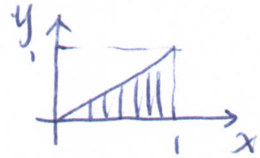
Name: ______________________ Student ID: ______________________

1. (6 marks) The joint Probability Density Function (PDF) of X and Y is given by

$$f_{X,Y(x,y)} = \begin{cases} 8xy & 0 \le y \le x \le 1 \\ 0 & otherwise \end{cases}$$

Find: a) marginal PDF $f_X(x)$, b) conditional PDF $f_X(x|B)$ where event B={$X \le 0.5$}, and c) conditional PDF $f_{Y|X(y|x)}$.



a) $f_X(x) = \int_0^x 8xy\,dy = 4x^3 \qquad 0 \le x \le 1$

b) $P(B) = \int_0^{\frac{1}{2}} 4x^3 dx = \frac{1}{16}$

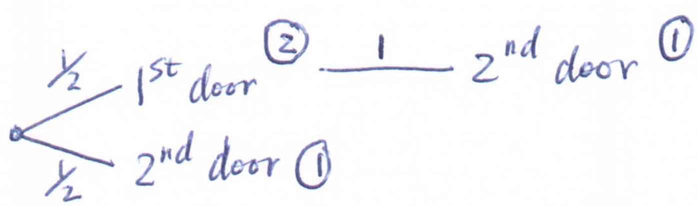
When $0 \le x \le 1/2$

$F_X(x|B) = P(X \le x | B) = \dfrac{P((X \le x) \cap (X \le \frac{1}{2}))}{P(B)} = 16 F_X(x)$

$f_X(x|B) = \begin{cases} 64x^3 & 0 \le x \le \frac{1}{2} \\ 0 & \text{o.w} \end{cases}$

c) $f_{Y|X}(y|x) = \begin{cases} \dfrac{8xy}{4x^3} & 0 \le y \le x \\ 0 & \text{o.w} \end{cases}$

2. (4 marks) A prisoner is trapped in a cell containing two doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second door takes him to freedom after one day of travel. Assuming that the prisoner is always equally likely to choose among those doors that he has not used, what is the expected number of days until he reaches freedom? (For instance, if the prisoner initially tries door 1, then when he returns to the cell, he will now select door 2.)



$E(X) = \frac{1}{2}(2+1) + \frac{1}{2} \times 1 = 2$