EE8103 – Random Processes, Fall 2011, Quiz 2

Name: ______________________ Student ID: ______________________

1. (4 marks) A sender sends messages to a receiver over a wireless channel. Each message is transmitted 4 times. A single transmission of a message is successful with probability 0.9.
a) Among 4 transmissions for a message, what is the probability that at least 3 transmissions are successful?
b) X is an indicator Random Variable defined as: $X = 1$ if at least 3 transmissions for a message are successful; otherwise $X = 0$. What is the PMF of X?

a) $P(k \geq 3) = P(k=3) + P(k=4)$

$= \binom{4}{3} 0.9^3 0.1^1 + \binom{4}{4} 0.9^4 0.1^0 = 0.9477$

b) $P(X) = \begin{cases} 0.9477, & X=1 \\ 0.0523, & X=0 \\ 0. & \text{o.w.} \end{cases}$

2. (6 marks) X is a continuous random variable with a Cumulative Distribution Function (CDF) as

$$F_X(x) = \begin{cases} 0, & x < -5, \\ (x+k)^2/100, & -5 \leq x < 5, \\ 1, & x \geq 5. \end{cases}$$

Find: a) the constant k, b) Probability Density Function (PDF) of X, c) the mean E(X), and d) the variance Var[X].

a) $\begin{cases} \frac{(-5+k)^2}{100} = 0 \\ \frac{(5+k)^2}{100} = 1 \end{cases} \Rightarrow k = 5$

b) $f_X(x) = \begin{cases} \frac{1}{50}(x+5), & -5 \leq x < 5 \\ 0, & \text{o.w} \end{cases}$

c) $E(X) = \int_{-5}^{5} \frac{x}{50}(x+5)\,dx = \frac{5}{3}$

d) $E(X^2) = \int_{-5}^{5} \frac{x^2}{50}(x+5)\,dx = \frac{25}{3}$

$Var(X) = E(X^2) - (E(X))^2 = \frac{50}{9}$