Chapter 2

2.1. The proof is as follows:

$$\begin{aligned}(x+y)\cdot(x+z) &= xx+xz+xy+yz\\ &= x+xz+xy+yz\\ &= x(1+z+y)+yz\\ &= x\cdot 1+yz\\ &= x+yz\end{aligned}$$

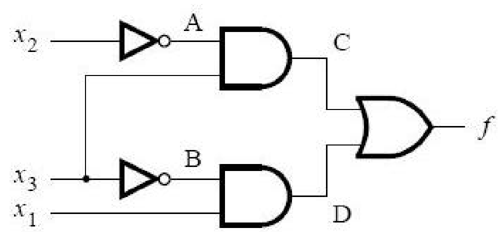
2.2. The proof is as follows:

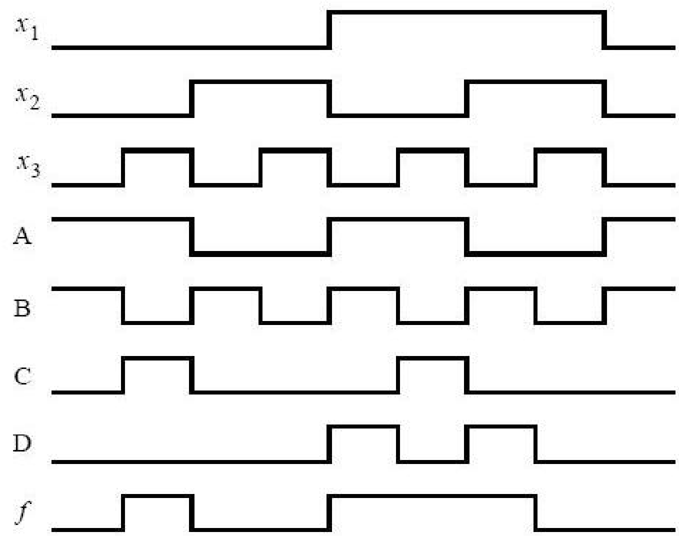
$$\begin{aligned}(x+y)\cdot(x+\overline{y}) &= xx+xy+x\overline{y}+y\overline{y}\\ &= x+xy+x\overline{y}+0\\ &= x(1+y+\overline{y})\\ &= x\cdot 1\\ &= x\end{aligned}$$

2.7. A possible approach for determining whether or not the expressions are valid is to try to manipulate the left and right sides of an expression into the same form, using the theorems and properties presented in section 2.5. While this may seem simple, it is an awkward approach, because it is not obvious what target form one should try to reach. A much simpler approach is to construct a truth table for each side of an expression. If the truth tables are identical, then the expression is valid. Using this approach, we can show that the answers are:

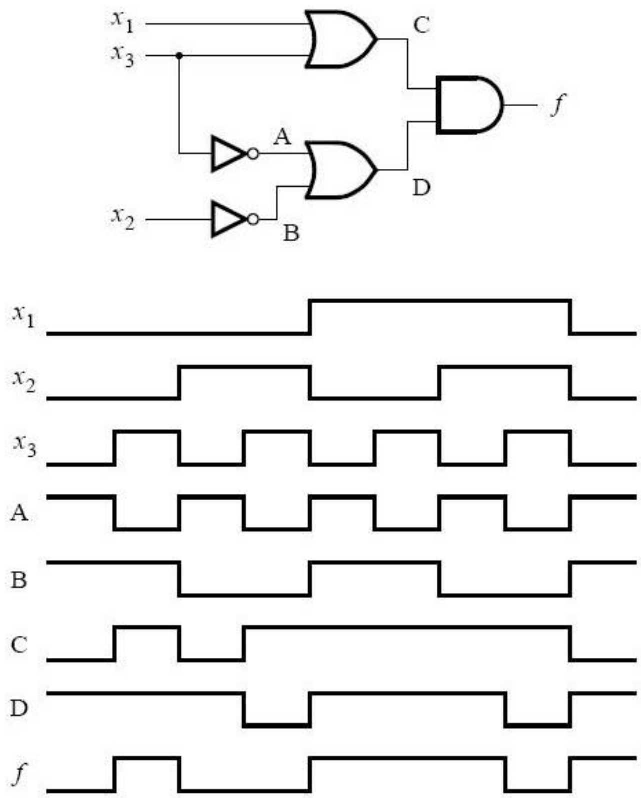
(a) Yes
(b) Yes
(c) No

2.8. Timing diagram of the waveforms that can be observed on all wires of the circuit:





2.9. Timing diagram of the waveforms that can be observed on all wires of the circuit:



2.10. Starting with the canonical sum-of-products for f get

$$\begin{aligned}
f &= \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2\overline{x}_3 + \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2x_3 + x_1x_2\overline{x}_3 + x_1x_2x_3 \\
&= x_1(\overline{x}_2\overline{x}_3 + \overline{x}_2x_3 + x_2\overline{x}_3 + x_2x_3) + x_2(\overline{x}_1\overline{x}_3 + \overline{x}_1x_3 + x_1\overline{x}_3 + x_1x_3) \\
&\quad + x_3(\overline{x}_1\overline{x}_2 + \overline{x}_1x_2 + x_1\overline{x}_2 + x_1x_2) \\
&= x_1(\overline{x}_2(\overline{x}_3 + x_3) + x_2(\overline{x}_3 + x_3)) + x_2(\overline{x}_1(\overline{x}_3 + x_3) + x_1(\overline{x}_3 + x_3)) \\
&\quad + x_3(\overline{x}_1(\overline{x}_2 + x_2) + x_1(\overline{x}_2 + x_2)) \\
&= x_1(\overline{x}_2 \cdot 1 + x_2 \cdot 1) + x_2(\overline{x}_1 \cdot 1 + x_1 \cdot 1) + x_3(\overline{x}_1 \cdot 1 + x_1 \cdot 1) \\
&= x_1(\overline{x}_2 + x_2) + x_2(\overline{x}_1 + x_1) + x_3(\overline{x}_1 + x_1) \\
&= x_1 \cdot 1 + x_2 \cdot 1 + x_3 \cdot 1 \\
&= x_1 + x_2 + x_3
\end{aligned}$$

2.11. Starting with the canonical product-of-sums for f can derive:

$$\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + x_3) \\
&= ((x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3))((x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)) \cdot \\
&\quad ((\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3))((\overline{x}_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + x_3)) \\
&= (x_1 + x_2 + x_3\overline{x}_3)(x_1 + \overline{x}_2 + x_3\overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3\overline{x}_3)(\overline{x}_1 + \overline{x}_2x_2 + x_3) \\
&= (x_1 + x_2)(x_1 + \overline{x}_2)(\overline{x}_1 + x_2)(\overline{x}_1 + x_3)
\end{aligned}$$

2.12. Derivation of the minimum sum-of-products expression:

$$\begin{aligned} f &= x_1x_3 + x_1\overline{x}_2 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\ &= x_1(\overline{x}_2 + x_2)x_3 + x_1\overline{x}_2(\overline{x}_3 + x_3) + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\ &= x_1\overline{x}_2x_3 + x_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\ &= x_1x_3 + (x_1 + \overline{x}_1)x_2x_3 + (x_1 + \overline{x}_1)\overline{x}_2\overline{x}_3 \\ &= x_1x_3 + x_2x_3 + \overline{x}_2\overline{x}_3 \end{aligned}$$

2.13. Derivation of the minimum sum-of-products expression:

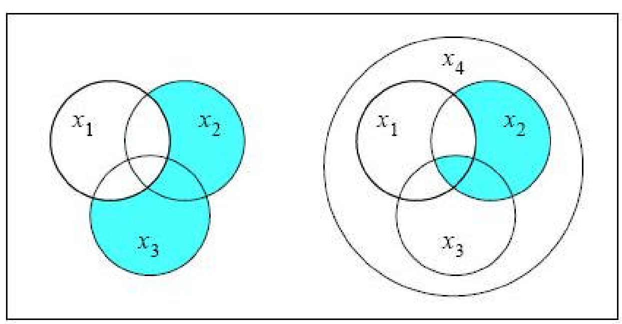
$$\begin{aligned} f &= x_1\overline{x}_2\overline{x}_3 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\ &= x_1\overline{x}_2\overline{x}_3(\overline{x}_4 + x_4) + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\ &= x_1\overline{x}_2\overline{x}_3\overline{x}_4 + x_1\overline{x}_2\overline{x}_3x_4 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\ &= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2(\overline{x}_3 + x_3)\overline{x}_4 + x_1x_2x_4 \\ &= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2\overline{x}_4 + x_1x_2x_4 \end{aligned}$$

2.15. Derivation of the minimum product-of-sums expression:

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)(x_1 + (\overline{x}_2 + x_3))(\overline{x}_1 + (\overline{x}_2 + x_3)) \\ &= (x_1 + x_2)(\overline{x}_2 + x_3) \end{aligned}$$

2.18. In Figure P2.1*a* it is possible to represent only 14 minterms. It is impossible to represent the minterms $\overline{x}_1\overline{x}_2x_3x_4$ and $x_1x_2\overline{x}_3\overline{x}_4$.
In Figure P2.1*b*, it is impossible to represent the minterms $x_1x_2\overline{x}_3\overline{x}_4$ and $x_1x_2x_3\overline{x}_4$.

2.19. Venn diagram for $f = \overline{x}_1\overline{x}_2x_3\overline{x}_4 + x_1x_2x_3x_4 + \overline{x}_1x_2$ is



2.20. The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1x_2\overline{x}_3 + x_1x_2x_3 \\ &= (\overline{x}_1 + x_1)x_2x_3 + x_1(\overline{x}_2 + x_2)\overline{x}_3 \\ &= x_2x_3 + x_1\overline{x}_3 \end{aligned}$$

2.21. The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1x_2\overline{x}_3 + x_1x_2x_3 \\ &= \overline{x}_1(\overline{x}_2 + x_2)x_3 + x_1(\overline{x}_2 + x_2)\overline{x}_3 + (\overline{x}_1 + x_1)x_2x_3 \\ &= \overline{x}_1x_3 + x_1\overline{x}_3 + x_2x_3 \end{aligned}$$

Another possibility is

$$f = \overline{x}_1x_3 + x_1\overline{x}_3 + x_1x_2$$

□

2.22. The simplest POS implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= ((x_1 + x_3) + x_2)((x_1 + x_3) + \overline{x}_2)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= (x_1 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \end{aligned}$$

2.23. The simplest POS implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)((\overline{x}_1 + x_3) + x_2)((\overline{x}_1 + x_3) + \overline{x}_2) \\ &= (x_1 + x_2)(\overline{x}_1 + \overline{x}_3) \end{aligned}$$

2.33. The truth table that corresponds to the timing diagram in Figure P2.4 is

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

The simplest SOP expression is derived as follows:

$$\begin{aligned} f &= \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2\overline{x}_3 + \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1x_2x_3 \\ &= \overline{x}_1(\overline{x}_2 + x_2)x_3 + \overline{x}_1x_2(\overline{x}_3 + x_3) + (\overline{x}_1 + x_1)x_2x_3 + x_1\overline{x}_2\overline{x}_3 \\ &= \overline{x}_1 \cdot 1 \cdot x_3 + \overline{x}_1x_2 \cdot 1 + 1 \cdot x_2x_3 + x_1\overline{x}_2\overline{x}_3 \\ &= \overline{x}_1x_3 + \overline{x}_1x_2 + x_2x_3 + x_1\overline{x}_2\overline{x}_3 \end{aligned}$$

2.47. The VHDL code is

```
ENTITY prob47 IS
    PORT ( x1, x2, x3, x4  : IN    STD_LOGIC ;
           f1, f2          : OUT   STD_LOGIC ) ;
END prob47 ;

ARCHITECTURE LogicFunc OF prob47 IS
BEGIN
    f1 <= (x1 AND NOT x3) OR (x2 AND NOT x3) OR
          NOT x3 AND NOT x4) OR (x1 AND x2) OR
          x1 AND NOT x4) ;
    f2 <= (x1 OR NOT x3) AND (x1 OR x2 OR NOT x4) AND
          x2 OR NOT x3 OR NOT x4) ;
END LogicFunc ;
```

2.48. The VHDL code is

```
ENTITY prob48 IS
    PORT ( x1, x2, x3, x4  : IN    STD_LOGIC ;
           f1, f2          : OUT   STD_LOGIC ) ;
END prob48 ;

ARCHITECTURE LogicFunc OF prob48 IS
BEGIN
    f1 <= ((x1 AND x3) OR (NOT x1 AND NOT x3)) OR
          ((x2 AND x4) OR (NOT x2 AND NOT x4)) ;
    f2 <= (x1 AND x2 AND NOT x3 AND NOT x4) OR
          (NOT x1 AND NOT x2 AND x3 AND x4) OR
          (x1 AND NOT x2 AND NOT x3 AND x4) OR
          (NOT x1 AND x2 AND x3 AND NOT x4);
END LogicFunc ;
```