} P &= (1-P_e)^{100} + \begin{pmatrix} 100 \\ 1 \end{pmatrix} P_e(1-P_e)^{99} + \begin{pmatrix} 100 \\ 2 \end{pmatrix} P_e^2(1-P_e)^{98} \begin{pmatrix} 100 \\ 3 \end{pmatrix} P_e^3(1-P_e)^{97} \quad (3.5) \\ &= (1-100P_e) + 100P_e(1-99P_e) + 4950P_e^2(1-98P_e) + 161700P_e^3(1-97P_e) \end{aligned}$$

Lathi, 10.2-1: Solution: Based on law of total probability,

$$P_y(0) = P_{x,y}(1,0) + P_{x,y}(0,0) = P_x(1)P_{y|x}(0|1) + P_x(0)P_{y|x}(0|0)$$

which is

$$P_y(0) = 0.6 \times 0.1 + 0.4[1 - P_{y|x}(1|0)] = 0.06 + 0.32 = 0.38$$

we can have,

$$P_y(1) = 1 - P_y(0) = 0.62$$

Lathi, 10.2-5: X is Gaussian with mean $\mu = 4$ and $\sigma_x = 3$, therefore,

1.

$$P(x \geq 4) = Q\left(\frac{4-\mu}{\sigma}\right) = Q\left(\frac{4-4}{3}\right) = Q(0) = 0.5$$

2.

$$P(x \geq 0) = Q\left(\frac{0-\mu}{\sigma}\right) = Q\left(\frac{0-4}{3}\right) = Q(-4/3) = 1 - Q(4/3) = 1 - 0.09176 = 0.9083$$

3.

$$P(x \geq -2) = Q\left(\frac{-2-\mu}{\sigma}\right) = Q\left(\frac{-2-4}{3}\right) = Q(-2) = 1 - Q(2) = 0.9773$$

Lathi, 13.5-2: The conditional probability density functions of the receiver decision RV are

$$f_{Y(t)|\text{“1”}} \sim N\left(3E, \frac{N_0}{2}E\right)$$

$$f_{Y(t)|\text{“0”}} \sim N\left(E, \frac{N_0}{2}E\right)$$

Therefore, the optimal decision threshold is $2E$. The probability of transmission error is

$$P_b = Q\left(\frac{a_2 - a_1}{2\sigma}\right) = Q\left(\frac{3E - E}{2\sqrt{E\,N_0/2}}\right) = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

Sklar, 1.6: Fourier transform of $R(\tau)$ must be a nonnegative function because $F[R(\tau)] = S(f)$; and the power spectral density, $S(f)$, must be a nonnegative function.

(a) It satisfies $x(\tau) = x(-\tau)$, $x(0) \geq x(\tau)$, but the Fourier transform of $x(\tau)$ is a positive and negative going function. Therefore, $x(\tau)$ cannot be a valid autocorrelation function.

(b) Since $x(\tau) \neq x(-\tau)$, therefore, $x(\tau)$ cannot be a valid autocorrelation function.

(c) It satisfies $x(\tau) = x(-\tau)$, but it doesn't satisfy $x(0) \geq x(\tau)$. Therefore, not a valid autocorrelation function.

(d) It satisfies $x(\tau) = x(-\tau)$, $x(0) \geq x(\tau)$, and Fourier transform of $x(\tau)$ is $2sinc^2 f\tau$, which is a non-negative function. Therefore, $x(\tau)$ is a valid autocorrelation function.

Sklar, 1.7:

(a) $X(f) = \delta(f) + \cos^2 2\pi f$. Yes, it can be a PSD function since (i) it is always real; (ii) $P_X(f) \geq 0$; (iii) $P_X(-f) = P_X(f)$.

(b) $X(f) = 10 + \delta(f - 10)$. No, it cannot be a PSD function. It satisfies the first two conditions (i) it is always real; (ii) $P_X(f) \geq 0$; but the third condition (iii) $P_X(-f) \neq P_X(f)$.

(c) $X(f) = \exp(-2\pi|f - 10|)$. No, it cannot be a PSD function. It satisfies the first two conditions (i) it is always real; (ii) $P_X(f) \geq 0$; but the third condition (iii) $P_X(-f) \neq P_X(f)$.

(d) $X(f) = \exp[-2\pi(f^2 - 10)]$. Yes, it can be a PSD function since (i) it is always real; (ii) $P_X(f) \geq 0$; (iii) $P_X(-f) = P_X(f)$.

Sklar, 3-4: Using equation

$$P_b = Q\left(\frac{a_2 - a_1}{2\sigma}\right) = Q\left(\frac{1 - (-1)}{2}\right) = Q(1) = 0.1587$$

Sklar, 3-5: Using equation:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

where $E_b = A^2 T$, and $A = 1$ for bipolar signalling. Since

$$P_b = Q(x) \leq 10^{-3}$$

we need $x = \sqrt{\frac{2E_b}{N_0}} \geq 3.09$. which implies $E_b/N_0 \geq 4.77$; while $N_0/2$ is given as 10^{-3}, hence,

$$E_b = T \geq 4.77 \times 2 \times 10^{-3}$$

and therefore,

$$R = \frac{1}{T} \leq 104.8 \quad bits/s$$

Sklar, 3-7: The optimal decision threshold is 0, therefore,

$$\begin{aligned} P_b &= P(z > 0|\text{“0”})P(\text{“0”}) + P(z < 0|\text{“0”})P(\text{“0”}) && (3.6) \\ &= P(s_1)\int_{-0.2}^{0} \frac{1}{2}\,dz + P(s_2)\int_{0}^{0.2} \frac{1}{2}\,dz = \left[\frac{1}{2}z\right]|_{-0.2}^{0} = \frac{0.2}{2} = 0.1 && (3.7) \end{aligned}$$

Sklar, 3-14: Signalling with NRZ pulses represents an example of antipodal signalling. Therefore, we have

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

since $Q(3.1) \approx 10^{-3}$, hence

$$\sqrt{\frac{2A^2(1/56000)}{10^{-6}}} = 3.1$$

we can solve that $A^2 = 0.268$. Thus if there were no signal power loss, the minimum power needed would be approximately 268 mW. With a 3-dB loss, 536 mW are needed.

Problem 12: (a) the structure of the optimal receiver is a matched filter (diagram refer lecture notes).

(b) Let T denote the pulse width of $s(t)$. The energy of $s(t)$ is

$$E = \int_0^T |s(t)|^2 dt$$

Then the conditional density of the decision variable $y(T)$ is

$$y_i(T) \sim N(m_1, \sigma^2) \quad \text{when there is a target}$$

$$y_2(T) \sim N(m_2, \sigma^2) \quad \text{when no target}$$

where m_1 is the pulse energy E and m_2 is zero because no target and $\sigma^2 = \frac{N_0}{2}E$. Decision threshold, $U = E/2$. The decision rule is:

$$\begin{cases} u \geq \alpha & \text{a target is present} \\ u < \alpha & \text{no target} \end{cases} \qquad (3.8)$$

given

$$\begin{cases} \text{target present} & \sim N(E, \sigma^2) \\ \text{no target} & \sim N(0, \sigma^2) \end{cases} \qquad (3.9)$$

where $\sigma^2 = \frac{N_0 E}{2}$.

(c) Probability of false alarm is

$$\begin{aligned} P\{\text{false alarm}\} &= P\{\text{target detected} \mid \text{no target}\} = P[y(T) > U| \text{ no target}] \quad (3.10) \\ &= P[y_2(T) > U] = Q\left(\frac{U}{\sigma}\right) = Q\left(\frac{E/2}{\sqrt{\frac{N_0 E}{2}}}\right) = Q\left(\sqrt{\frac{E}{2N_0}}\right) \end{aligned}$$

(d) Probability of detection is

$$\begin{aligned} P\{\text{detection}\} &= P\{\text{target detected} \mid \text{target present}\} \quad (3.11) \\ &= P[y_1(T) > U|\text{target}] = P[y_1(T) > U] = 1 - Q\left(\sqrt{\frac{E}{2N_0}}\right) \end{aligned}$$

Problem 13 see lecture notes.

4. ASSIGNMENT 4

4.1 Assignment 4 Problems

1. **Sklar, 4.7**

   Find the probability of bit error, P_B, for the coherent matched filter detection of the equally likely binary FSK signals

   $$s_1(t) = 0.5 \cos 2000\pi t$$

   and

   $$s_2(t) = 0.5 \cos 2020\pi t$$

   where the two-sided AWGN power spectral density is $N_0/2 = 0.0001$. Assume that the symbol duration is $T = 0.01$ s.

2. **Sklar, 4.8**

   Find the optimum (minimum probability of error) threshold γ_0, for detecting the equally likely signal $s_1(t) = \sqrt{2E/T} \cos \omega_0 t$ and $s_2(t) = \sqrt{E/2T} \cos(\omega_0 t + \pi)$ in AWGN, using a correlator receiver. Assume a reference signal of $\psi_1(t) = \sqrt{2/T} \cos \omega_0 t$

3. **Sklar, 4.9**

   A system using matched filter detection of equally likely BPSK signal, $s_1(t)) = \sqrt{2E/T} \cos \omega_0 t$ and $s_2(t)) = \sqrt{2E/T} \cos(\omega_0 t + \pi)$, operates in AWGN with a received E_b/N_0 of 6.8 dB. Assume that $E[z(T)] = \pm\sqrt{E}$.

   (a) Find the minimum probability of bit error, P_B, for this signal set and E_b/N_0.

   (b) If the decision threshold is $\gamma = 0.1\sqrt{E}$, find P_B.

4. **Sklar, 4.13**

   Consider a coherent orthogonal MFSK system with $M = 8$ having the equally likely waveforms $s_i(t) = A \cos 2\pi f_i t, i = 1, \cdots, M, 0 \leq t \leq T$, where $T = 0.2$ ms. The received carrier amplitude, A, is 1 mV, and the two-sided AWGN spectral density, $N_0/2$, is 10^{-11} W/Hz. Calculate the probability of bit error, P_B.

5. **Sklar, 4.14**

   A bit error probability of $P_B = 10^{-3}$ is required for a system with a data rate of 100 kbits/s to be transmitted over an AWGN channel using coherently detected MPSK modulation. The system bandwidth is 50 kHz. Assume that the system frequency transfer function is a raised cosine with a roll-off characteristic of $r = 1$ and that a Gray code is used for the symbol to bit assignment.

   (a) What E_s/N_0 is required for the specified P_B?

   (b) What E_b/N_0 is required?

6. **3.1** Determine whether or not $s_1(t)$ and $s_1(t)$ are orthogonal over the interval $(-1.5T_2 < t < 1.5T_2)$, where $s_1(t) = \cos(2\pi f_1 t + \phi_1)$ and $f_2 = 1/T_2$ for the following cases

   (a) $f_1 = f_2$ and $\phi_1 = \phi_2$

   (b) $f_1 = \frac{1}{3} f_2$ and $\phi_1 = \phi_2$

   (c) $f_1 = 2f_2$ and $\phi_1 = \phi_2$

   (d) $f_1 = \pi f_2$ and $\phi_1 = \phi_2$

   (e) $f_1 = f_2$ and $\phi_1 = \phi_2 + \pi/2$

   (f) $f_1 = f_2$ and $\phi_1 = \phi_2 + \pi$

7. **3.2**

   (a) Show that the three functions illustrated in Figure P3.1 are pairwise orthogonal over the interval (-2,2).

   (b) Determine the value of the constant A, that makes the set of functions in part (a) an orthonormal set.

   (c) Express the following waveform, $x(t)$, in terms of the orthonormal set of part (b)

$$x(t) = \begin{cases} 1 & \text{for } 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

8. **3.3** Consider the functions

$$\psi_1(t) = \exp(-|t|) \qquad \psi_2(t) = 1 - A\exp(-2|t|)$$

   Determine the constant, A, such that $\psi_1(t)$ and $\psi_2(t)$ are orthogonal over the interval $(-\infty, \infty)$.

4.2 Assignment 4 Solutions

Sklar, 3.1: (a) $f_1 = f_2$ and $\phi_1 = \phi_2$

$$\int_{-1.5T_2}^{1.5T_2} s_1(t) \cdot s_2(t)dt = \int_{-1.5T_2}^{1.5T_2} s_1^2(t)dt \neq 0$$

therefore, not orthogonal.

(b) $f_1 = \frac{1}{3}f_2$ and $\phi_1 = \phi_2$
Let $\phi_1 = \phi_2 = 0$,

$$\begin{aligned} \int_{-1.5T_2}^{1.5T_2} s_1(t) \cdot s_2(t)dt &= \frac{1}{2}\int_{-1.5T_2}^{1.5T_2} \cos 2\pi \left(\frac{2}{3}f_2\right) tdt + \frac{1}{2}\int_{-1.5T_2}^{1.5T_2} \cos 2\pi \left(\frac{4}{3}f_2\right) tdt && (4.1) \\ &= \frac{\sin 2\pi}{4/3\pi(1/T_2)} + \frac{\sin 4\pi}{8/3\pi(1/T_2)} = 0 && (4.2) \end{aligned}$$

therefore, orthogonal.

(c) $f_1 = 2f_2$ and $\phi_1 = \phi_2$ Let $\phi_1 = \phi_2 = 0$,

$$\int_{-1.5T_2}^{1.5T_2} s_1(t) \cdot s_2(t)dt = \frac{1}{2}\int_{-1.5T_2}^{1.5T_2} (\cos 2\pi f_2 t + \cos 6\pi f_2 t)dt = 0$$

therefore, orthogonal.

(d) $f_1 = \pi f_2$ and $\phi_1 = \phi_2$ Let $\phi_1 = \phi_2 = 0$,

$$\int_{-1.5T_2}^{1.5T_2} s_1(t) \cdot s_2(t)dt = \frac{1}{2}\int_a^b \cos(\pi - 1)2\pi f_2 tdt + \frac{1}{2}\int_a^b \cos(\pi + 1)2\pi f_2 tdt \neq 0$$

therefore, not orthogonal.

(e) $f_1 = f_2$ and $\phi_1 = \phi_2 + \pi/2$

$$\int_a^b \sin 2\pi f_2 t \cdot \cos 2\pi f_2 tdt = 0$$

therefore, orthogonal.

(f) $f_1 = f_2$ and $\phi_1 = \phi_2 + \pi$ Let $\phi_1 = 0$,

$$\int_a^b \cos^2(2\pi f_2 t)\, dt \neq 0$$

not orthogonal.

Sklar, 3.2 (a):

$$\begin{aligned} \int_{-2}^{2} \psi_1(t)\psi_2(t)dt &= \int_{-2}^{-1} (-A)(-A)dt + \int_{-1}^{0} (A)(-A)dt + \int_{0}^{1} (A)(A)dt + \int_{1}^{2} (-A)(A)dt \\ &= A^2 - A^2 + A^2 - A^2 = 0 \end{aligned}$$

$$\int_{-2}^{2}\psi_1(t)\psi_3(t)dt = \int_{-2}^{-1}(-A)(-A)dt + \int_{-1}^{0}(A)(-A)dt + \int_{0}^{1}(A)(-A)dt + \int_{1}^{2}(-A)(-A)dt \quad (4.3)$$
$$= A^2 - A^2 - A^2 + A^2 = 0$$

$$\int_{-2}^{2}\psi_2(t)\psi_3(t)dt = \int_{-2}^{0}(-A)(-A)dt + \int_{0}^{2}(A)(-A)dt = 2A^2 - 2A^2 = 0$$

(b):

$$\int_{-2}^{2}\psi_3^2(t)dt = \int_{-2}^{0}A^2dt = 2A^2 + 2A^2 = 4A^2$$

To be orthonormal, $4A^2 = 1$ which implies $A = 1/2$.

(c): $x(t) = \psi_2(t) - \psi_3(t)$

Sklar, 3.3: the correlation between $\psi_1(t)$ and $\psi_2(t)$ is

$$\begin{aligned} R &= \int_{-\infty}^{0} e^t(1 - Ae^{2t})\,dt + \int_{0}^{\infty} e^{-t}(1 - Ae^{-2t})\,dt \\ &= \int_{-\infty}^{0}(e^t - Ae^{3t})\,dt + \int_{0}^{\infty}(e^{-t} - Ae^{-3t})\,dt \\ &= \left[e^t - \frac{Ae^{3t}}{3}\right]_{-\infty}^{0} + \left[-e^{-t} + \frac{Ae^{-3t}}{3}\right]_{0}^{\infty} = 1 - \frac{A}{3} - [-1 + \frac{A}{3}] = 2 - \frac{2A}{3} \end{aligned}$$

In order to make $\psi_1(t)$ and $\psi_2(t)$ orthogonal, we need $R = 0$, which solves $A = 3$.

4.7 $E_b = ST = \frac{(0.5)^2}{2}(0.01)$

$= 0.00125$ Joule

$$\rho = \frac{1}{E_b}\int_0^T s_1(t)\, s_2(t)\, dt$$

$$= \frac{1}{E_b}\int_0^T 0.5\cos(2\pi\, 1000t)\; 0.5\cos(2\pi\, 1010t)\, dt$$

$$= \frac{0.25}{0.00125}\int_0^{0.01} \frac{1}{2}\left[\cos 2\pi\, 10t + \cos 2\pi\, 2010t\right] dt$$

$$= 100\left[\frac{\sin 2\pi\, 10t}{20\pi} + \frac{\sin 2\pi\, 2010t}{4020\pi}\right]_0^{0.01}$$

$$\rho = [0.935 + 0.005] = 0.94$$

$$P_B = Q\left(\sqrt{\frac{E_b(1-\rho)}{N_0}}\right) = Q\left(\sqrt{\frac{0.00125\,(0.06)}{0.0002}}\right)$$

$$= Q(0.612)$$

$$P_B = 0.27$$

The error is much greater than if the tone spacing required for coherent orthogonal signaling, $\frac{1}{2T} = 50$ Hz had been used, instead of the 10 Hz specified.

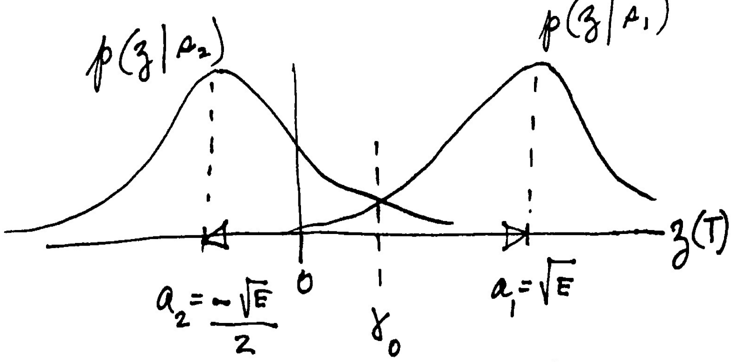
4.8 $s_1(t) = \sqrt{2E/T} \cos \omega_0 t$

$s_2(t) = \sqrt{E/2T} \cos(\omega_0 t + \pi)$

From Equations (4.21) to (4.23), we can characterize these waveforms using the basis function, $\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t$.

$s_1(t) = \sqrt{E} \; \psi_1(t)$

$s_2(t) = -\frac{1}{2}\sqrt{E} \; \psi_1(t)$



$$\gamma_0 = \frac{a_1 + a_2}{2} = \frac{\sqrt{E} + \left(-\frac{1}{2}\sqrt{E}\right)}{2} = \frac{\sqrt{E}}{4}$$

$$\gamma_0 = \sqrt{E/2}$$

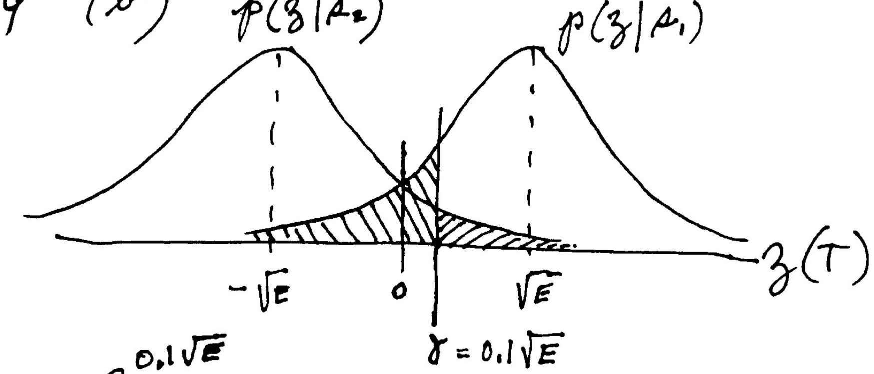
4.9 (a) $P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

$E_b/N_0 = 6.8 \text{ dB} = 4.786$

$P_B = Q\left(\sqrt{2 \times 4.786}\right) = Q(3.09)$

From Table B.1, $P_B = 10^{-3}$

4.9 (b)



$$P_B = \frac{1}{2}\int_{-\infty}^{0.1\sqrt{E}} p(z|s_1)\,dz + \frac{1}{2}\int_{0.1\sqrt{E}}^{\infty} p(z|s_2)\,dz$$

$$= \frac{1}{2}\int_{-\infty}^{0.1\sqrt{E}} \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-\sqrt{E}}{\sigma_0}\right)^2\right] dz$$

$$+ \frac{1}{2}\int_{0.1\sqrt{E}}^{\infty} \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z+\sqrt{E}}{\sigma_0}\right)^2\right] dz$$

Let $u_1 = \dfrac{z-\sqrt{E}}{\sigma_0}$; $\sigma_0\, du_1 = dz$

let $u_2 = \dfrac{z+\sqrt{E}}{\sigma_0}$; $\sigma_0\, du_2 = dz$

$$P_B = \frac{1}{2}\int_{-\infty}^{\frac{-0.9\sqrt{E}}{\sigma_0}} \frac{1}{\sqrt{2\pi}} e^{-\frac{u_1^2}{2}} du_1 + \frac{1}{2}\int_{\frac{1.1\sqrt{E}}{\sigma_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u_2^2}{2}} du_2$$

For the symmetrical Gaussian function,

$$\int_{-\infty}^{-x} e^{-\frac{u^2}{2}} du = \int_{x}^{\infty} e^{-\frac{u^2}{2}} du$$

$$P_B = \frac{1}{2} Q\left(\frac{0.9\sqrt{E}}{\sigma_0}\right) + \frac{1}{2} Q\left(\frac{1.1\sqrt{E}}{\sigma_0}\right)$$

For binary matched filter detection, we can write $E = E_b$ and $\sigma_0^2 = N_0/2$.

$$P_B = \frac{1}{2} Q\left(0.9\sqrt{\frac{2E_b}{N_0}}\right) + \frac{1}{2} Q\left(1.1\sqrt{\frac{2E_b}{N_0}}\right)$$

$$E_b/N_0 = 6.8 \text{ dB} = 4.786$$

$$P_B = \frac{1}{2} Q\left(0.9 \times 3.09\right) + \frac{1}{2} Q\left(1.1 \times 3.09\right)$$

$$P_B = \frac{1}{2} Q\left(2.78\right) + \frac{1}{2} Q\left(3.4\right)$$

$$= \frac{1}{2}\left(0.0027\right) + \frac{1}{2}\left(0.0003\right)$$

$$= 1.4 \times 10^{-3}$$

4.13 $P_E = (M-1)\,Q\left(\sqrt{\frac{E_s}{N_0}}\right)$

$E_s = \frac{A^2}{2}T = \frac{(10^{-3})^2}{2} \times 0.2 \times 10^{-3} = 10^{-10}$

$P_E = (8-1)\,Q\left(\sqrt{\frac{10^{-10}}{2\times 10^{-11}}}\right) = 7Q(2.236)$

Using Table B.1, $P_E = 7 \times 0.0127$
$= 8.89 \times 10^{-2}$

$P_B = \frac{2^{k-1}}{2^k - 1} P_E = \frac{2^2}{2^3-1} P_E = \frac{4}{7} P_E$

$= 5 \times 10^{-2}$

4.14 (a) With roll-off $r = 1$, and no ISI,
$W_{DSB} = (1+r)R_s$

$50\text{ kHz} = 2R_s\,;\quad R_s = 25\text{k symbols/s}$

$k = \log_2 M = \frac{R}{R_s} = \frac{100\text{ kbits/s}}{25\text{ ksymbols/s}} = 4$

$\therefore M = 16$

Since a Gray code is used, $P_B \approx \frac{P_E}{\log_2 M}$

$P_E = (\log_2 M)\,P_B = 4 \times 10^{-3}$

$P_E = 2Q\left[\left(\sqrt{\frac{2E_s}{N_0}}\right)\sin\left(\frac{\pi}{M}\right)\right] = 4 \times 10^{-3}$

$Q(x) = 2 \times 10^{-3}$

From Table B.1, $x = 2.88$

$$\left(\sqrt{\frac{2E_s}{N_0}}\right) \sin\left(\frac{\pi}{M}\right) = 2.88$$

$$\sqrt{\frac{2E_s}{N_0}} = \frac{2.88}{\sin\left(\frac{\pi}{16}\right)} = \frac{2.88}{0.19509} = 14.76$$

$$\frac{E_s}{N_0} = 108.9 = 20.4\,\text{dB}$$

(b) $$\frac{E_b}{N_0} = \frac{108.9}{k} = \frac{108.9}{4} = 27.2 = 14.3\,\text{dB}$$

5. EXPERIMENT 1: SAMPLING THEORY

1. Objectives:

- In this experiment you will investigate Sampling Theorem.

2. Prelab Assignment:

Given signal $x(t) = sinc(t)$:

1. Find out the Fourier transform of $x(t)$, $X(f)$, sketch them.
2. Find out the Nyquist sampling frequency of $x(t)$.
3. Given sampling rate f_s, write down the expression of the Fourier transform of $x_s(t)$, $X_s(f)$ in terms of $X(f)$.
4. Let sampling frequency $f_s = 1\text{Hz}$. Sketch the sampled signal $x_s(t) = x(kT_s)$ and the Fourier transform of $x_s(t)$, $X_s(f)$.
5. Let sampling frequency $f_s = 2\text{Hz}$. Repeat 4.
6. Let sampling frequency $f_s = 0.5\text{Hz}$. Repeat 4.
7. Let sampling frequency $f_s = 1.5\text{Hz}$. Repeat 4.
8. Let sampling frequency $f_s = 2/3\text{Hz}$. Repeat 4.
9. Design a Matlab function to calculate the Fourier transform of a sampled signal $x_s(t)$, $X_s(f) = \sum_k x(kT_s) \exp(-j \cdot 2\pi f \cdot kT_s)$. This is necessary in your experiments.

NOTE: In Matlab and this experiment, sinc(t) is defined as $\text{sinc}(t) = \sin(\pi t)/(\pi t)$. Under this definition: $\text{sinc}(2Wt) \rightarrow 1/(2W)\ \text{rect}(f/2W)$.

3. Procedure:

1. Design Matlab programs to illustrate items 4-8 in Prelab. You need to plot all the graphs.

2. Compare your results with your sketches in your Prelab assignment and explain them.

6. EXPERIMENT 2: BINARY SIGNALLING FORMATS

1. Objectives:

In this experiment you will investigate how binary information is serially coded for transmission at baseband frequencies. In particular, you will study:

- line coding methods which are currently used in data communication applications;
- power spectral density functions associated with various line codes;
- causes of signal distortion in a data communications channel;
- effects of intersymbol interference (ISI) and channel noise by observing the eye pattern.

2. Prelab Assignment:

1. Given the binary sequence $b = \{1, 0, 1, 0, 1, 1\}$, sketch the waveforms representing the sequence b using the following line codes:

   (a) unipolar NRZ;

   (b) polar NRZ;

   (c) unipolar RZ;

   (d) bipolar RZ;

   (e) manchester.

   Assume unit pulse amplitude and use binary data rate $R_b = 1\,\text{kbps}$.

2. Determine and sketch the power spectral density (PSD) functions corresponding to the above line codes. Use $R_b = 1\,\text{kbps}$. Let $f_1 > 0$ be the location of the first spectral null in the PSD function. If the transmission bandwidth B_T of a line code is determined by f_1, determine B_T for the line codes in question 1 as a function of R_b.

3. Procedure:

A. Binary Signalling Formats: Line Code Waveforms

Binary 1's and 0's such as in pulse-code modulation (PCM) systems, may be represented in various serial bit signalling formats called *line codes*. In this section you will study signalling formats and their properties.

A.1 You will use the MATLAB function *wave_gen* to generate waveforms representing a binary sequence:

```
wave_gen( binary_sequence, 'line_code_name', Rb )
```

where R_b is the binary data rate specified in bits per second (bps). If you use the function *wave_gen* with the first two arguments only, it will default to the binary data rate set by the variable *binary_data_rate*, which is 1,000 bps. Create the following binary sequence:

$\gg \quad b = [1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1];$

Generate the waveform representing b, using unipolar NRZ line code with $R_b = 1\,\text{kbps}$ and display the waveform x.

$\gg \quad x = \text{wave\_gen}(b, \texttt{'unipolar\_nrz'}, 1000);$

$\gg \quad \text{waveplot}(x)$

A.2 Repeat step A.1 for the following line codes:

- polar NRZ (`'polar_nrz'`);
- unipolar RZ (`'unipolar_rz'`);
- bipolar RZ (`'bipolar_rz'`);
- manchester (`'manchester'`).

You may want to simplify your command line by using:

```
waveplot( wave_gen( b, 'line_code_name')
```

Since you will compare waveforms at the same R_b, you can use the function *wave_gen* with only two arguments.

Q2.1 For the above set of line codes determine which will generate a waveform with no *dc* component regardless of binary sequence represented. Why is the absence of a *dc* component of any practical significance for the transmission of waveforms?

A.3 Power spectral density (PSD) functions of line codes: Generate a 1,000 sample binary sequence:

```
>> b = binary(1000);
```

Display the PSD function of each line code used in part A.1:

```
>> psd(wave_gen(b,'line_code_name'));
```

Let:

f_{p1}: first spectral peak; f_{n1}: first spectral null
f_{p2}: second spectral peak; f_{n2}: second spectral null

such that all $f_{(.)} > 0$. Record your observations in Table 2.1.

Table (2.1)

$R_b =$	f_{p1}	f_{n1}	f_{p2}	f_{n2}	B_T
unipolar NRZ					
polar NRZ					
unipolar RZ					
bipolar NRZ					
manchester					

Location of the first spectral null determines transmission bandwidth B_T.

A.4 To illustrate the dependence of the PSD function on the underlying binary data rate, use the manchester line code and vary R_b:

```
>> psd(wave_gen(b,'manchester',Rb))
```

where `Rb` $\in$ {5 kbps, 10 kbps, 20 kbps}. You may replace manchester by any other line code used in part A.1. Observe the location of spectral peaks and nulls and relate them to R_b.

Q2.2 For a baseband data communications channel with usable bandwidth of 10 kHz, what is the maximum binary data rate for each of the line codes examined in part A.1.

B. Channel Characteristics

In this part you will simulate characteristics of a communications channel.

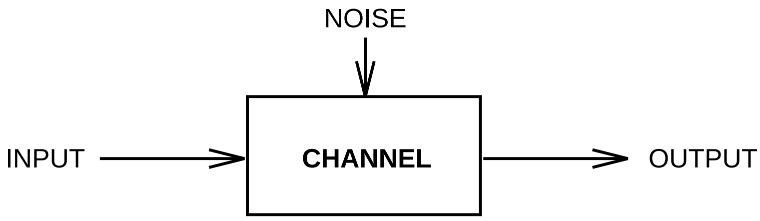


Fig 2.1 Channel model

The MATLAB function that represents the channel response is *channel* which is called with the following arguments:

```
channel( input, gain, noise_power, bandwidth )
```

B.1 Create a 10 sample binary sequence b and generate a waveform representing b in polar NRZ signalling format. Use $R_b = 1$ kbps.

$\gg$ $b = \texttt{binary}(10);$

$\gg$ $\texttt{x} = \texttt{wave\_gen}(\texttt{b},'\texttt{polar\_nrz}', 1000);$

From your observation in part A, determine the transmission bandwidth B_T of x:

$B_T =$ Hz

B.2 Consider a baseband data transmission channel with unity gain and additive white Gaussian noise (AWGN) where the noise power is 10^{-2} W and the channel bandwidth is 4.9 kHz. Transmit waveform x over this channel. Display the channel input and output waveforms:

$\gg$ $\texttt{y} = \texttt{channel}(\texttt{x}, 1, 0.01, 4900);$

$\gg$ $\texttt{subplot}(211), \texttt{waveplot}(\texttt{x});$

$\gg$ $\texttt{subplot}(212), \texttt{waveplot}(\texttt{y});$

If the signalling format is polar NRZ at $R_b = 1$ kbps, estimate the transmitted sequence from the display of the channel output waveform.

$\hat{\texttt{b}} =$

Compare your estimate with the original sequence b.

B.3 Effect of channel noise on the transmitted waveform: Gradually increase the channel noise power while keeping the channel bandwidth at 4.9 kHz and observe changes in the channel output.

≫ `y = channel(x, 1, sigma, 4900);`

≫ `waveplot(y);`

where `sigma` $\in \{0.1, 0.5, 1, 2, 5\}$. At what noise power level, does the channel output waveform becomes indistinguishable from noise?

B.4 You can also observe effects of increasing channel noise power by looking at the PSD of the channel output waveform.

≫ `b = binary(1000);`

≫ `x = wave_gen(b,'polar_nrz', 1000);`

≫ `clf; subplot(121); psdf(x);`

≫ `subplot(122); psdf(channel(x, 1, 0.01, 4900));`

≫ hold on;

≫ `subplot(122); psdf(channel(x, 1, 1, 4900));`

≫ `subplot(122); psdf(channel(x, 1, 5, 4900));`

Q2.3 Since the channel noise is additive and uncorrelated with the channel input, determine an expression that will describe the PSD of the channel output in terms of the input and noise PSD functions.

B.5 Effects of channel bandwidth on transmitted waveform: Distortion observed in the time display of the channel output is due to finite bandwidth of the channel and due to noise. To study distortion due to channel bandwidth only, set noise power to zero and regenerate the channel output waveform:

≫ hold off; clf;;

≫ `b = binary(10);`

≫ `x = wave_gen(b,'polar_nrz', 1000);`

≫ `subplot(211), waveplot(x);`

```
>> subplot(212), waveplot(channel(x, 1, 0, 4900));
```

B.5 Investigate effects of channel bandwidth on the output waveform.

```
>> subplot(212), waveplot(channel(x, 1, 0, bw));
```

where bw $\in$ {3000, 2000, 1000, 500}. Observe the delay in the output waveform due to filtering characteristics of the channel. Plot the input and output waveforms. Determine the appropriate sampling instants for the decoding of the waveform for the case bw = 500.

C. Eye Diagram

Effects of channel filtering and noise can be best seen by observing the output waveform in the form of an *"eye diagram"*. The eye diagram is generated with multiple sweeps where each sweep is triggered by a clock signal and the sweep width is slightly larger than the binary data period $T_b = 1/R_b$. In this simulation the eye diagram is based on a sweep width of $2T_b$.

C.1 Generation of Eye Diagram:

```
>> b = [1 0 0 1 0 1 1 0];

>> x = wave_gen(b,'polar_nrz', 1000);

>> clf;

>> subplot(221), waveplot(x);

>> subplot(223), eye_diag(x);
```

The eye diagram for the waveform x represents what you should expect to see for an undistorted signal. To observe how the eye diagram is generated and to observe effects of signal distortion as the signal x is transmitted over a finite bandwidth channel with no noise component:

```
>> y = channel(x, 1, 0, 4000);

>> subplot(222), waveplot(y);

>> subplot(224), eye_diag(y, -1);
```

If the second argument to the function *eye_diag* is negative, you have to hit the Return key for the next trace to be displayed. This will assist you to understand how the eye diagram is generated.

C.2 Key parameters to be measured with an eye diagram are shown below.

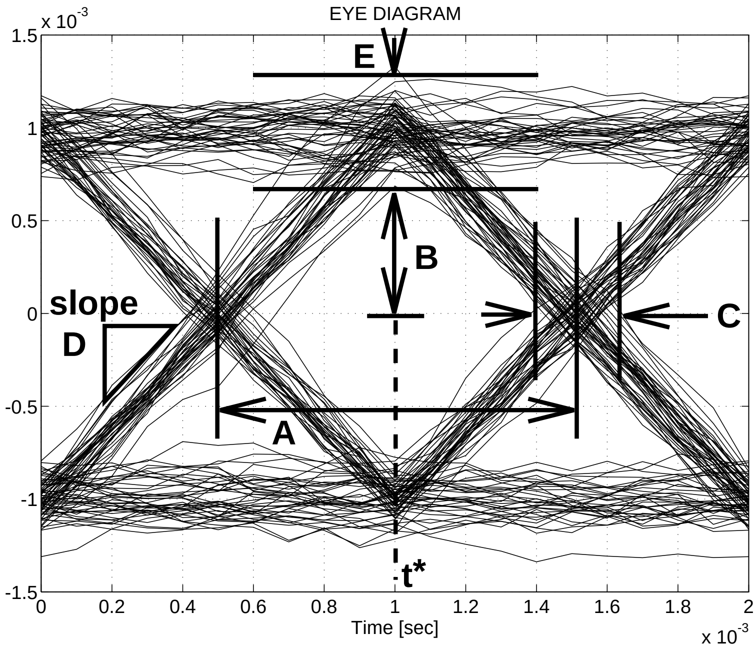


Fig 2.2 Interpretation of the eye pattern

A time interval over which the waveform can be sampled;

B margin over noise;

C distortion of zero crossings;

D slope: sensitivity to timing error;

E maximum distortion;

$\mathbf{t}^*$ optimum sampling instant measured with respect to the time origin. If the binary data period is T_b, then the waveform will be sampled at t^*, $t^* + T_b$, $t^* + 2T_b$, $\ldots$ for signal detection.

Generate the eye diagram from a polar NRZ waveform at the channel output for values of noise variance `s2` and channel bandwidth `bw` shown in Table 2.2. Record t^*, A and B for each set of `s2` and `bw`.

≫ `clf`;

≫ `b = binary(100));`

≫ `x = wave_gen(b,'polar_nrz', 1000);`

Table 2.2

Polar NRZ Line Code				
s2	bw	t^*	**A**	**B**
0.01	3000			
	2000			
	1000			
0.02	4000			
0.08				
0.10				

$\gg$ `eye_diag(channel(x, 1, s2, bw));`

C.3 Repeat step C.2 for manchester line code and record your results in Table 2.3.

Table 2.3

Manchester Line Code				
s2	bw	t^*	**A**	**B**
0.01	3000			
	2000			
	1000			
0.02	4000			
0.08				
0.10				

Q2.4 When you compare the eye diagrams from C.2 and C.3 for `s2` = 0.01 and `bw` = 1000, for which line code do you observe a "reasonable" eye diagram? Explain the difference in terms of the respective line code properties.

C.4 Generate eye diagrams as in step C.2 for polar RZ and unipolar RZ and unipolar NRZ line codes and observe how the line code dictates the shape and the symmetry of the eye diagram.

7. EXPERIMENT 3: MATCHED FILTER AND BIT ERROR RATE (BER)

1. Objectives:

In this experiment you will investigate the signal detection process by studying elements of a receiver and of the decoding process. In particular you will:

- investigate the characteristics of matched filters;
- study performance of various receiver structures based on different receiver filters by measuring probability of bit error;
- use the eye diagram as an investigative tool to set parameters of the detection process.

2. Prelab Assignment:

1. A matched filter is to be designed to detect the rectangular pulse

$$r(t) = rect\Big(\frac{t - T_b/2}{T_b}\Big), \quad \text{with } T_b = 1 \text{ msec.}$$

   (a) Determine the impulse response of the matched filter.

   (b) Determine the output of the matched filter if $r(t)$ is the input.

   (c) Repeat parts (a) and (b) for a triangular pulse of 10 msec duration.

2. Let $Y(t) = X(t) + n(t)$, represent the waveform at the output of a channel. $X(t)$ is a polar NRZ waveform with unit pulse amplitude and binary data rate R_b of 1 kbps. $n(t)$ is a white noise process with PSD function:

$$S_n(f) = N_o/2 = 0.5 \times 10^{-4} \text{ W/Hz.}$$

   If $Y(t)$ is applied to a matched-filter receiver:

   (a) Determine the rms value of $n(t)$ and the peak signal amplitude at the output of the matched filter.

(b) Determine E_b, the average energy of $X(t)$ in a bit period.

(c) Determine the probability of bit error $P_e = Q(\sqrt{2E_b/N_o})$.

3. If $Y(t)$ in Question 2 is applied to a RC-filter with frequency response:

$$H_{rc}(f) = \frac{1}{1 + j2\pi fRC},$$

with $RC = 1/(2000\pi)$,

(a) Determine the peak signal amplitude and rms value of the noise at the filter output;

(b) Determine the probability of bit error P_e, if $X(t)$ were to be detected by a receiver based on the RC-filter.

3. Procedure:

A. Characteristics of Matched Filters

A.1 Generate a rectangular pulse with unit pulse amplitude and 1 msec pulse duration.

```
>> r = wave_gen(1,'polar_nrz',1000);
```

A.2 Display `r` and the impulse response of a matched filter based on `r`.

```
>> subplot(311),waveplot(r);
>> subplot(312),match('polar_nrz');
```

A.3 Observe the matched filter output if `r` is applied to its input.

```
>> rm = match('polar_nrz',r);
>> subplot(313),waveplot(rm);
```

Q3.1 Determine the time when the filter output reaches its maximum value. How is this time related to the waveform `r`?

A.4 Repeat parts A.1–A.3 for a triangular pulse with 10 msec pulse width and unit peak amplitude.

```
>> r = wave_gen(1,'triangle',100);
>> clf;subplot(311),waveplot(r);
>> subplot(312),match('triangle');
```

```
>> rm = match('triangle', r);

>> subplot(313), waveplot(rm);
```

Q3.2 If the triangular pulse width is changed to 1 msec, determine the peak amplitude of the matched filter output?

A.5 Repeat parts A.1–A.3 for a manchester pulse with 10 msec pulse width and unit peak amplitude. Predict the matched filter impulse response and matched filter output. Verify your predictions using MATLAB functions.

A.6 Generate a polar NRZ waveform that represents the 5-sample binary sequence [1 0 0 1 0]. The binary data rate R_b is 1 kbps and the pulse amplitude A is 1 V.

```
>> x5 = wave_gen([1  0  0  1  0], 'polar_nrz', 1000);

>> clf, subplot(211), waveplot(x5);
```

Record the waveform `x5`

A.7 Apply `x5` to a matched filter. Record output.

```
>> subplot(212), waveplot(match('polar_nrz', x5));
```

Q3.3 Construct the waveform at a matched filter output if the input is a unipolar NRZ waveform that represents the binary sequence [1 0 0 1 0].

B. Signal Detection

B.1 Generate a 10-sample binary sequence and a waveform that represents this binary sequence in polar NRZ signalling format.

```
>> b10 = binary(10);

>> x10 = wave_gen(b10, 'polar_nrz', 1000);

>> subplot(211), waveplot(x10);
```

B.2 Apply `x10` to a channel with 4.9 kHz bandwidth and AWGN where the noise power is 2 W. Display the channel output waveform `y10`:

```
>> y10 = channel(x10, 1, 2, 4900);
```

```
≫ subplot(212), waveplot(y10);
```

Decode the binary sequence from the waveform `y10`:

$\widehat{\texttt{b10}} =$

B.3 Apply `y10` to a matched filter. Display the output waveform `z10`:

```
≫ z10 = match('polar_nrz', y10);
≫ subplot(212), waveplot(z10);
```

B.4 Let T_b be the binary data period. Sample the output of the matched filter at $k\,T_b$, $k = 1, \ldots, 10$ and apply the following decision rule:

$$\widehat{b_k} = \begin{cases} 0, & \text{if sample value} > 0; \\ 1, & \text{if sample value} < 0; \end{cases}$$

where $\widehat{b_k}$ is the estimated value of the k*th* element of the binary sequence `b10`. Apply this decision rule on the matched filter output `z10`:

$$\widehat{\texttt{b10}} =$$

Compare your decoded sequence with the original sequence `b10`:

Q3.4 Comment on whether it is easier to decode the transmitted binary sequence directly from the channel output `y10` or from the matched filter output `z10`. If sampling instants other than those specified above are used, the probability of making a decoding error will be larger. Why?

C. <u>Matched-Filter Receiver</u>

C.1 Generate a 2,000-sample binary sequence `b` and a polar NRZ waveform based on `b`:

```
≫ b = binary(2000);
≫ x = wave_gen(b, 'polar_nrz');
```

Apply `x` to a channel with 4.9 kHz bandwidth and channel noise power of 0.5 W. Let `y` be the channel output waveform.

```
≫ y = channel(x, 1, 0.5, 4900);
```

C.2 Apply `y` to a matched filter. Display the eye diagram of the matched filter output `z`.

≫ `z = match('polar_nrz', y);`

≫ `eye_diag(z);`

From the eye diagram, determine the optimum sampling instants and threshold value `v_th` for the detector to decode the transmitted binary sequence `b`. Sampling instants for the matched filter output are measured with respect to the time origin. For example, if the binary data period is T_b and the `sampling_instant` parameter is set to t_i, then the detector will sample the signal at t_i, $t_i + T_b$, $t_i + 2T_b$, $\ldots$ etc.

`v_th =` V.

`sampling_instant =` sec.

Use `v_th` and `sampling_instant` in the detector which will operate on the matched filter output. Record the resulting probability of bit error P_e (BER) in Table 3.1.

≫ `detect(z, v_th, sampling_instant, b);`

Table 3.1

$\sigma_n^2(W)$	P_e empirical	P_e theoretical
0.5		
1.0		
1.5		
2.0		

C.3 Repeat C.1–C.2 for channel noise power of 1, 1.5, and 2 W without displaying the eye diagram of the matched filter output `z`. Record P_e results in Table 3.1. **Remark:** In Experiment 2 you have observed that the optimum sampling instants and the threshold value are independent of channel noise power. Therefore, you can use the optimum sampling instants determined in part C.2 to decode the matched filter output for different channel noise power levels.

C.4 If different sampling instants other than the optimum values are used, the resulting BER will be larger. You can observe this by decoding the binary sequence using values for the `sampling_instant` parameter that are 0.9 and 0.5 times the optimal value used in part C.3.

Q3.5 Evaluate theoretical probability of bit error values for all cases considered above and record in Table 3.1. Note that the PSD function of a white noise process can be determined as:

$$S_n(f) = \frac{N_o}{2} = \frac{\sigma_n^2}{2 \times \text{system bandwidth}},$$

where the system bandwidth in this experiment is 4.9 kHz.

D. Low-Pass Filter Receiver

D.1 Apply a rectangular pulse to a first-order RC-filter of 1 kHz bandwidth. Display the filter output and measure the peak amplitude A_r:

```
≫ r = wave_gen(1,'unipolar_nrz'); r_lpf = rc(1000,r);
≫ subplot(211); waveplot(r);
≫ subplot(212); waveplot(r_lpf);
```

$A_r =$ V.

D.2 Generate 2,000 samples from a zero-mean white noise sequence of 0.5 W power. Apply the noise sequence to the RC-filter. Record the rms value of the output noise power.

```
≫ n = gauss(0,0.5,2000);
≫ meansq(rc(1000,n));
```

$\sigma_n^2 =$ W.

Q3.6 From the results in parts D.1 and D.2, determine the ratio A_r/σ_n, where A_r is the peak signal amplitude measured in D.1 and σ_n is the rms value of the output noise. If `y` in part C.1 is applied to a receiver which uses the above RC-filter, determine the resulting BER.

D.3 Regenerate `y` from part C.1. Apply `y` to the RC-filter. Display the eye diagram of the output waveform `z_lpf`.

```
≫ y = channel(x,1,0.5,4900);
≫ z_lpf = rc(1000,y);
≫ clf, eye_diag(z_lpf);
```

D.4 From the eye diagram, determine the optimum sampling instant and threshold value. Decode the binary sequence form `z_lpf`.

```
≫ detect(z_lpf,v_th,sampling_instant,b);
```

Compare the resulting BER with the BER evaluated in step C.2.

D.5 Repeat part D.4 for the channel noise power of 1, 1.5, and 2 W. Record results in Table 3.2.

Table 3.2

$\sigma_n^2(W)$	P_e	
	BW = 1.0 kHz	BW = 0.5 kHz
0.5		
1.0		
1.5		
2.0		

D.6 Repeat parts D.3 – D.5 for a first-order RC-filter with 500 Hz bandwidth. Record the resulting BER values in Table 3.2.

```
≫ z_lpf = rc(500, y);
≫ eye_diag(z_lpf);
≫ detect(z_lpf, v_th, sampling_instant, b);
```

Q3.7 Explain why the BER resulting from a low-pass filter of 500 Hz bandwidth is smaller than the BER resulting from a low-pass filter of 1 kHz bandwidth. Will the BER be further decreased if a low-pass filter of 100 Hz bandwidth is used?

8. EXPERIMENT 4: DIGITAL MODULATION

1. Objectives:

In this experiment you will apply concepts of baseband digital transmission and analog continuous wave modulation to the study of band-pass digital transmission. You will examine:

- generation of digital modulated waveforms;
- *coherent* (synchronous) and *noncoherent* (envelope) detection of modulated signals;
- system performance in the presence of corrupting noise.

2. Prelab Assignment:

1. Consider the binary sequence $b = [\ 1\ 0\ 0\ 1\ 0\ ]$. Let the bit rate R_b be 1 kbps and let the peak amplitude of all digital modulated waveforms be set to 1 V.
   (a) Sketch the ASK waveform representing the binary sequence b using a carrier frequency of 5 kHz.
   (b) Sketch the PSK waveform representing the binary sequence b using a carrier frequency of 5 kHz.
   (c) Let the *mark* and *space* frequencies used by an FSK modulator be set to 3 and 6 kHz, respectively. Sketch the resulting FSK waveform representing the binary sequence b.
2. Sketch the power spectral density function for each of the modulated signals in Question 1.
3. If an ASK signal is applied to the input of a coherent detector shown in Fig. 4.1, sketch the waveforms at the output of each block.

3. Procedure:

In this experiment, the binary data rate R_b is 1 kbps and peak modulated signal amplitude is 1 V. The bit period $T_b = 1/R_b$ is represented by 100 samples.

A. Generation of Modulated Signals

Amplitude-Shift Keying (ASK)

A.1 Generate a binary sequence with the first 5 bits [1 0 0 1 0]:

```
≫ b = [1 0 0 1 0 binary(45)];
```

A.2 To generate the ASK signal, `sa`, with a carrier frequency of 8 kHz:

- generate a unipolar NRZ signal `xu`, from the sequence `b`;
- mix `xu` with the output of an oscillator operating at 8 kHz.

```
≫ xu = wave_gen(b,'unipolar_nrz');
≫ sa = mixer(xu, 8000);
```

A.3 Display the first 5 bits of `xu` and `sa` in the binary sequence `b`. Compare the two waveforms.

```
≫ subplot(211), waveplot(xu, 5);
≫ subplot(212), waveplot(sa, 5);
```

Also display and record the respective PSD functions over the frequency interval [0, 20 kHz].

```
≫ fr = [0, 20000];
≫ subplot(211), psd(xu, fr);
≫ subplot(212), psd(sa, fr);
```

Phase-Shift Keying (PSK)

A.4 To generate the PSK signal `sp`, with a carrier frequency of 8 kHz:

- generate a polar NRZ signal `xp`, from the sequence `b`;
- mix `xp` with the output of an oscillator operating at 8 kHz.

```
≫ xp = wave_gen(b,'polar_nrz');
≫ sp = mixer(xp, 8000);
```

A.5 Display the first 5 samples of the waveforms `xp` and `sp`:

```
≫ subplot(211), waveplot(xp, 5);
≫ subplot(212), waveplot(sp, 5);
```

What is the phase difference between `sp` and the carrier $\sin(2\pi f_c t)$ during the first and second bit periods?

A.6 Display the PSD functions of `xp` and `sp` over the frequency interval [0, 20 kHz]. Record main characteristics of each PSD function.

```
≫ fr = [0, 20000];
≫ subplot(211), psd(xp, fr);
≫ subplot(212), psd(sp, fr);
```

Frequency-Shift Keying (FSK)

A.7 To generate the *continuous phase* FSK signal `sf`, with *mark* and *space* frequencies of 4 and 8 kHz, respectively:

- generate a polar NRZ signal from the sequence `b`;
- mix `xp` apply the polar waveform to the input of a voltage controlled oscillator (VCO). In this experiment the VCO has the free-running frequency set to 6 kHz and has frequency sensitivity of -2 kHz/V.

```
≫ xf = wave_gen(b,'polar_nrz');
≫ sf = vcom(xf);
```

A.8 Display waveforms `xf` and `sf` for $0 < t < 5\,T_b$.

```
≫ subplot(211), waveplot(xp, 5);
≫ subplot(212), waveplot(sf, 5);
```

Display and record the PSD function of the FSK signal.

```
≫ clf;
≫ psd(sf, fr);
```

Q4.1 How can you generate an FSK signal from two ASK signals? For a system where efficient bandwidth utilization is required, which modulation scheme would you prefer?

B. Coherent and Noncoherent Detection

Coherent Detection

B.1 A coherent detector for ASK and PSK signals is depicted in Fig. 4.1.

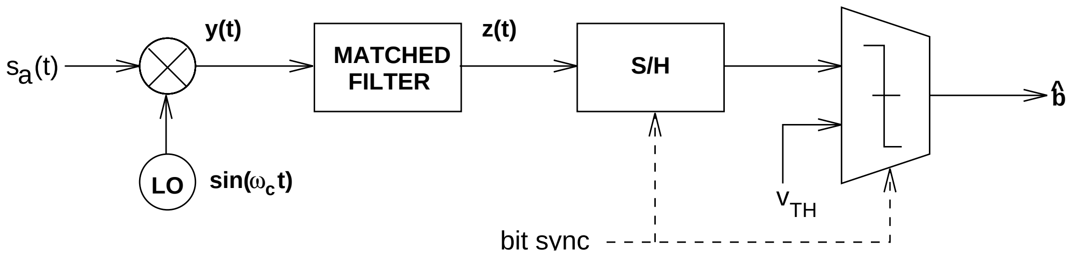


Fig. 4.1 Coherent Detector

To demodulate the ASK signal `sa`, first multiply `sa` by a locally generated carrier which has the same frequency and phase as the carrier used in generating `sa`. Display the waveform `ya` at the output of the multiplier for the first five bit periods. Also display and record the corresponding PSD function over the interval `fr`.

```
≫ ya = mixer(sa, 8000);
≫ clf, subplot(211), waveplot(ya, 5);
≫ subplot(212), psd(ya, fr);
```

B.2 Apply `ya` to a matched filter and record its output for $0 < t < 5\,T_b$.

```
≫ za = match('unipolar_nrz', ya);
≫ subplot(212), waveplot(za, 5);
```

Q4.2 Determine the impulse response of the matched filter. Note that `za` is similar to the output of the matched filter for a unipolar NRZ signal. Why?

B.3 The major difficulty in implementing a coherent detector is carrier synchronization. In order to achieve optimum performance, the local oscillator should have the same phase and frequency as the incoming carrier. Phase or frequency deviation will result in degradation of detection performance.

To observe the effect of phase error, demodulate `sa` using a local oscillator whose output is $\sin(2\pi f_c + \phi)$. Here, ϕ is the phase error measured with respect to the carrier. Record the peak signal amplitude at the matched filter output for each phase error shown in Table 4.1.

```
≫ phase_error = 0;
```

```
≫ ya = mixer(sa, 8000, phase_error));
≫ za = match('unipolar_nrz', ya);
≫ subplot(212), waveplot(za, 5);
```

Table 4.1

Phase Error	Peak Amplitude [V]
0°	
20°	
60°	
80°	
120°	

Q4.3 Recall that the BER resulting from the detection of a signal in the presence of noise, is a function of peak signal amplitude at the receiver filter output. Determine from the results displayed in Table 4.1 which phase error will result in smallest BER.

B.4 Demodulate `sa` with 60° and 120° phase errors. Decode the matched filter output to recover the first five bits of the sequence `b`. Record each decoded sequence and comment on the difference.

B.5 To observe the effect of frequency deviation in demodulating an ASK signal, demodulate `sa` with a local oscillator set to 7,900 Hz. Display and compare the demodulated signals `ya` and `ya1`.

```
≫ ya1 = match('unipolar_nrz', mixer(sa, 7900));
≫ subplot(211), waveplot(ya, 5);
≫ subplot(212), waveplot(ya1, 5);
```

Could the original binary sequence be recovered from `ya1`? Consider a second case where the local oscillator frequency is set to 7,985 Hz. Demodulate `sa` and generate the matched filter output:

```
≫ ya2 = match('unipolar_nrz', mixer(sa, 7985));
≫ subplot(211), waveplot(ya, 5), subplot(212), waveplot(ya2, 5);
```

Determine the frequency of the envelope of the matched filter output.

Q4.4 Consider an ASK signal $s_a(t)$ with carrier frequency of f_c. If $s_a(t)$ is demodulated by multiplying with the output of a local oscillator set to f_o, such that $f_o \neq f_c$, the envelope of detector matched filter output is modulated by a sinusoid. Determine the frequency of this modulating signal as a function of f_c and f_o.

C. System Performance Under Noise

Coherent Detection

C.1 Generate an ASK signal representing a 500-sample binary sequence:

```
≫ b = [1 0 0 1 0 binary(495)];
≫ sa = mixer(wave_gen(b,'unipolar_nrz'), 8000);
```

C.2 Apply `sa` to a channel with unity gain, channel noise $\sigma_n^2 = 1$ W, and of sufficient bandwidth such that no distortion is introduced to the signal. Display the ASK signal `sa` and the channel output `y` for $0 < t < 5T_b$.

```
≫ y = channel(sa, 1, 1.5, 49000);
≫ subplot(211), waveplot(sa, 5);
≫ subplot(212), waveplot(y, 5);
```

C.3 Use a coherent detector to demodulate `y`. Display the eye diagram of the matched filter output.

```
≫ zm = match('unipolar_nrz', mixer(y, 8000));
≫ clf, eye_diag(zm);
```

From the eye diagram, determine optimum sampling instants and the threshold value. Apply `zm` to the decision circuit, and record the resulting probability of bit error.

```
≫ detect(zm, vth, sampling_instant, b);
```

Q4.5 Compute the theoretical probability of bit error for the case considered above. Recall that the PSD function of the channel noise is

$$S_n(f) = \frac{N_o}{2} = \frac{\sigma_n^2}{2 \times \text{system bandwidth}}.$$

The system bandwidth in this experiment is 50 kHz.