EE8103 – Random Processes, Fall 2011, Quiz 4

Name: ______________________ Student ID: ______________________

1. (5 marks) The daily temperature sequence is modeled as a random process: $Y_n = 16\cos(\frac{2\pi n}{360}) + 4X_n$, where $X_1, X_2, \ldots,$ is an Independent and Identically Distributed (I.I.D.) random sequence with $E[X_n] = 0$ and $Var[X_n] = 1$.

Find: the autocovariance function $C_Y[m, k]$.

Solution:

$$C_Y[m,k] = E(Y_m Y_{m+k}) - E(Y_m)E(Y_{m+k})$$

$$= E\left[(16\cos(\tfrac{2\pi m}{360}) + 4X_m)(16\cos\tfrac{2\pi(m+k)}{360} + 4X_{m+k})\right]$$

$$- E(16\cos(\tfrac{2\pi m}{360}) + 4X_m)\,E(16\cos(\tfrac{2\pi(m+k)}{360}) + 4X_{m+k})$$

$$= 16E(X_m X_{m+k})$$

$$= \begin{cases} 16, & k=0 \\ 0, & k\neq 0 \end{cases}$$

2. (5 marks) Coin 1 has probability 0.7 of coming up heads, and coin 2 has probability 0.6 of coming up heads. We flip a coin each day. If the coin flipped today comes up head, then we select coin 1 to flip tomorrow, and if it comes up tail, then we select coin 2 to flip tomorrow. Define state 0 as head, and state 1 as tail.

Find: a) the transition probability matrix, and b) in a long run, what percentage of the results are heads?

Solution:

The transition probability matrix is given by $P = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$

$$0.7\pi_0 + 0.6\pi_1 = \pi_0$$

$\pi_0 + \pi_1 = 1$

Therefore, $\pi_0 = 2/3$